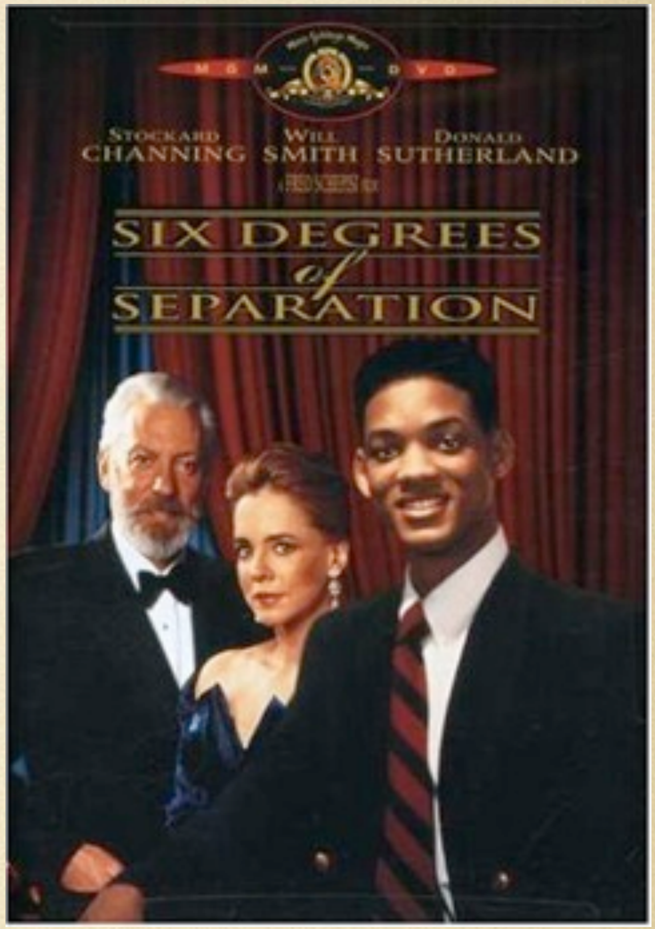
# Distributed Algorithms

Jared Saia

# 6 Degrees




**Ouisa Kitteridge**: *"I read somewhere that everybody on this planet is separated by only six other people. Six degrees of separation between us and everyone else on this planet. The President of the United States, a gondolier in Venice, just fill in the names. I find it extremely comforting that we're so close. I also find it like Chinese water torture, that we're so close because you have to find the right six people to make the right connection."*

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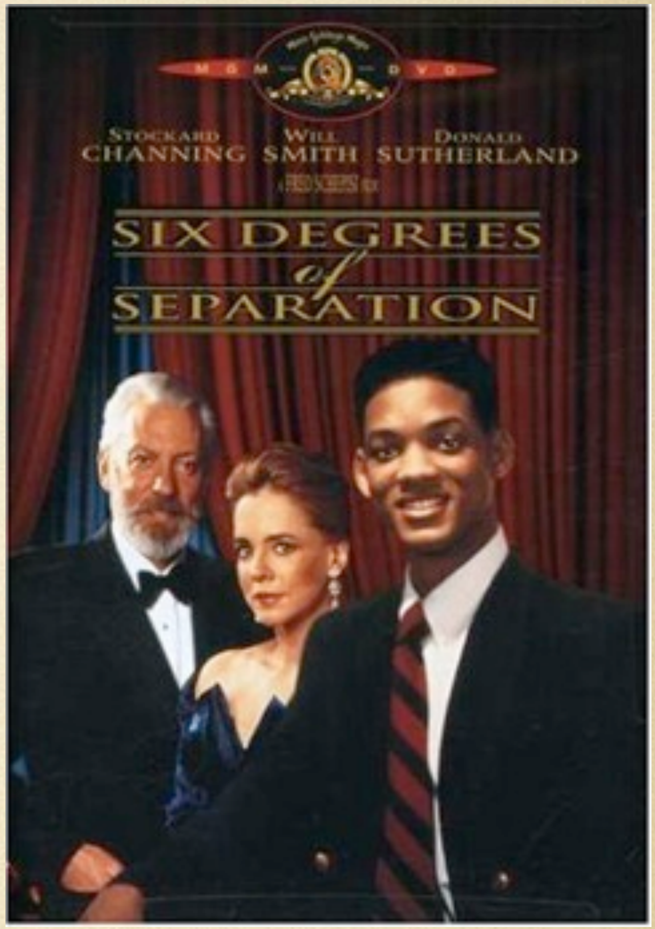


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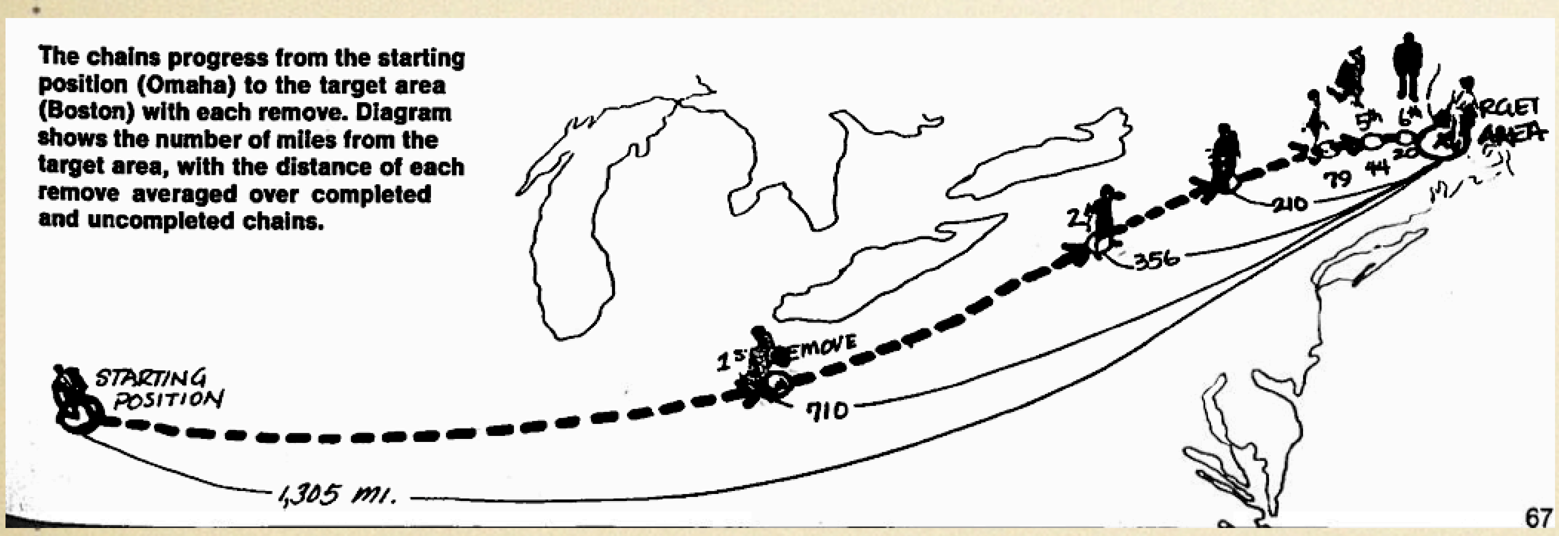
**Tess**: *"He offered you parts in Cats? I thought you hated Cats. You said it was an all time low in a lifetime of theatre going. You said, "Aeschylus did not invent the theatre to have it end up a bunch of chorus kids in cat suits prancing around wondering which of them will go to kitty-cat heaven."*

# Milgram's Experiment

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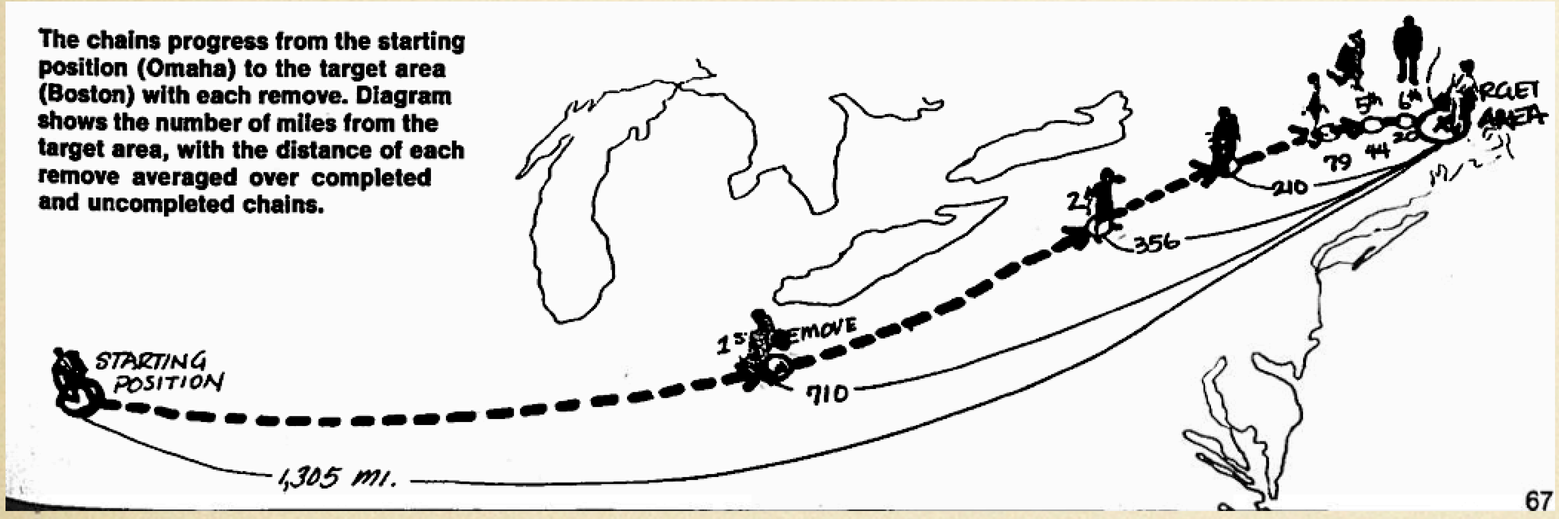


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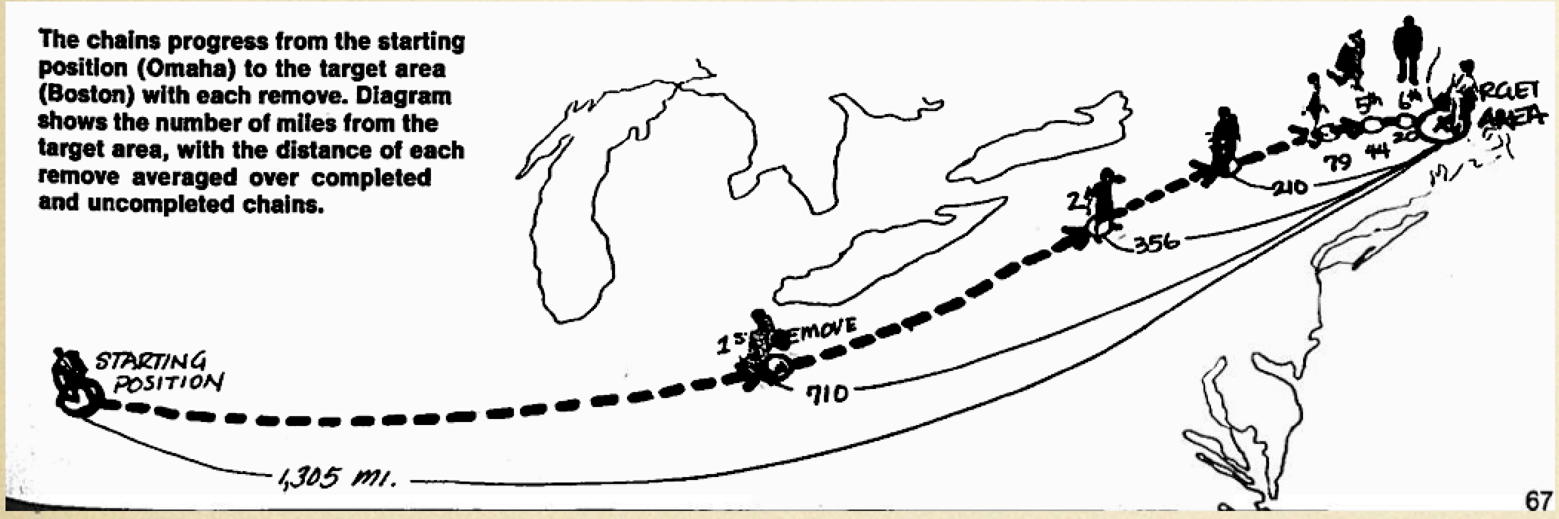


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Recent: ~6 hops to route via email (Watts, '01)

# Social Network Properties

- 1) Shortest paths are small

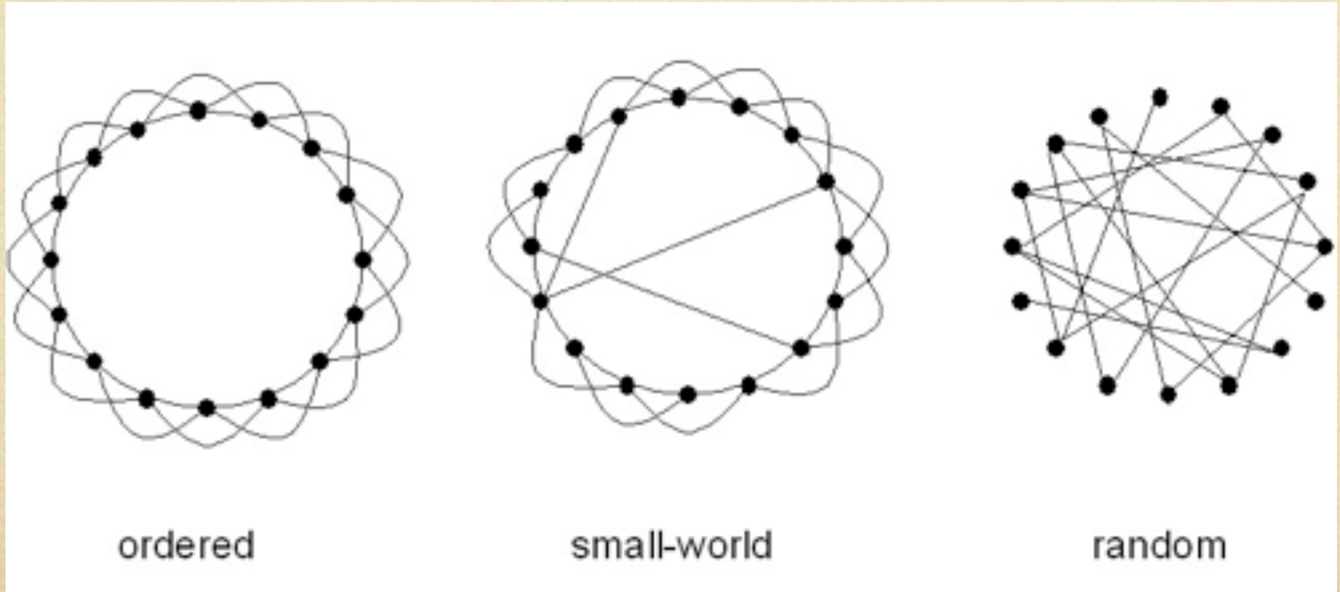
*“Six degrees of separation ... I find it extremely comforting that we're so close.”*

- 2) Local Clusters

*“Keep your friends close and your enemies closer” - Machiavelli*

# Watts-Strogatz Model

- "Small World" model ensures both:
  - Short paths (logarithmic)
  - Many clusters



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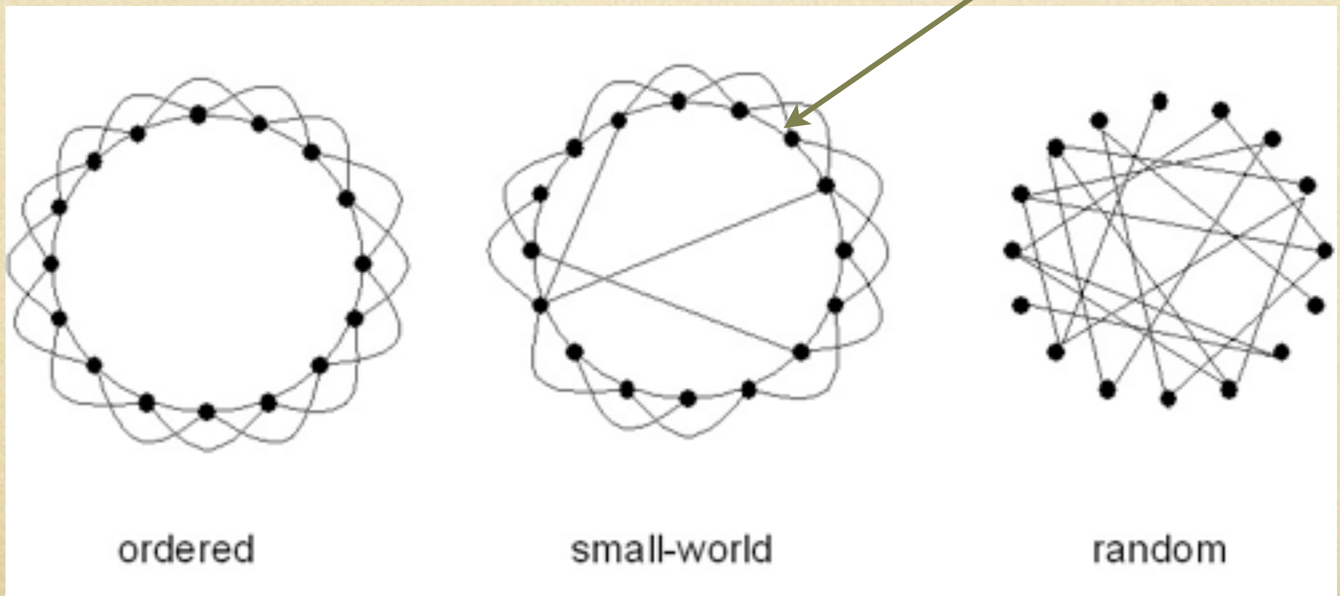
Small World is ordered + random



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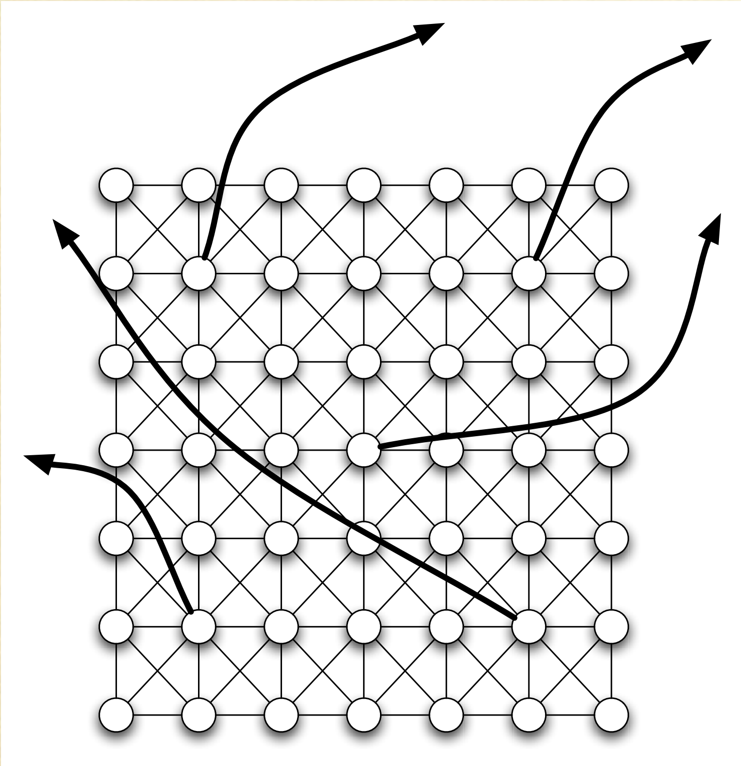

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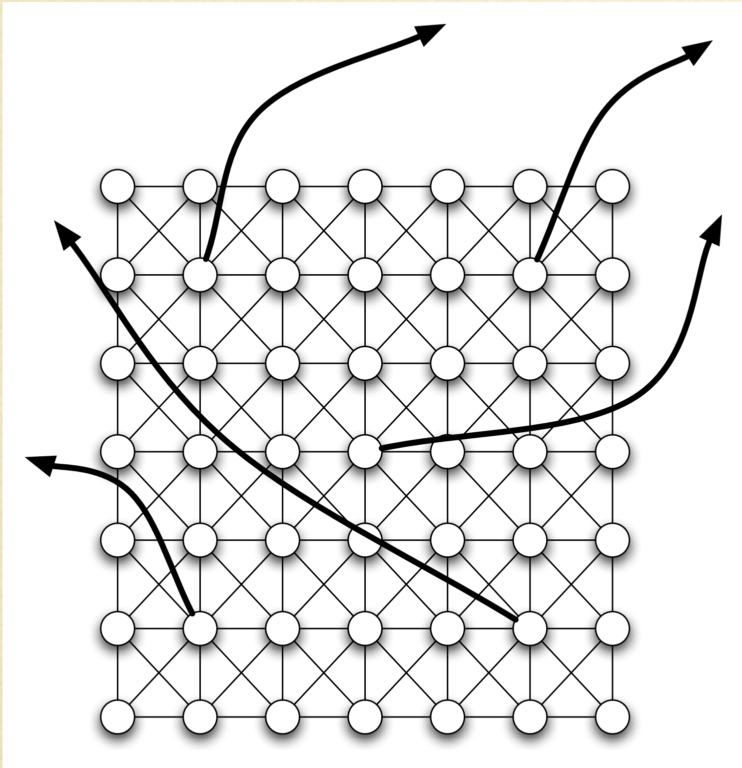

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Can show: 2) All distances at most logarithmic.

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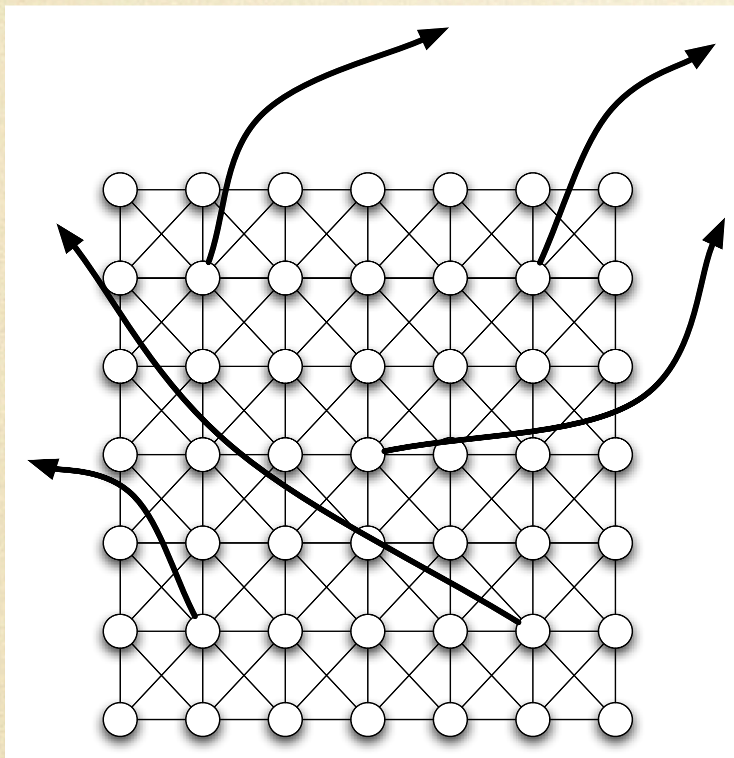

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Node selected uniformly at random

# A Problem

*“Six degrees of separation ... I find it extremely comforting that we're so close... I also find it like Chinese water torture, that we're so close because **you have to find the right six people to make the right connection.**”*

Knowing there **exist** six people is very different than **finding** those six people

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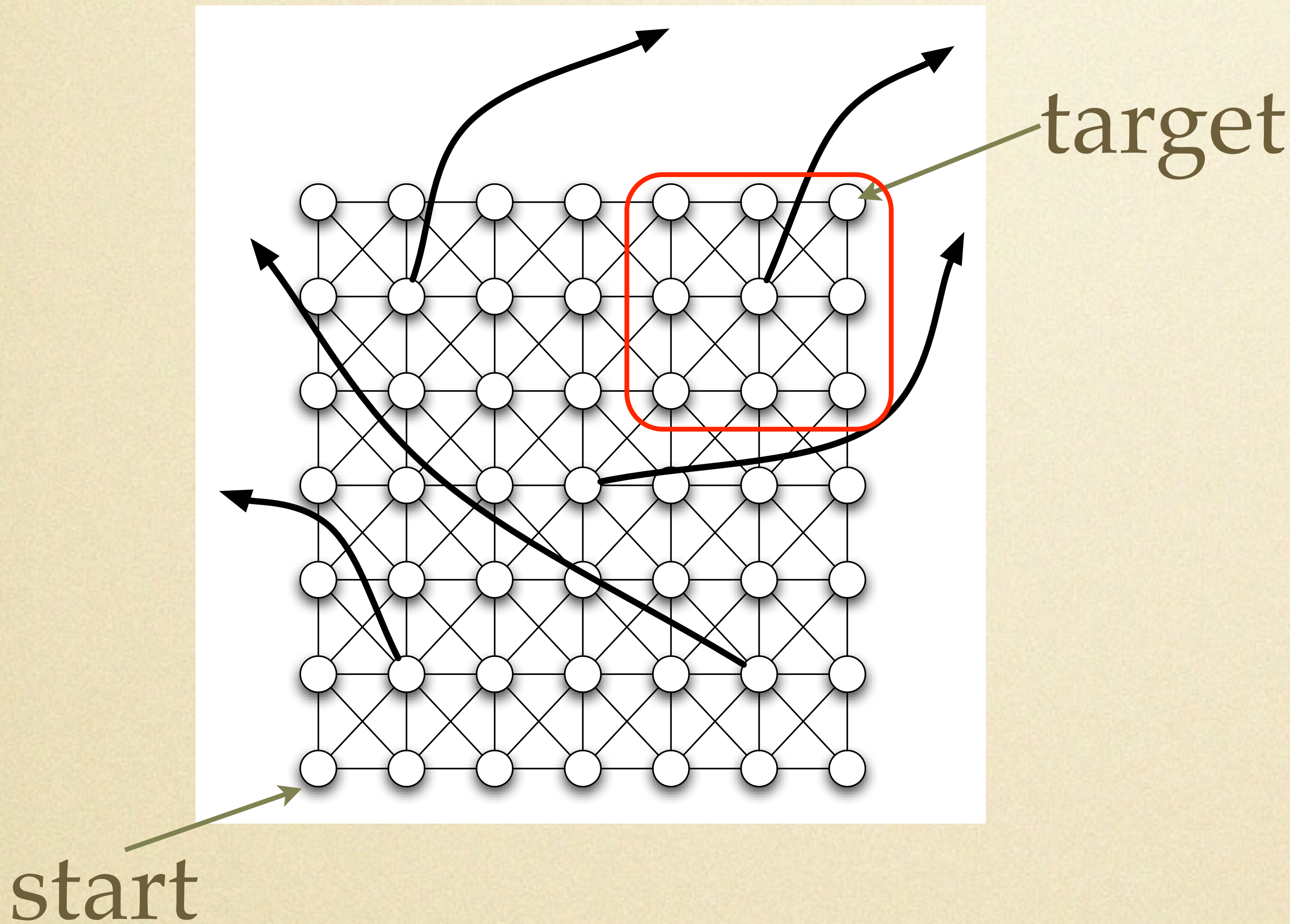
*"Six degrees of separation ... I find it extremely comforting that we're so close... I also find it like Chinese water torture, that we're so close because **you have to find the right six people to make the right connection.**"*

Knowing there **exist** six people is very different than **finding** those six people

In fact, Watts-Strogatz is wrong! It doesn't account for **finding** the six people.

# A Problem

n nodes in grid



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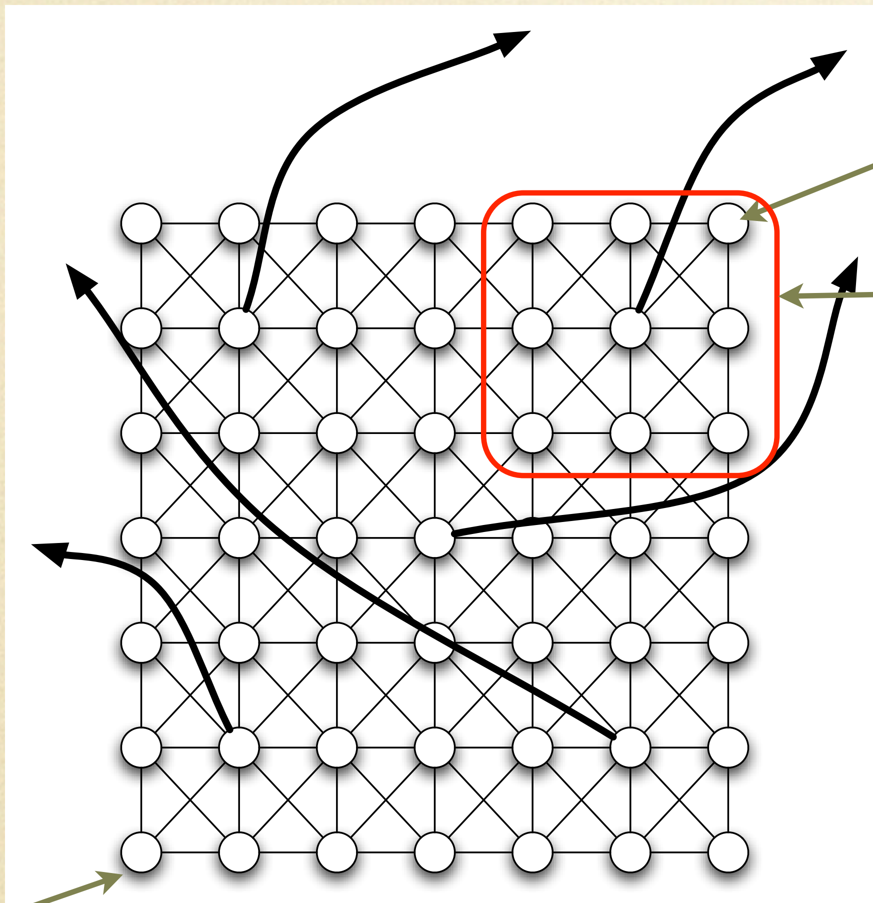

target

n nodes in grid

Region containing
√n nodes

start

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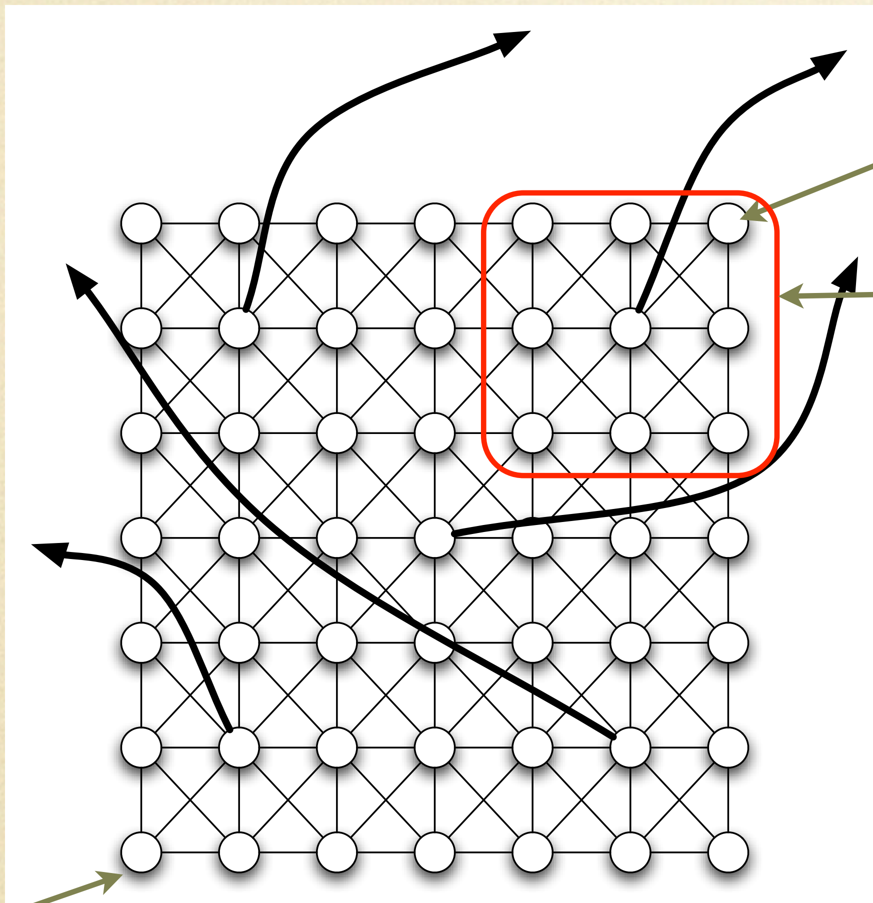

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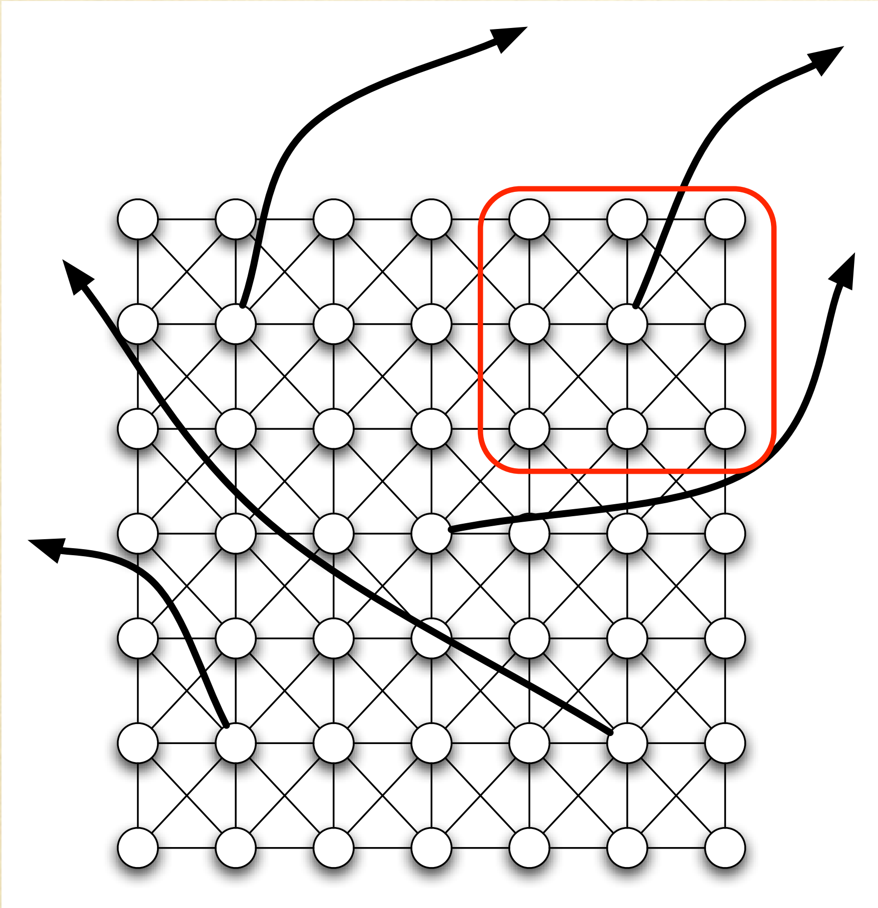

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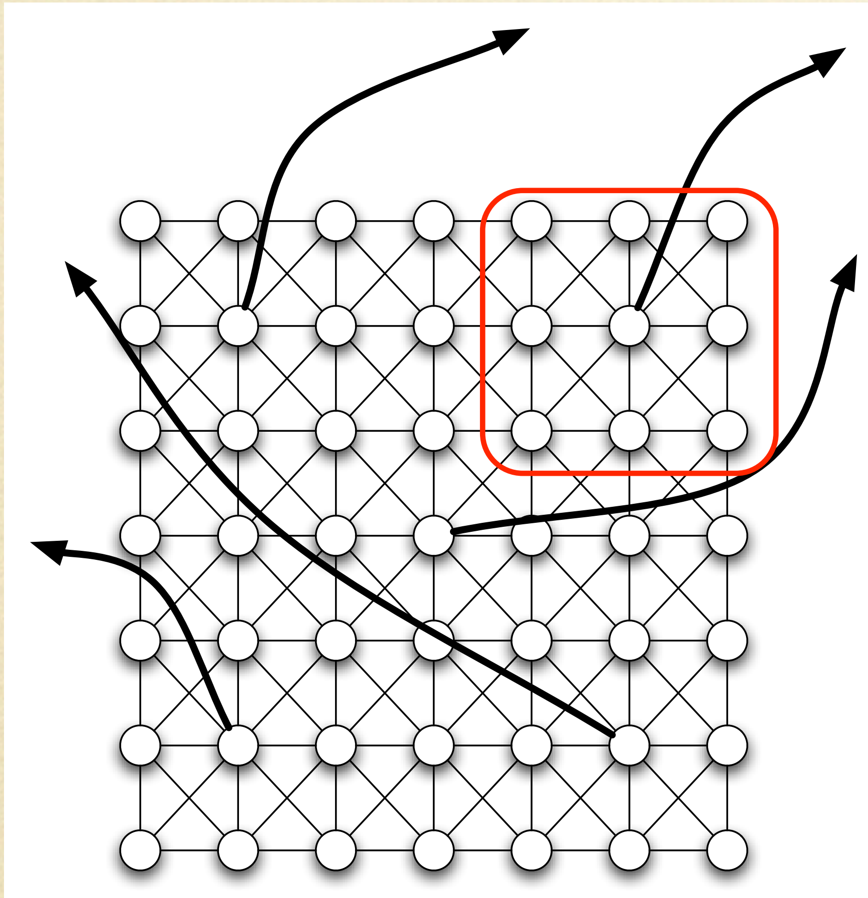

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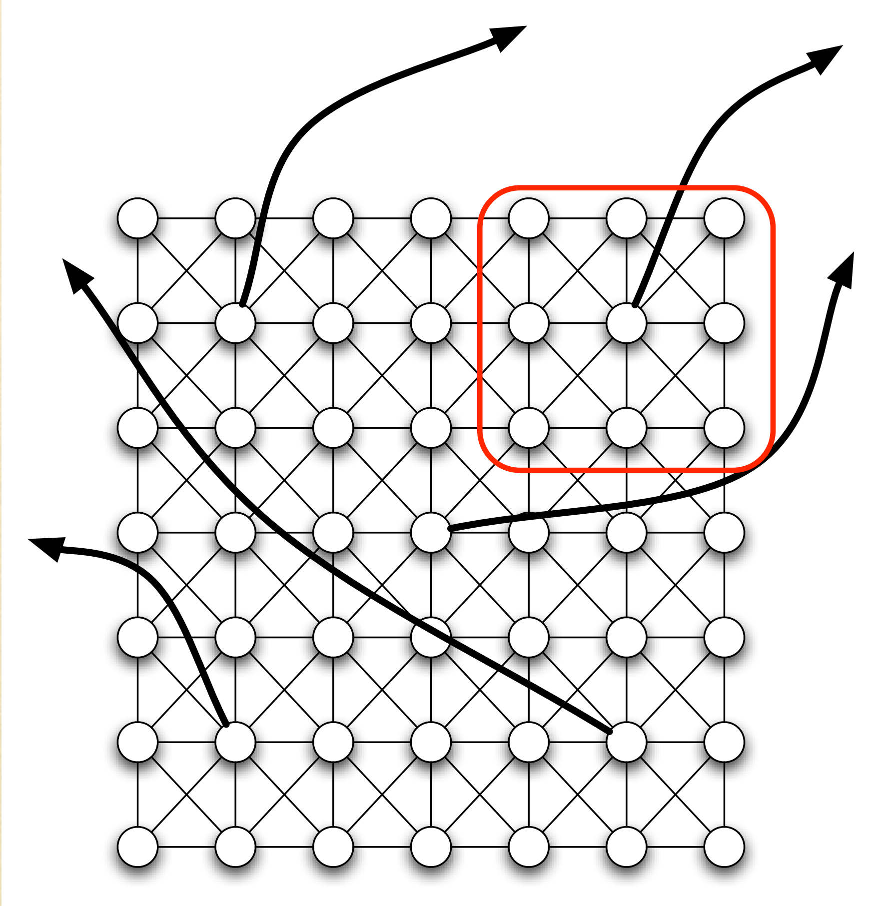

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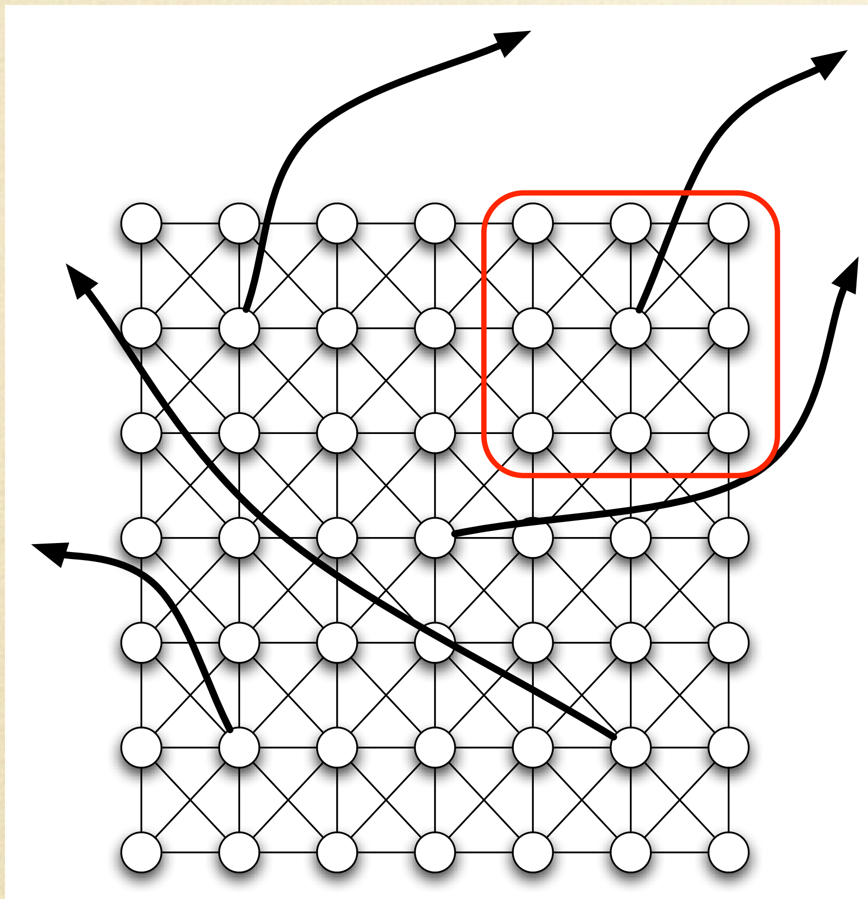

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Expect to have to visit √n nodes before finding a long link which falls in red square!

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Need much quicker routing!!!

# Kleinberg Model

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**Watts-Strogatz:**
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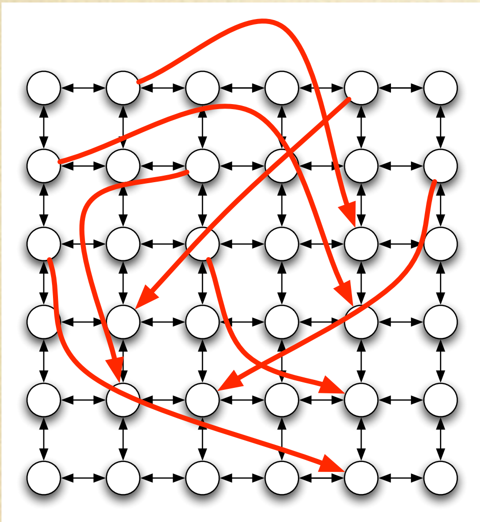

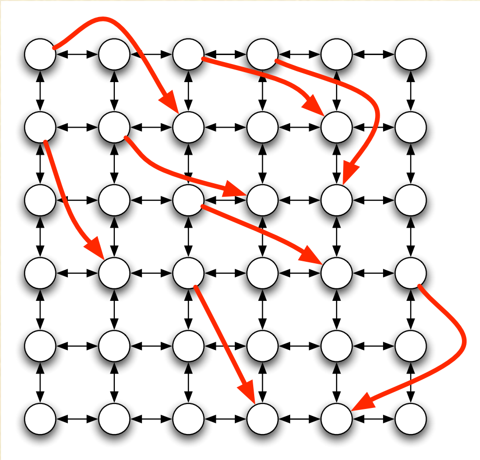

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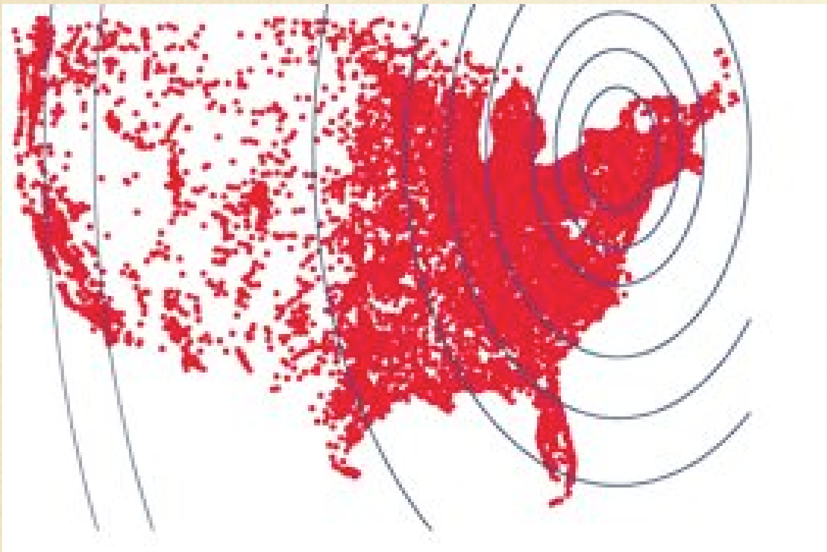
# Kleinberg

- Result: In Kleinberg model, can route from any start node to any goal node in essentially $\log^2 n$ hops!

# Data



Population density of LiveJournal network (Liben-Nowell et al. '05)

Rank addresses variations in population density

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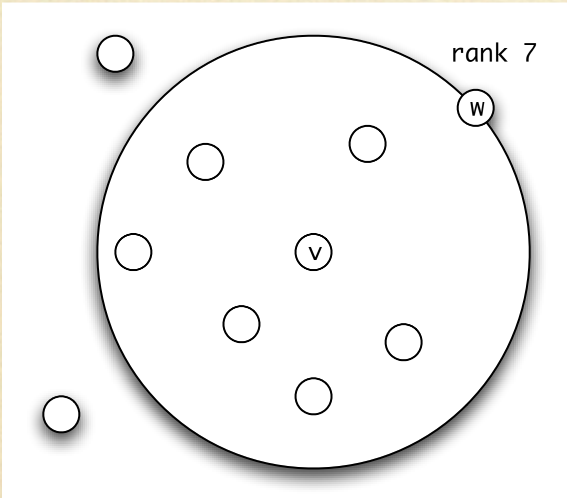


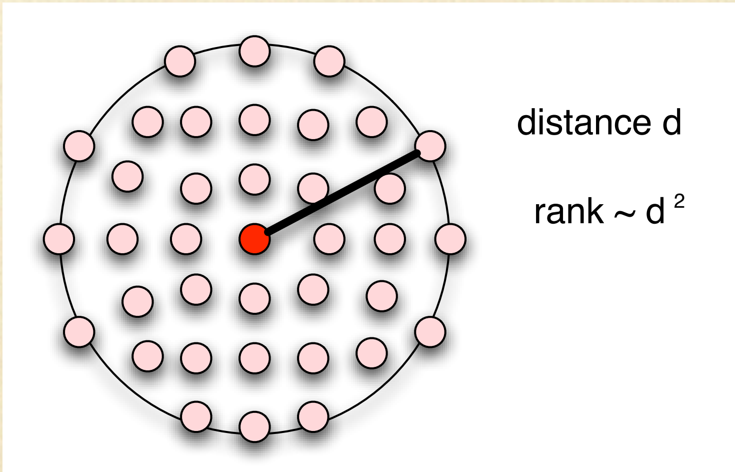


Rank addresses variations in population density

General Case: prob. of link to node w/ rank r is $\propto 1/r$

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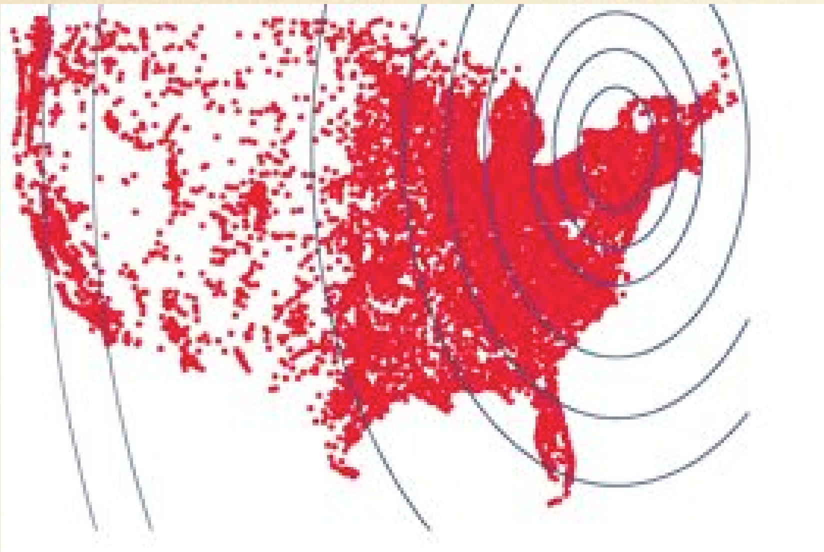

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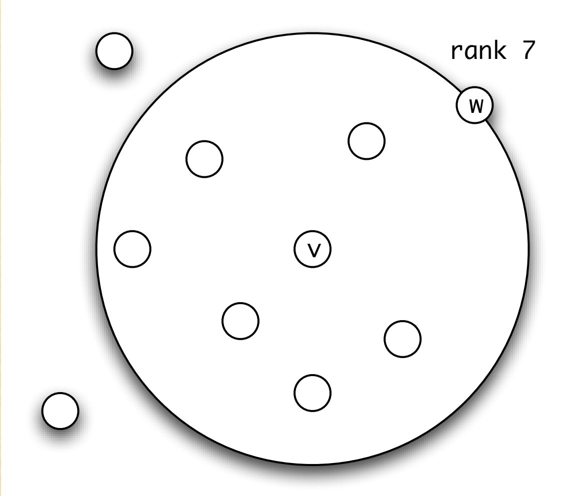


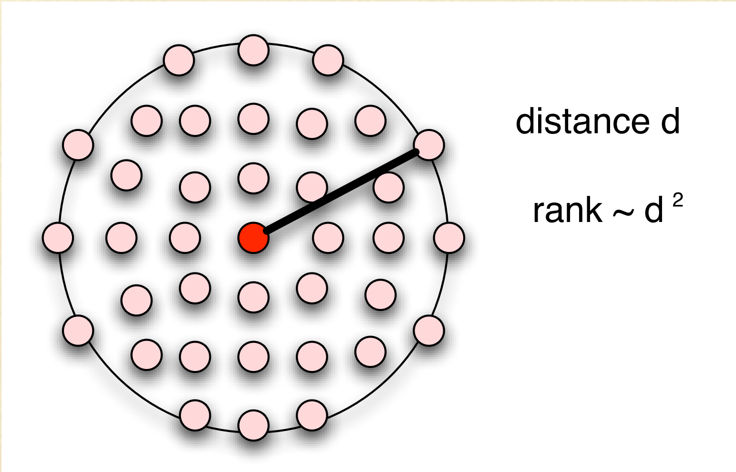


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Acknowledgement: Many of the figures in this talk are from the book *Networks, Crowds and Markets: Reasoning about a Highly Connected World* by David Easley and Jon Kleinberg

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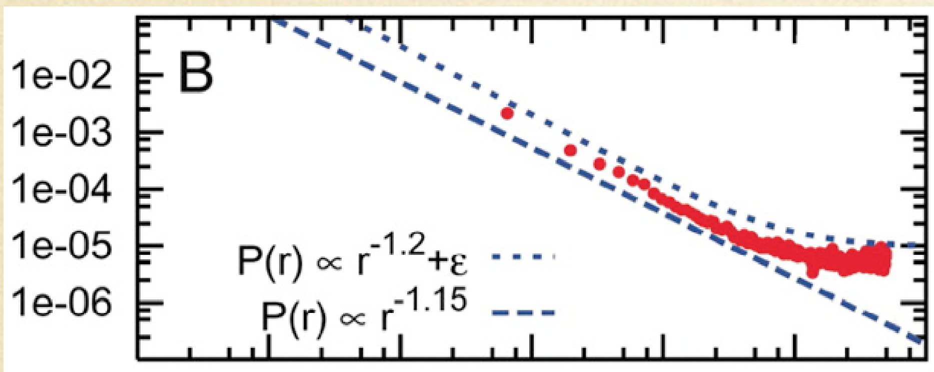
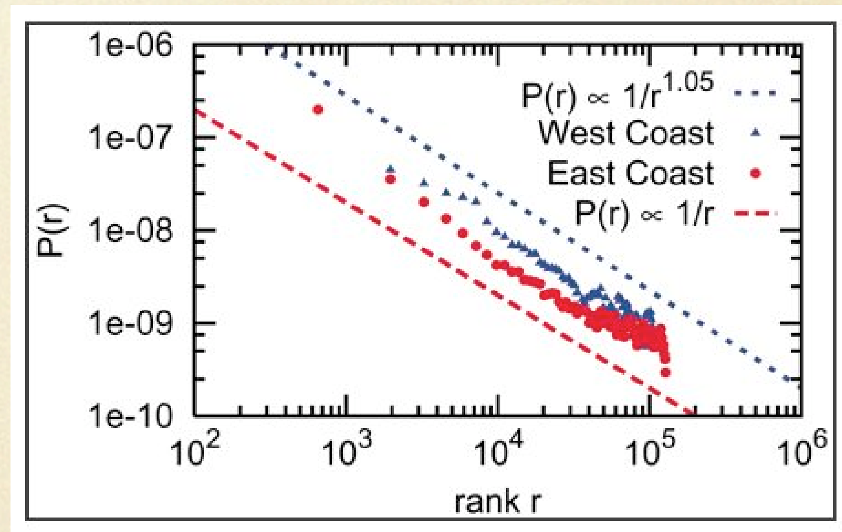


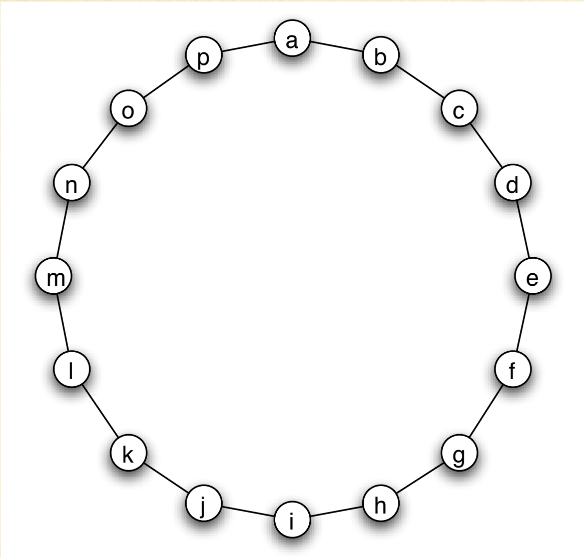

(a) *Rank-based friendship on LiveJournal*

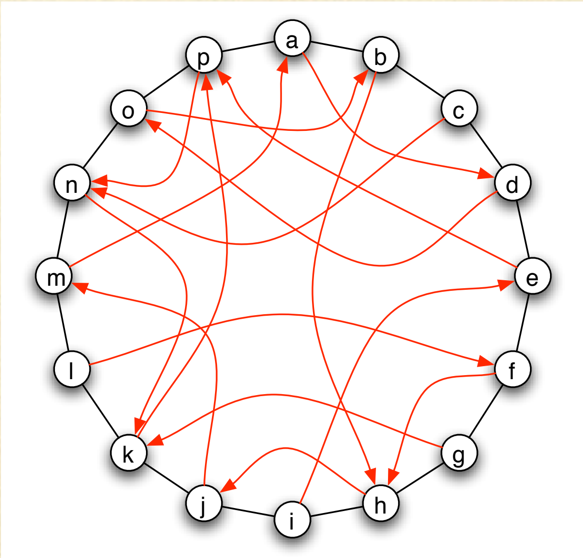



(b) *Rank-based friendship: East and West coasts*

Observed probability fits very close to 1/r

# Ring

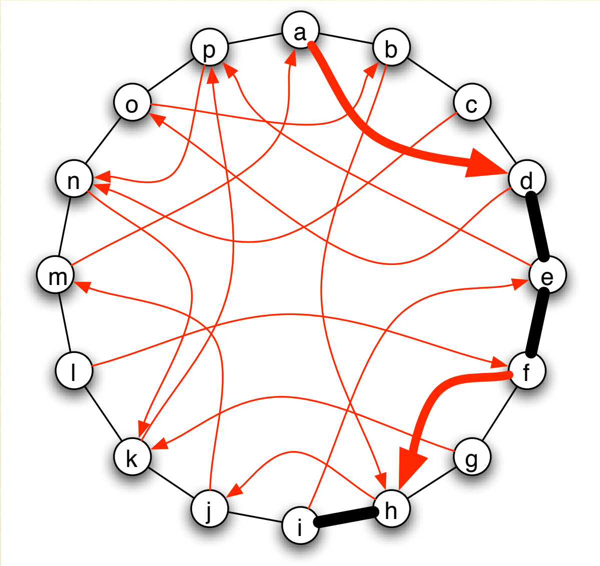



Easier to do analysis on a ring (but same techniques work for a grid)

Random link to x will now happen with probability $\propto 1/(\text{distance to x})$

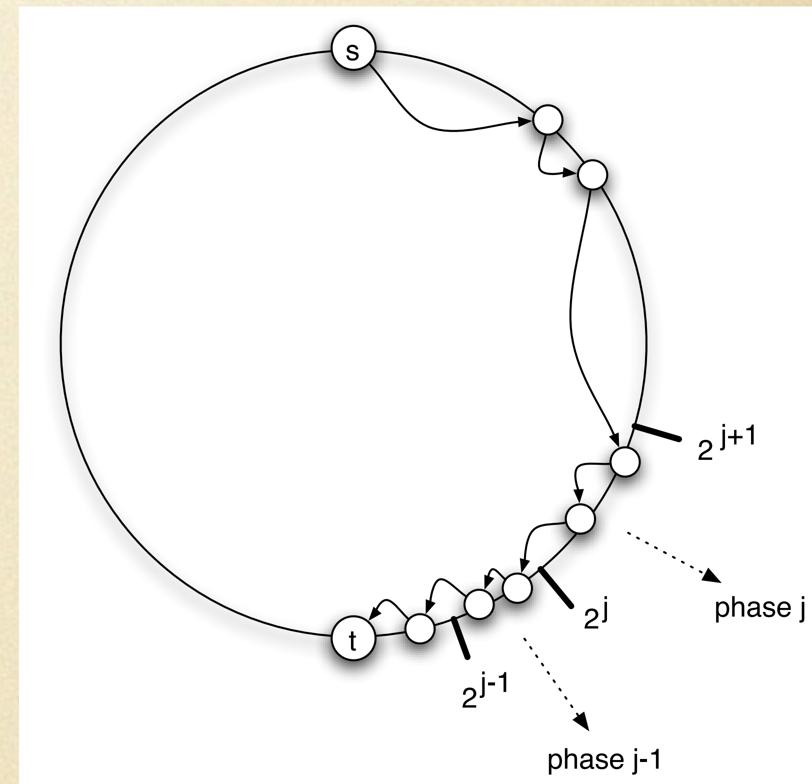
# Ring vs Grid




Algorithm: Current message holder forwards message to person it knows who is closest to target

# Analysis

We'll say we're in **phase j** of the algorithm when distance from target is between $2^j$ and $2^{j-1}$



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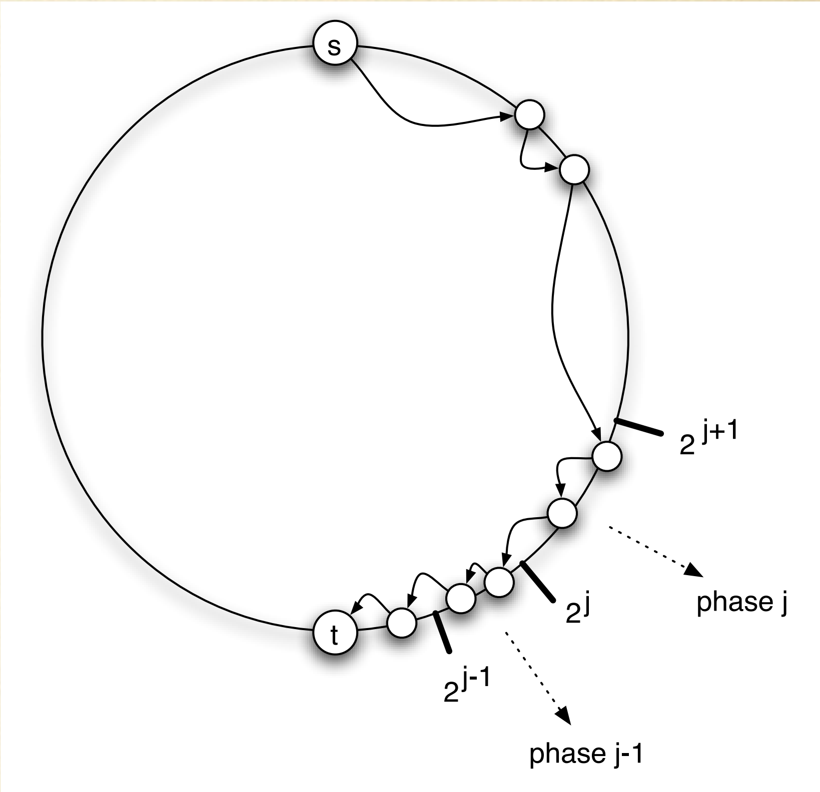

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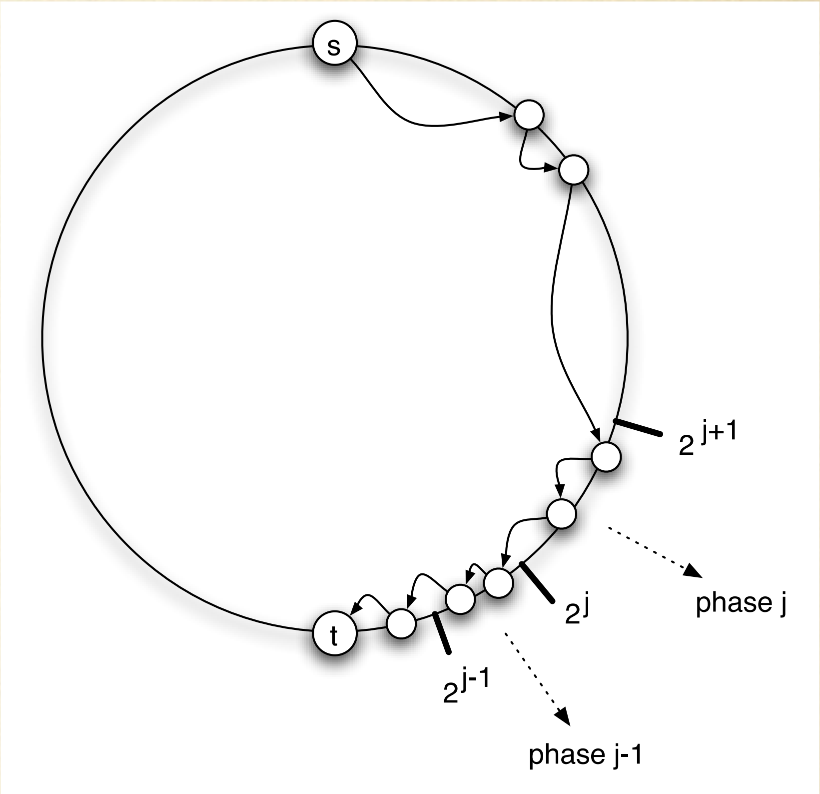


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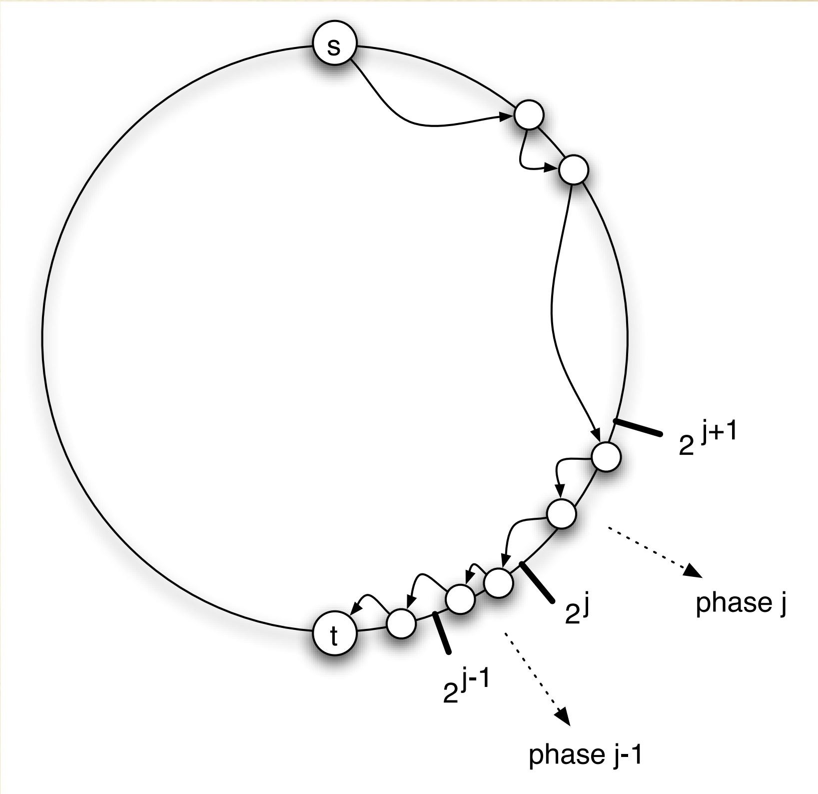


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By Linearity of Expectation!

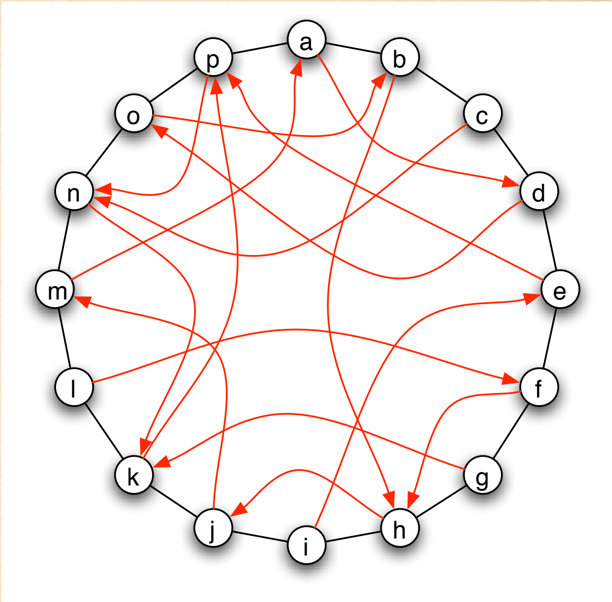
# Analysis

$$E(X) = E(X_1) + E(X_2) + \ldots + E(X_{\log n})$$

Now we "just" need to calculate $E(X_i)$, the expected number of hops in phase i

To do this, we calculate the probability that a single random link allows us to end phase i

# Probabilities

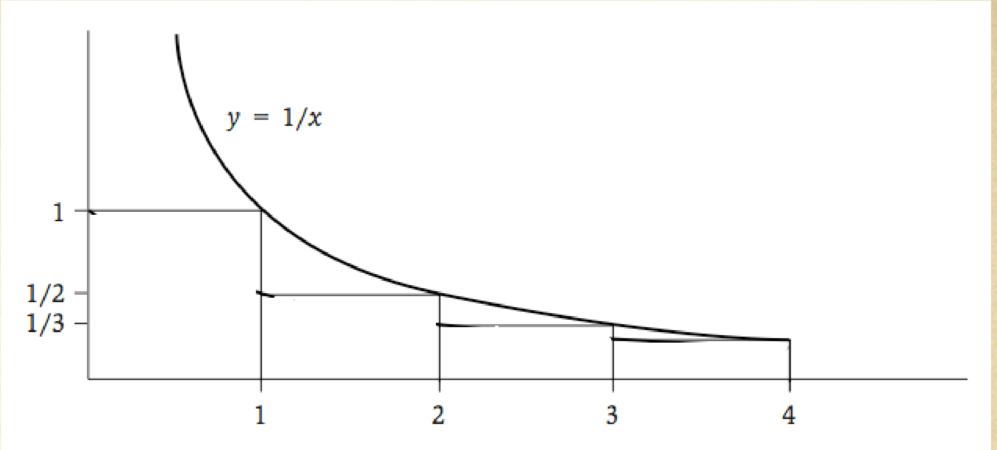



Recall: Random link to from u to v occurs with probability $\propto 1/(\text{distance from u to v})$

Normalizing constant Z is the sum over all v of 1/(distance from u to v)

$$Z \leq 2(1 + 1/2 + 1/3 + 1/4 + \ldots 1/(n/2))$$

# Z

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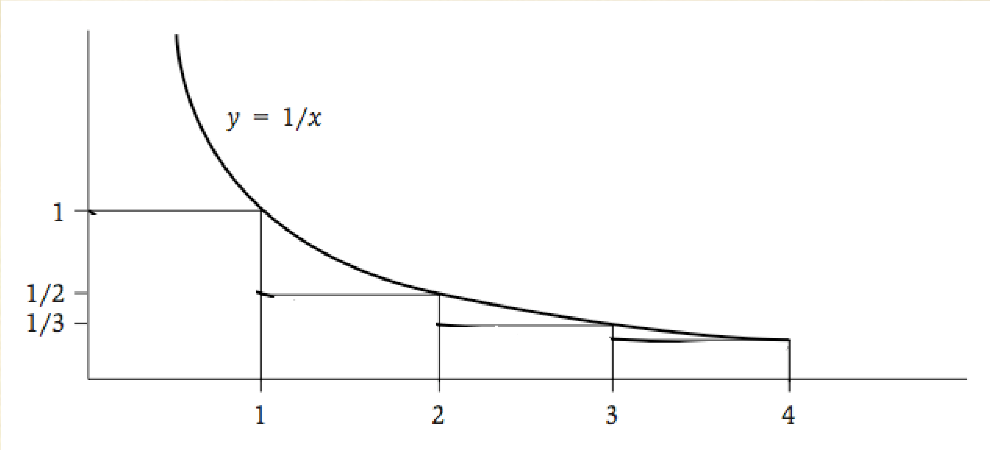


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But:

$$(1 + 1/2 + 1/3 + 1/4 + \ldots 1/k) \leq 1 + \int_1^k \frac{1}{x} dx$$

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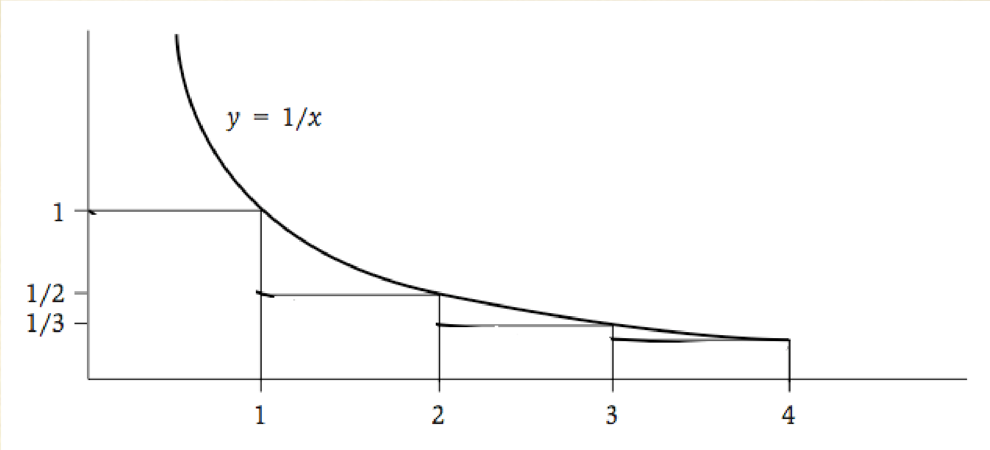


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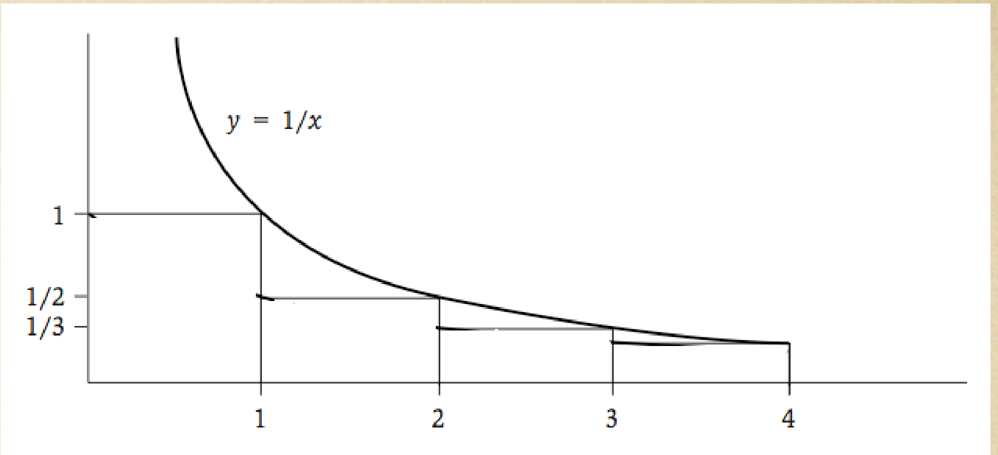


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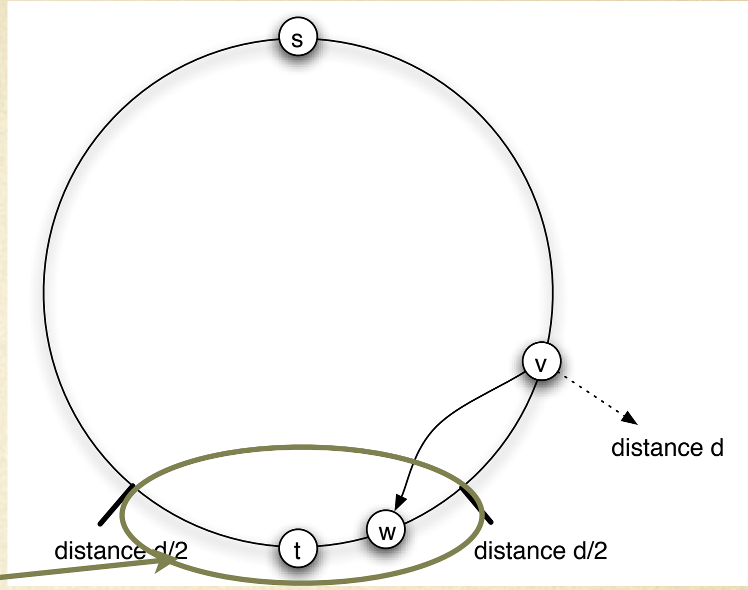
Let d(u,v) = distance from u to v. Then prob. u links to v is

$$\frac{1}{Z}d(u,v)^{-1} \geq \frac{1}{2\log n}d(u,v)^{-1}$$

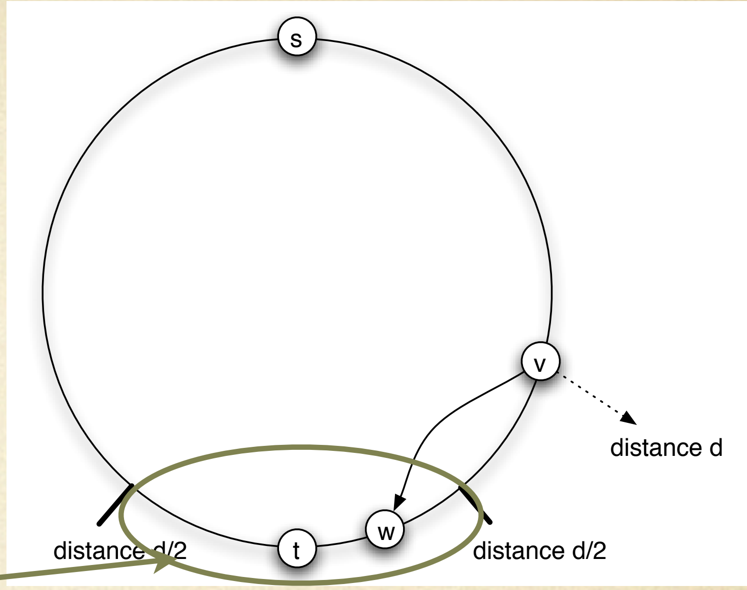
Only remaining task is to add up these probabilities over all vertices v that will let us exit the current phase

# Probabilities




d+1 nodes within distance d/2 of t

# Probabilities

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Prob. of hitting particular node v in there at least:

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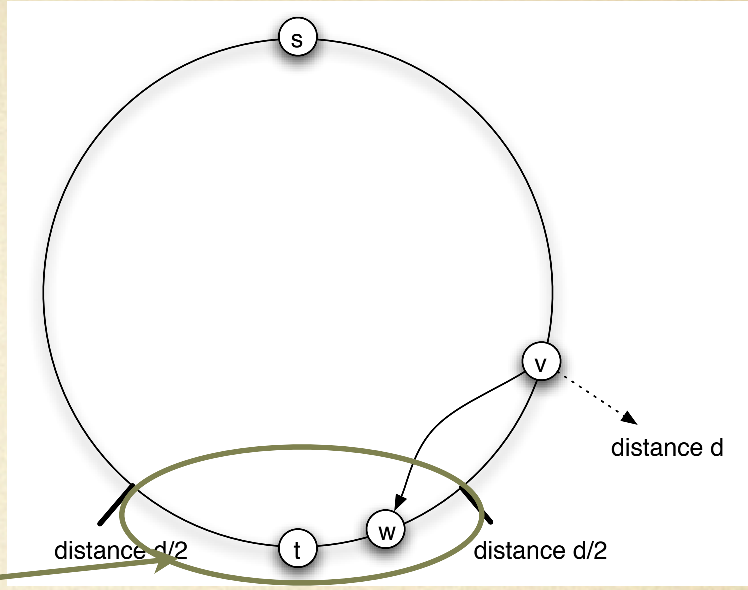


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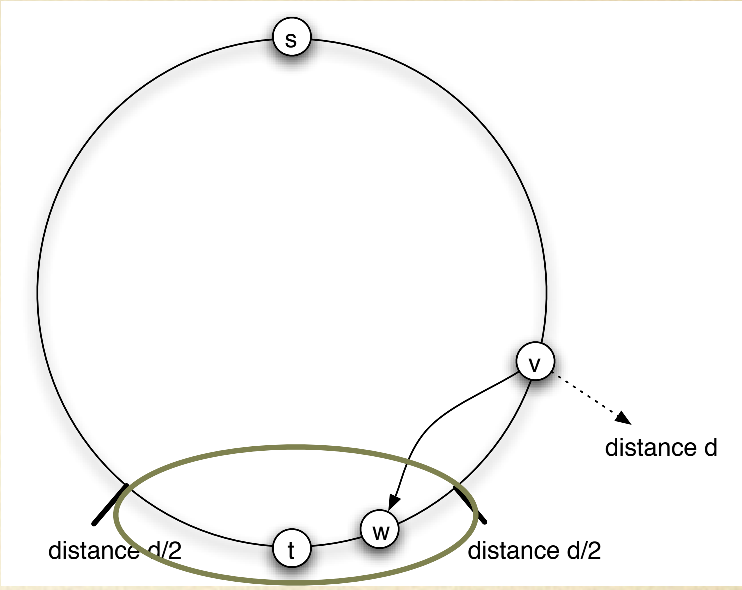
d+1 total nodes; prob. of hitting one is at least:

$$(d+1)\frac{1}{3d\log n} = \frac{1}{3\log n}$$

# Phases

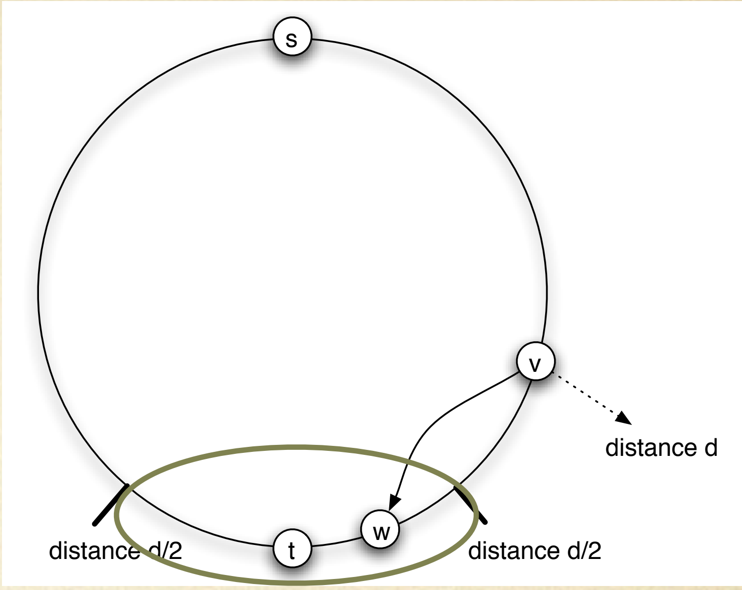



So we're walking around in phase j

Every time we see a random edge, it has prob. at least **1/(3 log n)** of taking us to next phase

Q: How long do we expect to walk before finding one of these special edges?

# Phases

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A: 1/p

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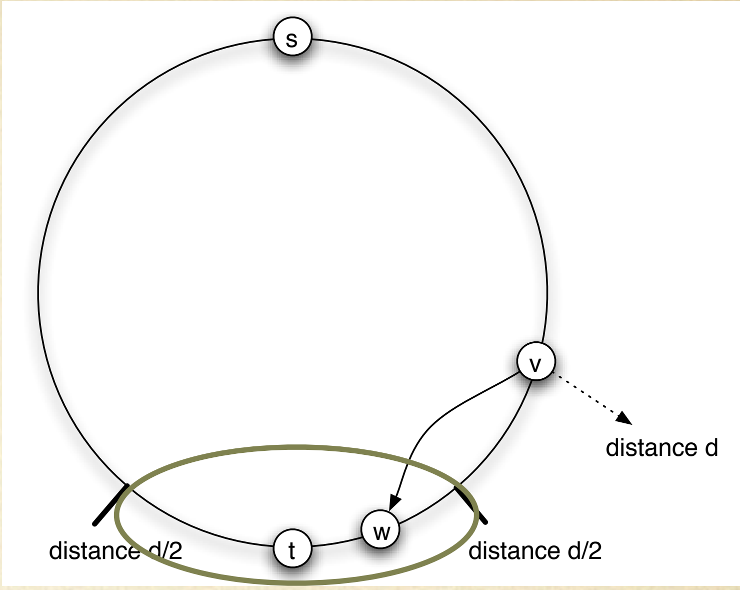


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A: 1/p

$$E(X) = p*1 + (1-p)(1 + E(X))$$

# Wrapup

Recall: $E(X) = E(X_1) + E(X_2) + \ldots + E(X_{\log n})$

Thus: $E(X) \leq 3 \log n + 3 \log n + \ldots + 3\log n$

$\leq 3 \log^2 n$

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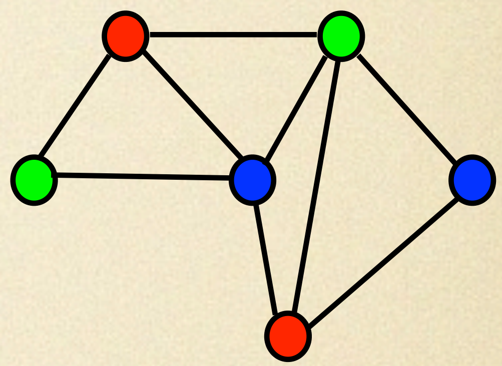
The End!

Or is It???

# Open Questions

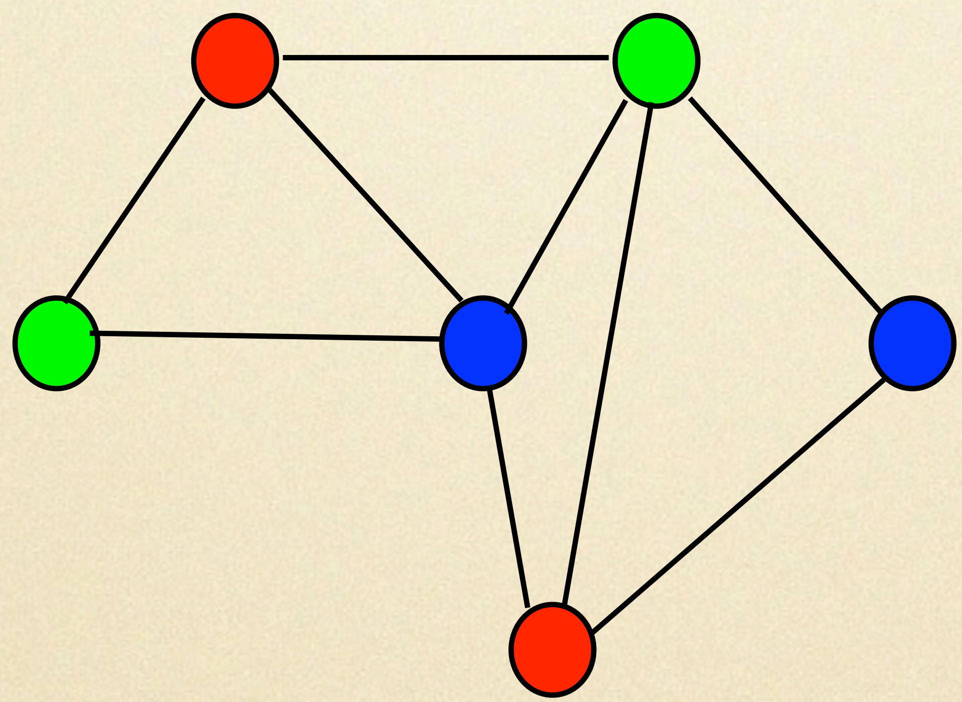
- Why do friendship links have the Kleinberg exponent?
- Why should routing speed determine the way in which we make friends?
- Why do we have friends?

# Graph Coloring



- Must color each node in a graph (network)
- A coloring is **valid** if any pair of nodes that are linked have different colors
- Goal: Find a valid coloring using the smallest number of colors

# Graph Coloring



Example graph and valid 3 coloring

# Graph Coloring

- Unlike shortest paths, coloring is computational hard even when centralized
- Sudoku is a graph coloring problem (with some colors already fixed)
- How?

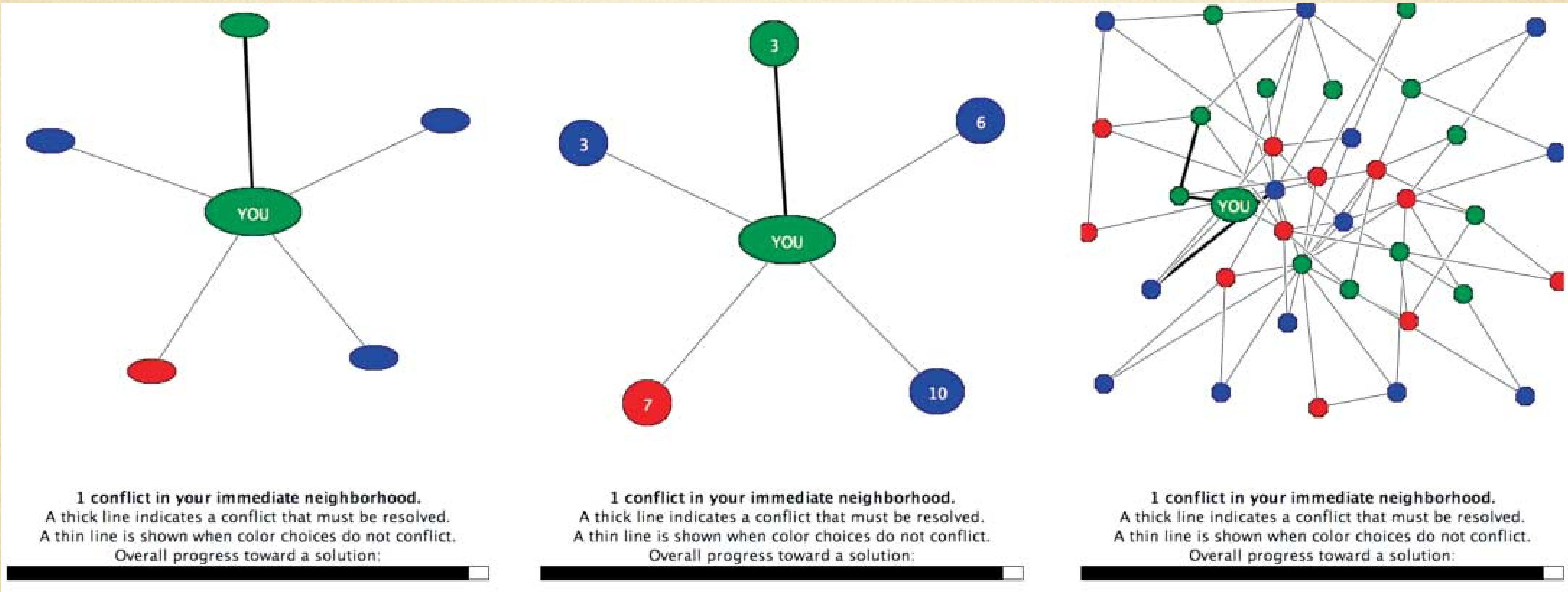
# Distributed Coloring

- Division of resources in social networks
- Nodes are people, links represent friendships
- Colors are resources
- Goal: Assign resources to people so that friends don't fight over the same resource
- **Distributed**: Each node knows only local neighborhood

# Example Resources

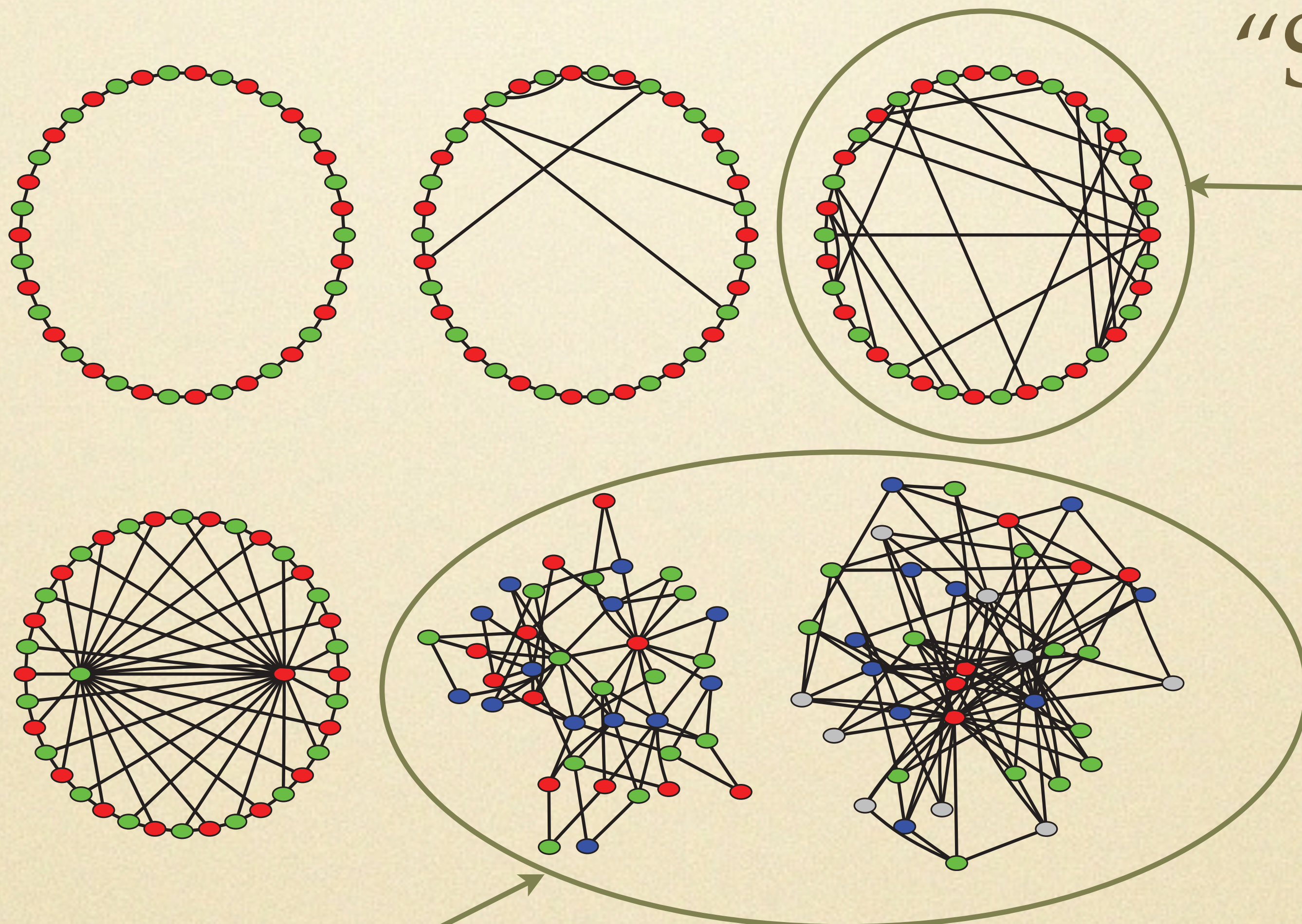
- **Time**: scheduling talks in conference rooms
- **Economic**: pursuing different expertise/ markets by people/companies
- **Political**: pursuing different political offices
- **Technological**: selecting a channel unused by close parties in a wireless network

# An Experiment




Kearns et al. '06 ran a distributed coloring experiment on people

# Graphs Used



“Small World” (Watts-Strogatz)

Kleinberg?!?

Preferential attachment

# Empirical Results

| | Graph statistics | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Colors required (No.) | Min. links (No.) | Max. links (No.) | Avg. links (No.) | SD | Avg. distance (No. of links) | Avg. experiment duration (s) and fraction solved | | Distributed heuristic (No. of color changes) |
| Simple cycle | 2 | 2 | 2 | 2 | 0 | 9.76 | 144.17 | 5/6 | 378 |
| 5-chord cycle | 2 | 2 | 4 | 2.26 | 0.60 | 5.63 | 121.14 | 7/7 | 687 |
| 20-chord cycle | 2 | 2 | 7 | 3.05 | 1.01 | 3.34 | 65.67 | 6/6 | 8265 |
| Leader cycle | 2 | 3 | 19 | 3.84 | 3.62 | 2.31 | 40.86 | 7/7 | 8797 |
| Pref. att., $\nu = 2$ | 3 | 2 | 13 | 3.84 | 2.44 | 2.63 | 219.67 | 2/6 | 1744 |
| Pref. att., $\nu = 3$ | 4 | 3 | 22 | 5.68 | 4.22 | 2.08 | 154.83 | 4/6 | 4703 |

Small world easy

Preferential attachment hard

# Maximal Independent Set

- To solve distributed graph coloring, we first address a simpler problem:
- **Independent Set:** A set of nodes in a network, such that there is no edge between any pair in the set
- An independent set is **maximal** if no nodes can be added

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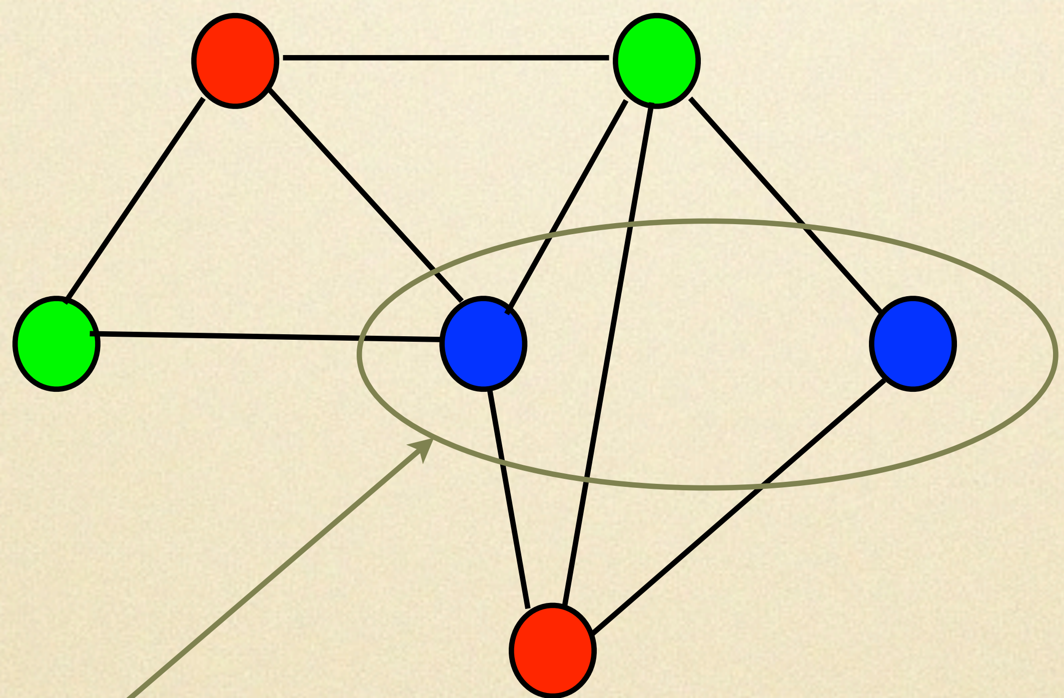
Acknowledgement: Much of the discussion here is based on lecture notes by Roger Wattenhoffer at http://www.dcg.ethz.ch/lectures/podc/

# Maximal Independent Set



This is a maximal independent set

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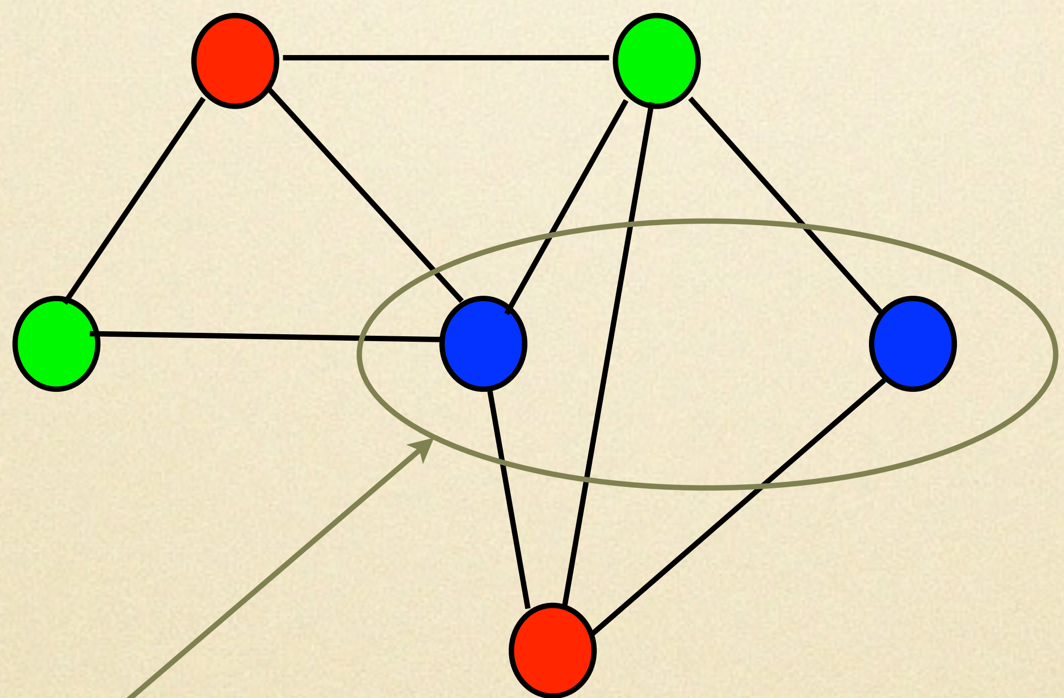
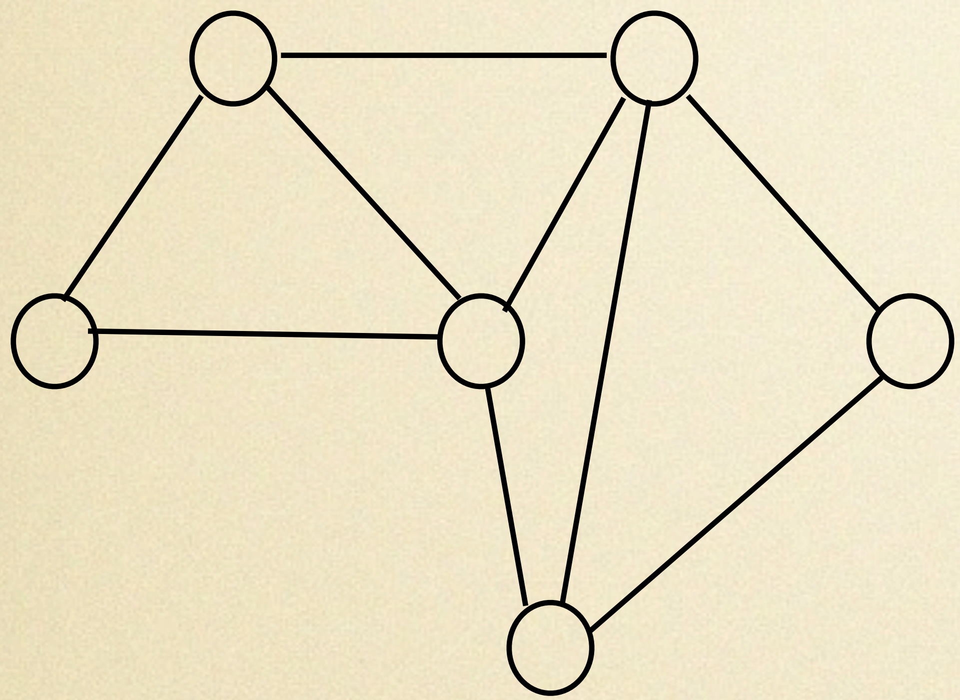


This is a maximal independent set

Note: all nodes in an independent set can be colored with the same color

# An MIS Algorithm



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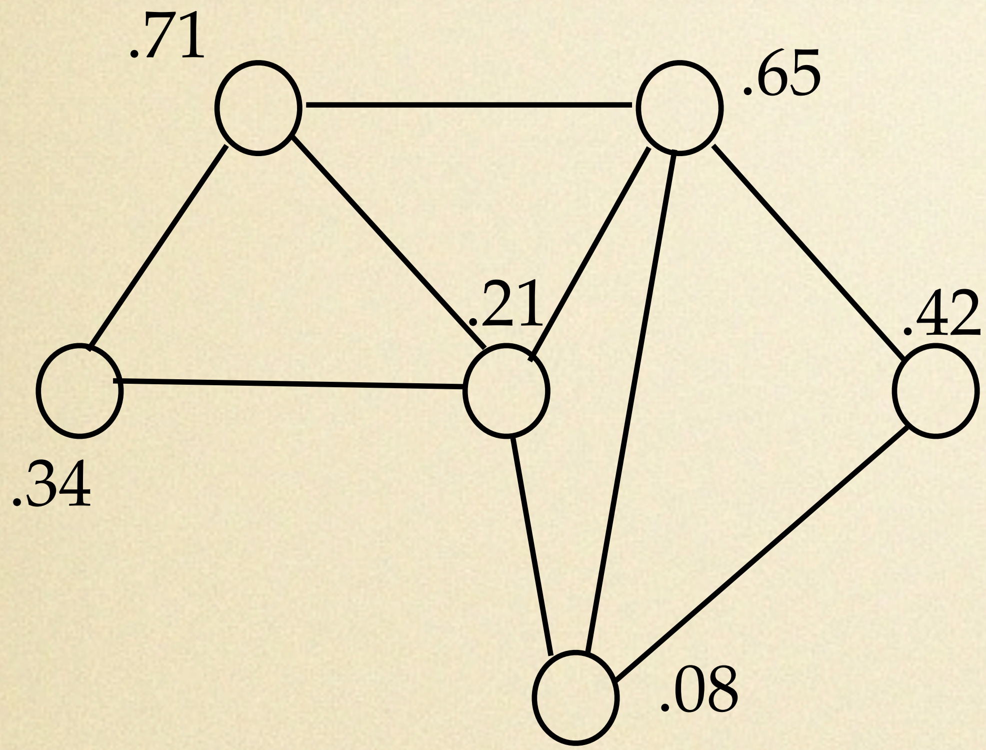


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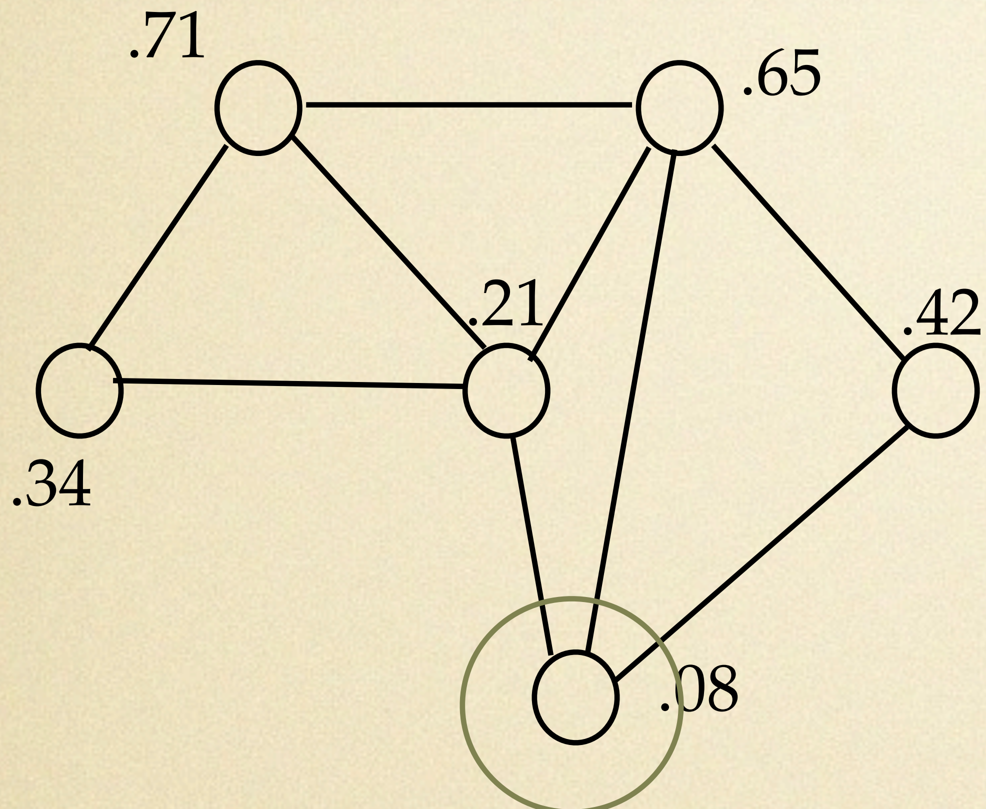


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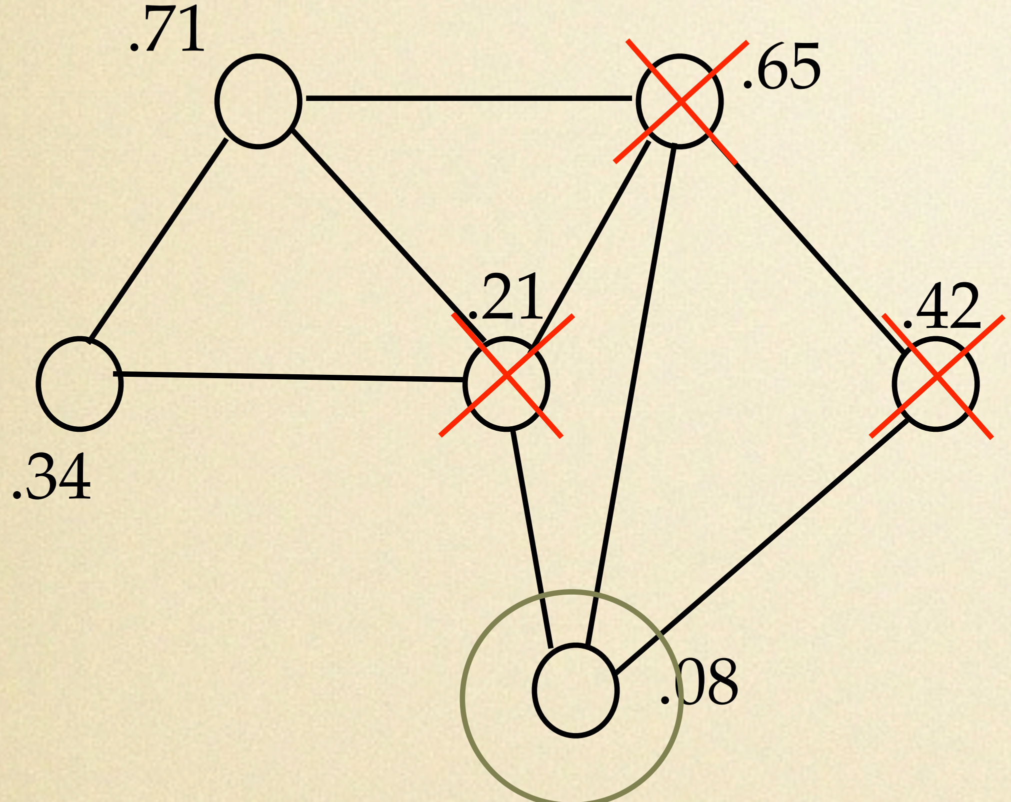


1) Each node v chooses a random value, r(v), in [0,1] and sends it to its neighbors

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3) If v or a neighbor entered the MIS, it terminates (removing all edges); otherwise go back to step 1

# Some Facts

- The algorithm always finds a MIS
- The algorithm terminates since in each loop, at least one node is added
- Q: How fast is the algorithm?

# Analysis

- We'll show that, in expectation, half of the edges are removed in each loop of the algorithms
- This implies that number of loops is only log m where m is number of edges, n number of nodes
- Since $m \leq n^2$ , we know that $\log m \leq 2 \log n$
- We'll let d(x) be the "degree of x" i.e. number of edges incident to x

# A Clever Trick

Let $v \Rightarrow w$ be the event that $r(v) \leq r(w)$ **and** $r(v) \leq r(x)$ for all neighbors $x$ of $w$

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Since for any edge (s,t), at most one event $X_{* \Rightarrow s}$ and at most one event $X_{* \Rightarrow t}$ can happen.

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Now all that remains is to compute E(X)

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$$= m$$

# Recap

- We've shown that $E(X) = m$
- We also shown that the number of edges removed in each loop is at least $X/2$
- Implies that we expect **half** the edges to be removed in each loop
- Thus, we expect only log m iterations of the loop!

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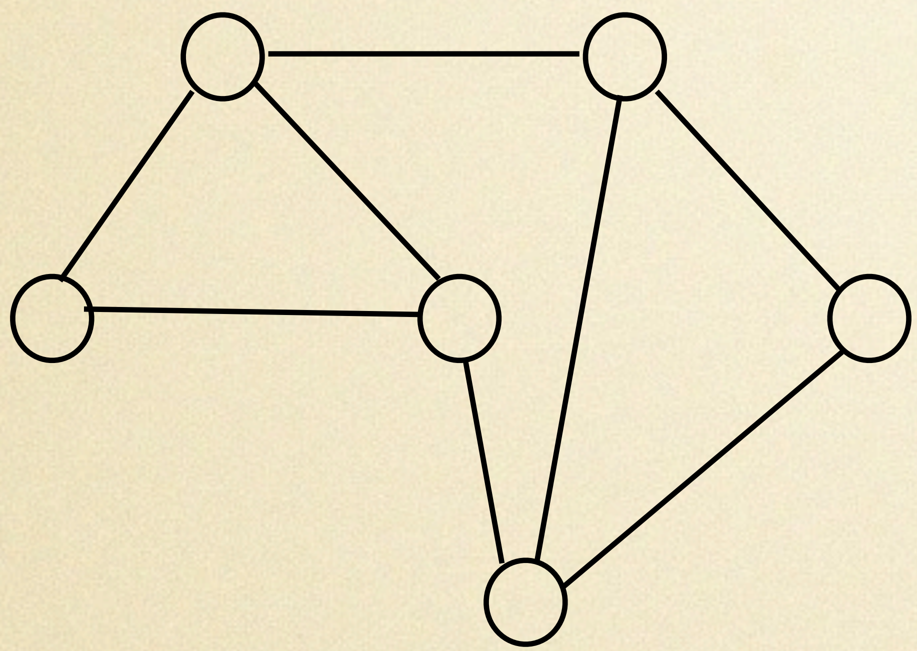
Also: hey, what about graph coloring?

# Create New Graph



1) Each node v makes d(v)+1 clones. All clones of v are linked together

2) If u and v neighbors, then for all i, the i-th clone of u is linked to the i-th clone of v

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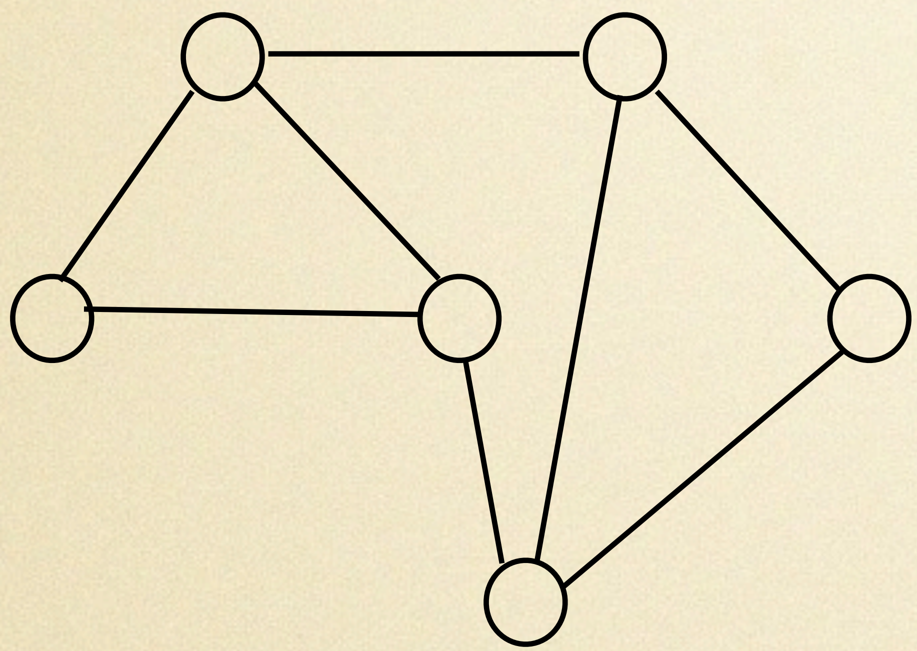

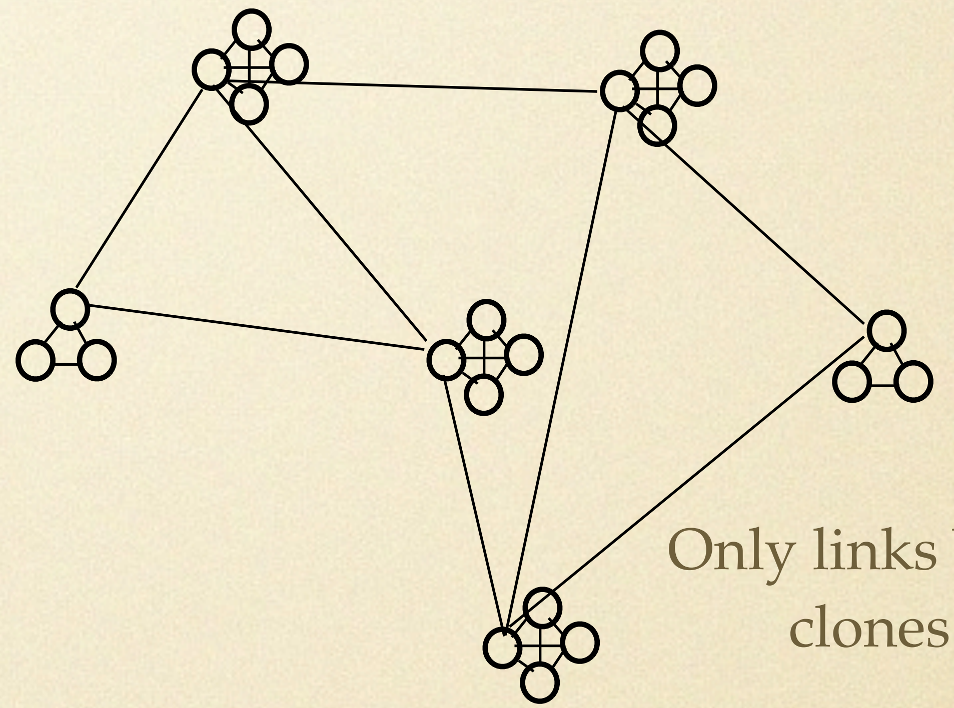

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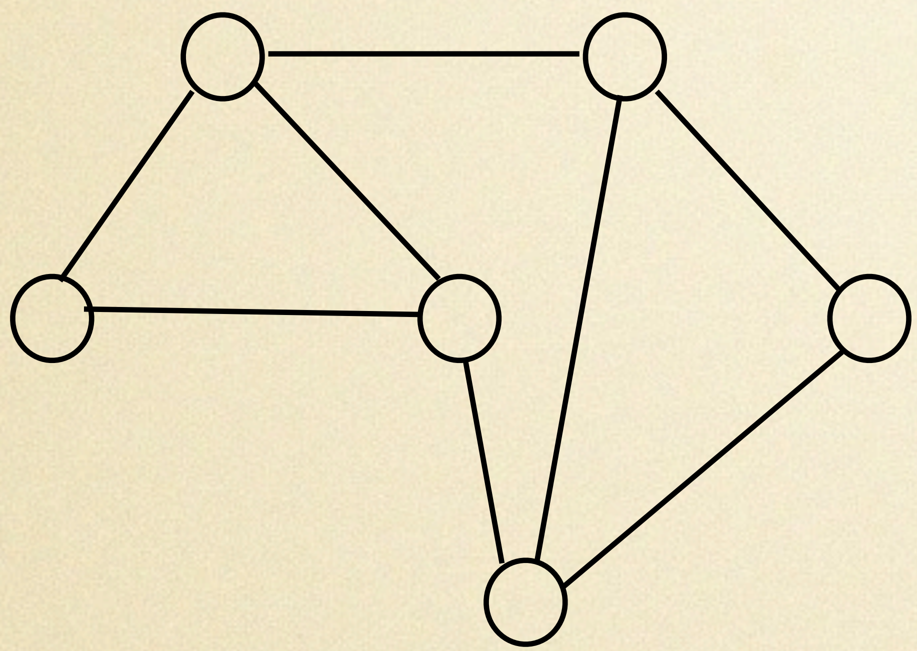
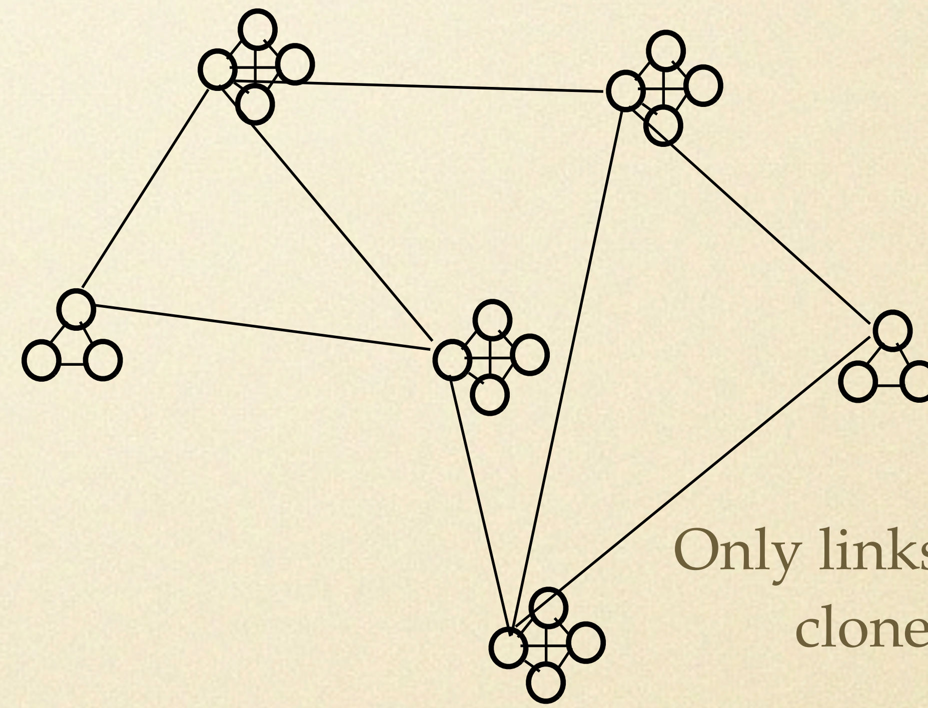




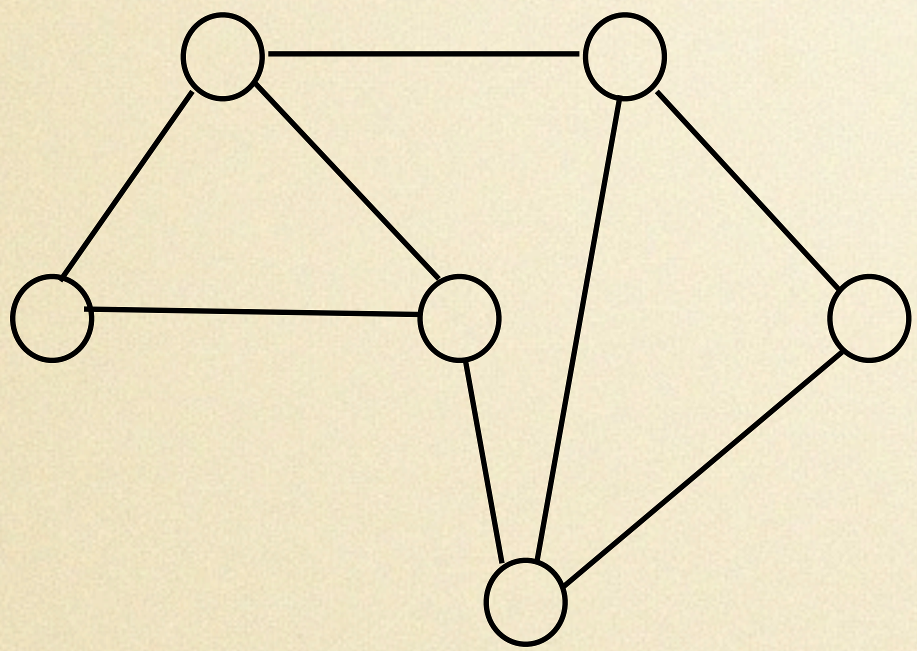
Only links between 1st clones shown

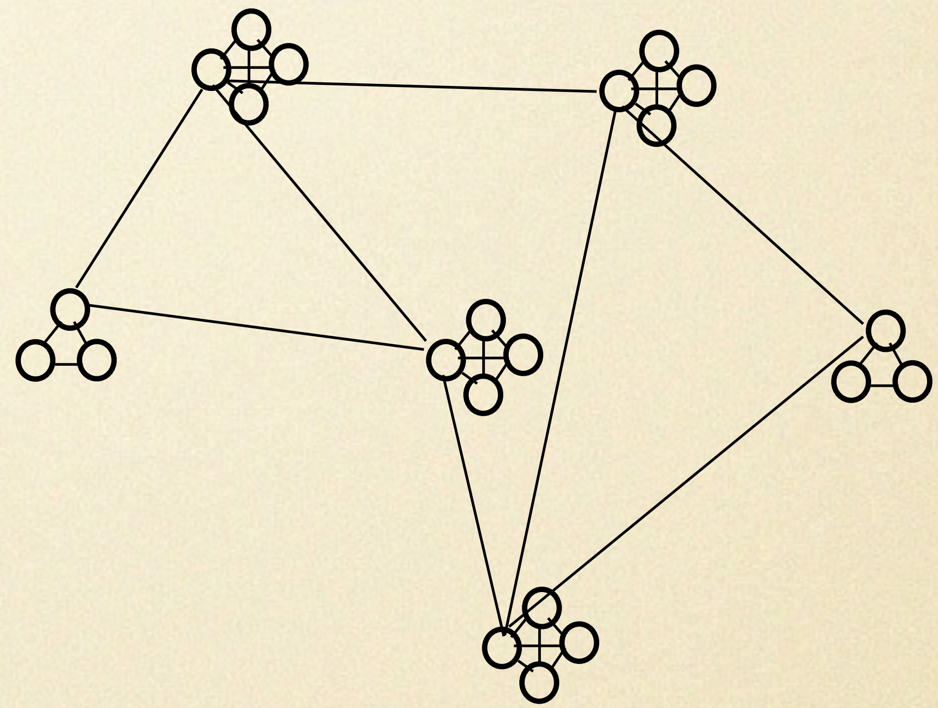
1) Each node v makes d(v)+1 clones. All clones of v are linked together

2) If u and v neighbors, then for all i, the i-th clone of u is linked to the i-th clone of v

3) We now run the MIS algorithm on the new graph. If the i-th clone of v is in the MIS, v is colored i!

# A Coloring Algorithm





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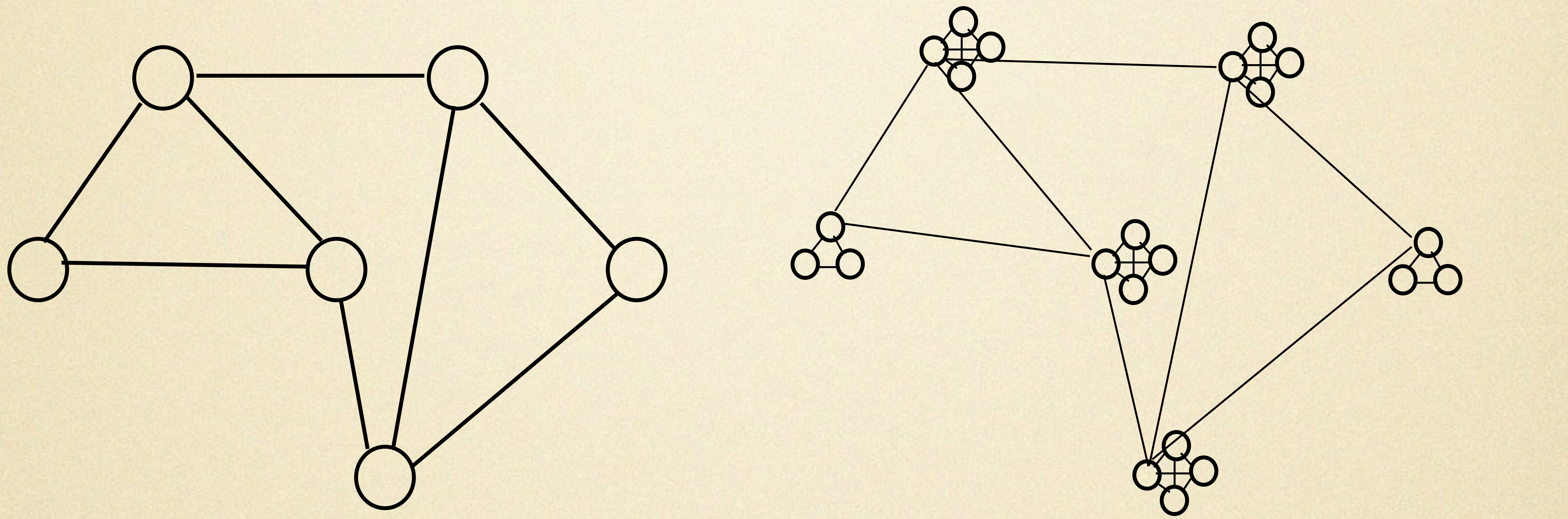

Fact 1: For any node v, at most one clone is in the MIS

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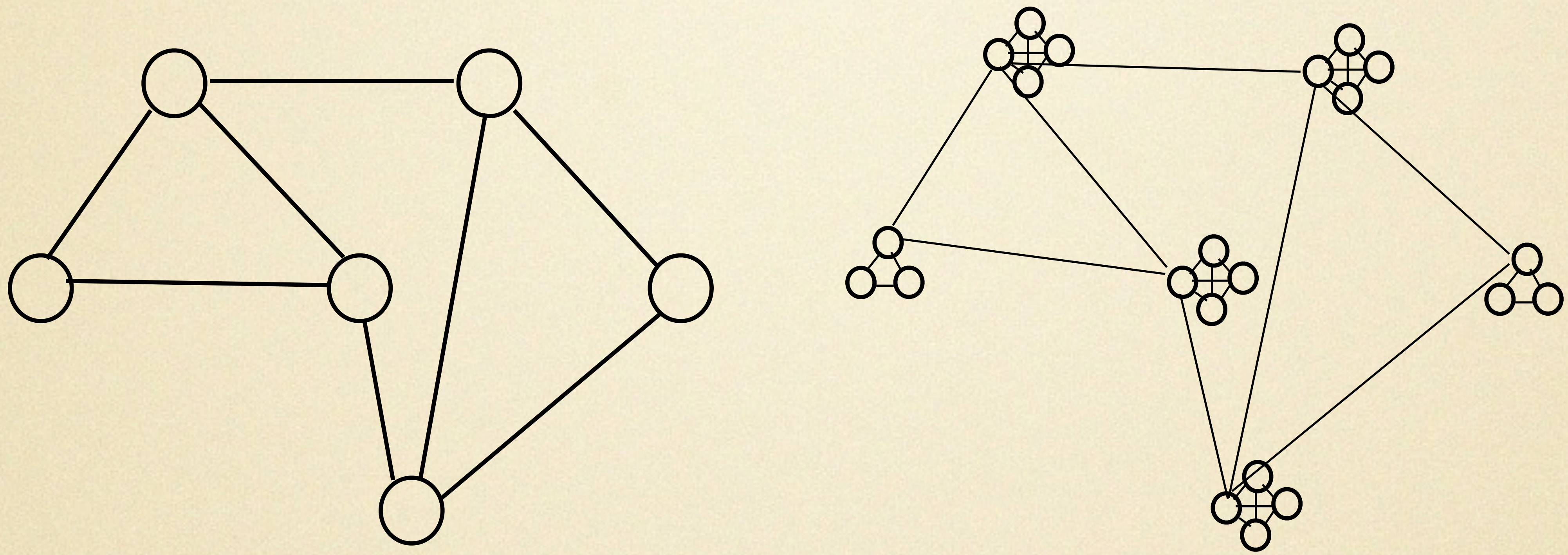

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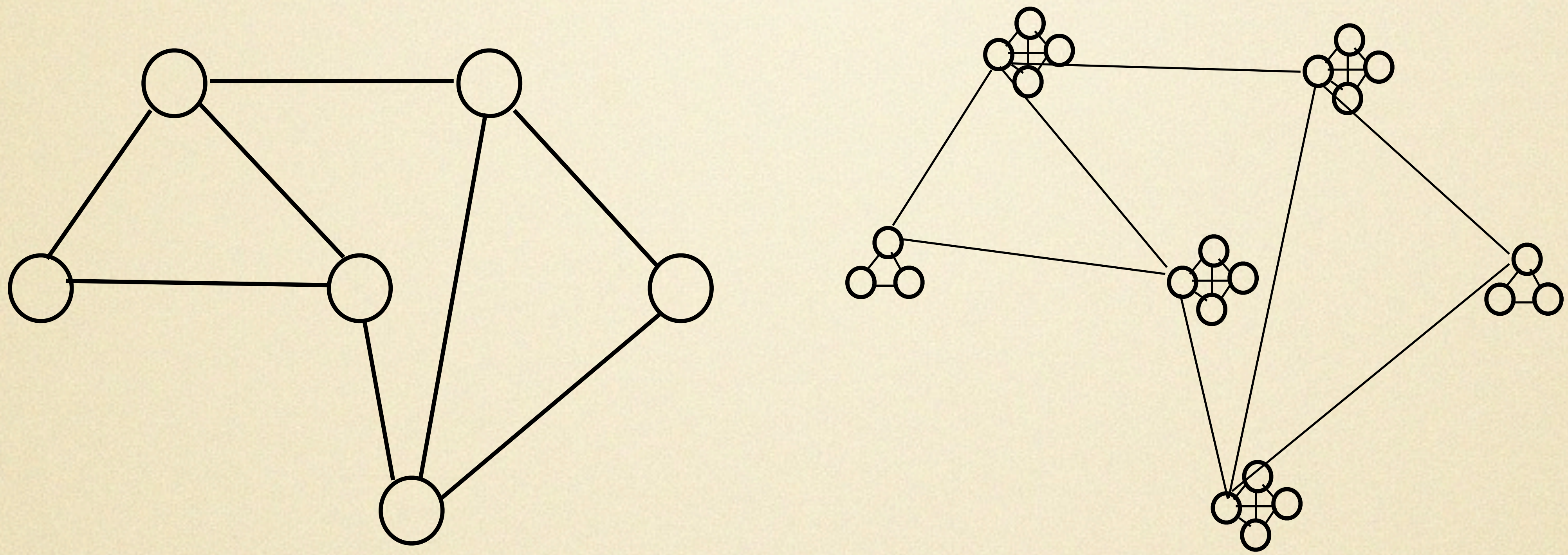
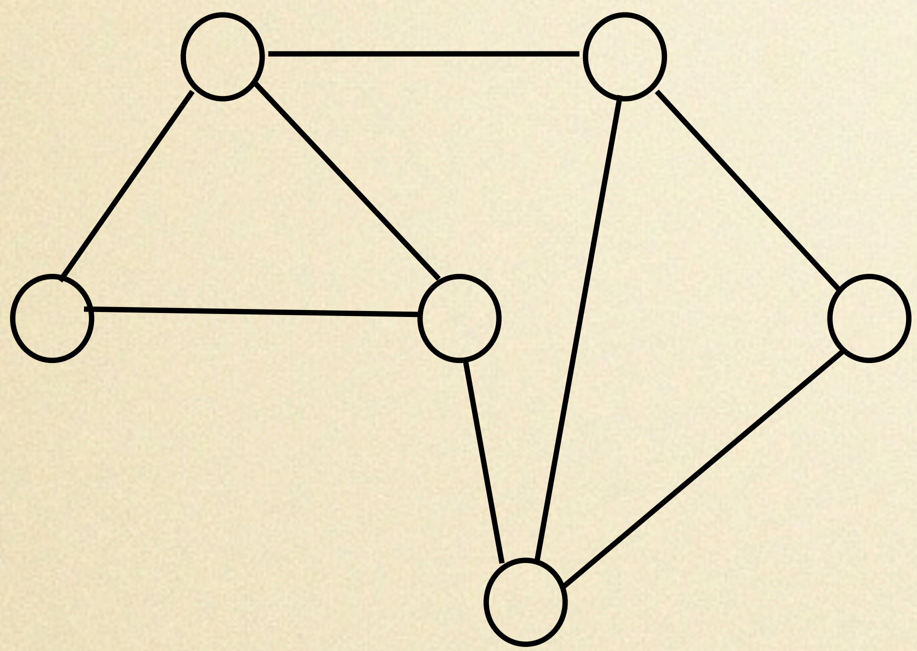


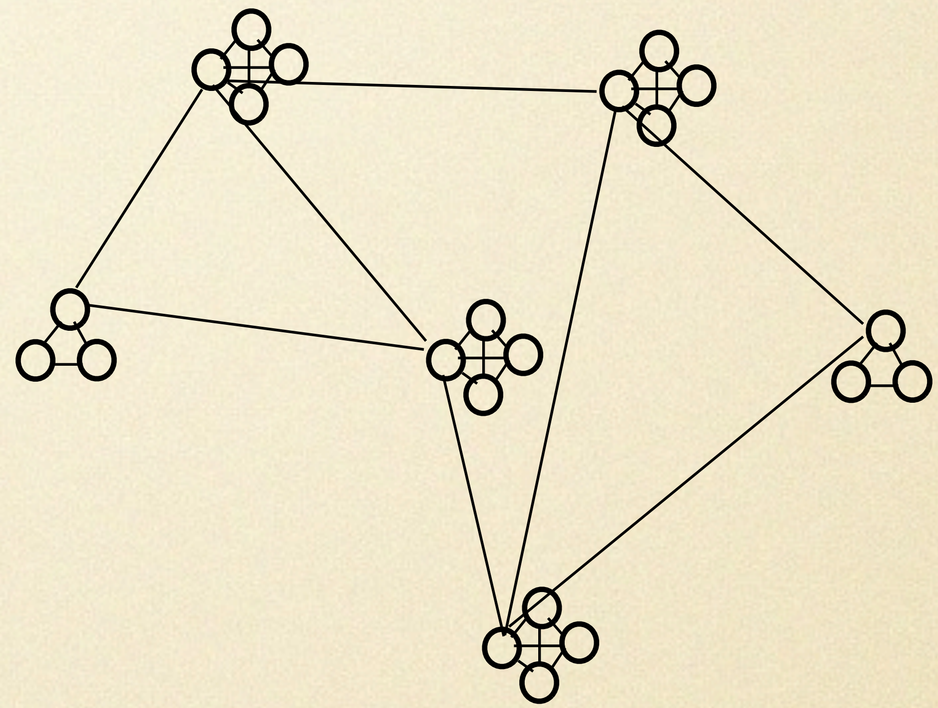
Fact 1: For any node v, at most one clone is in the MIS

Fact 2: For any node v, at **least** one clone is in the MIS

Fact 3: The running time is logarithmic since the new graph has at most $m^2$ edges

# Wrapup





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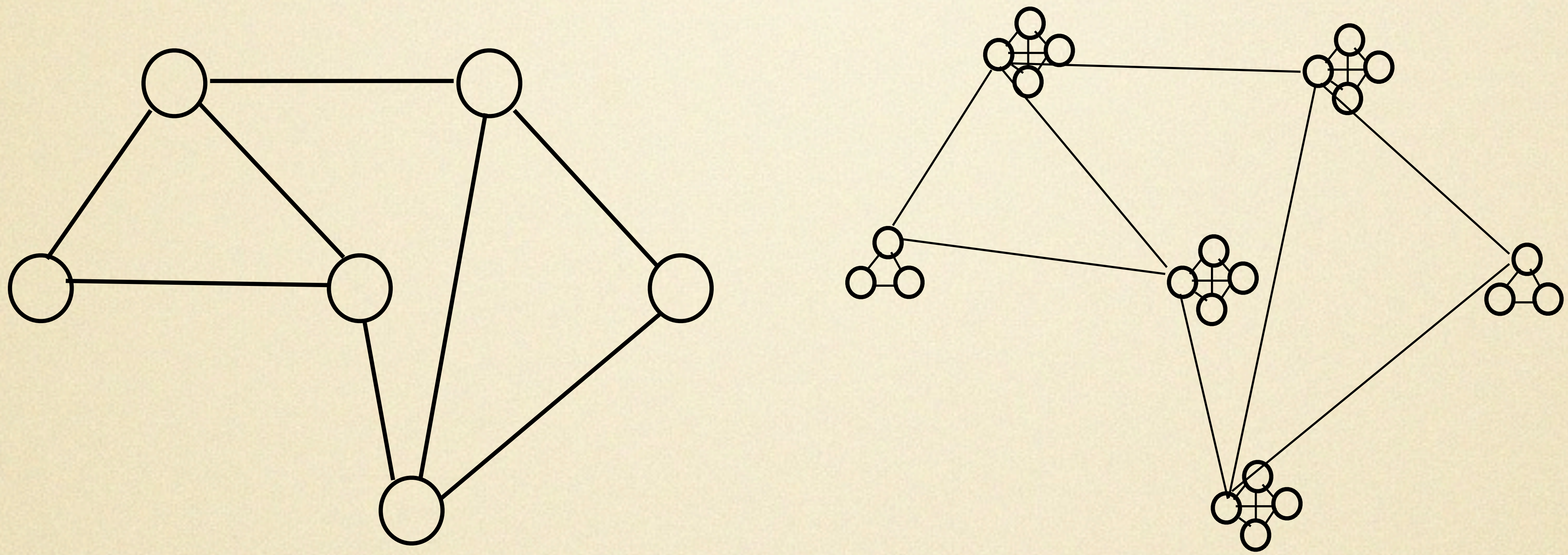

The algorithm colors any graph with Δ+1 colors, where Δ is the maximum degree of a node

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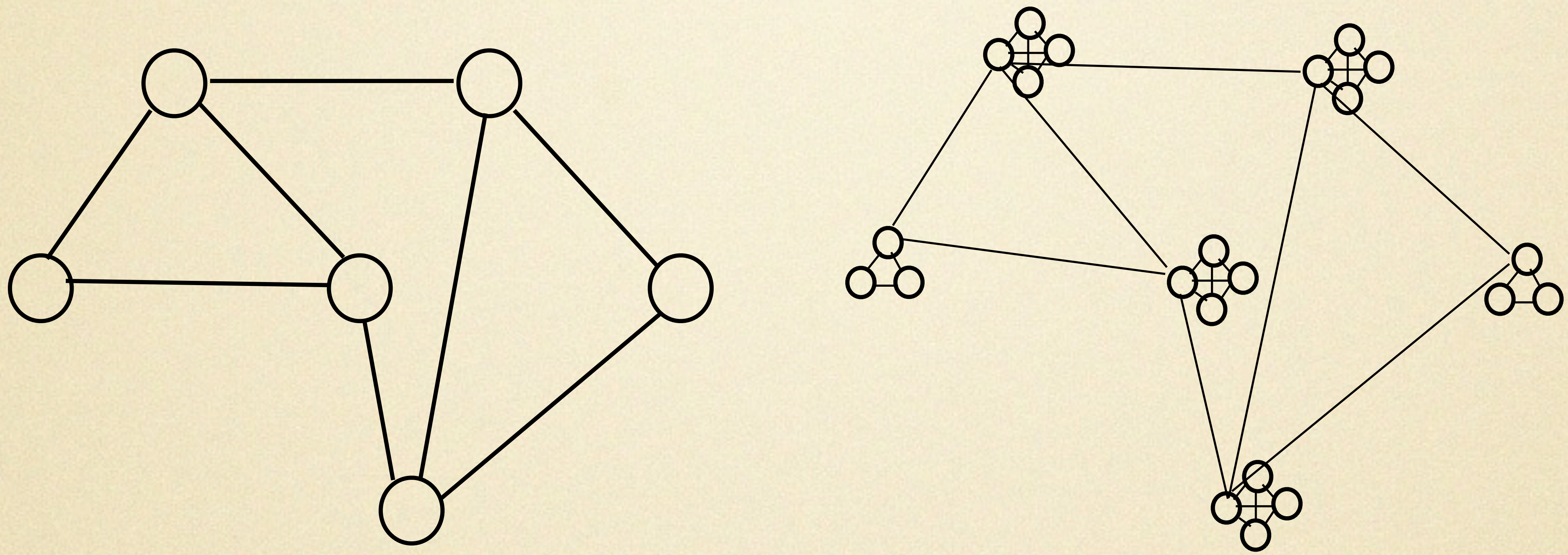

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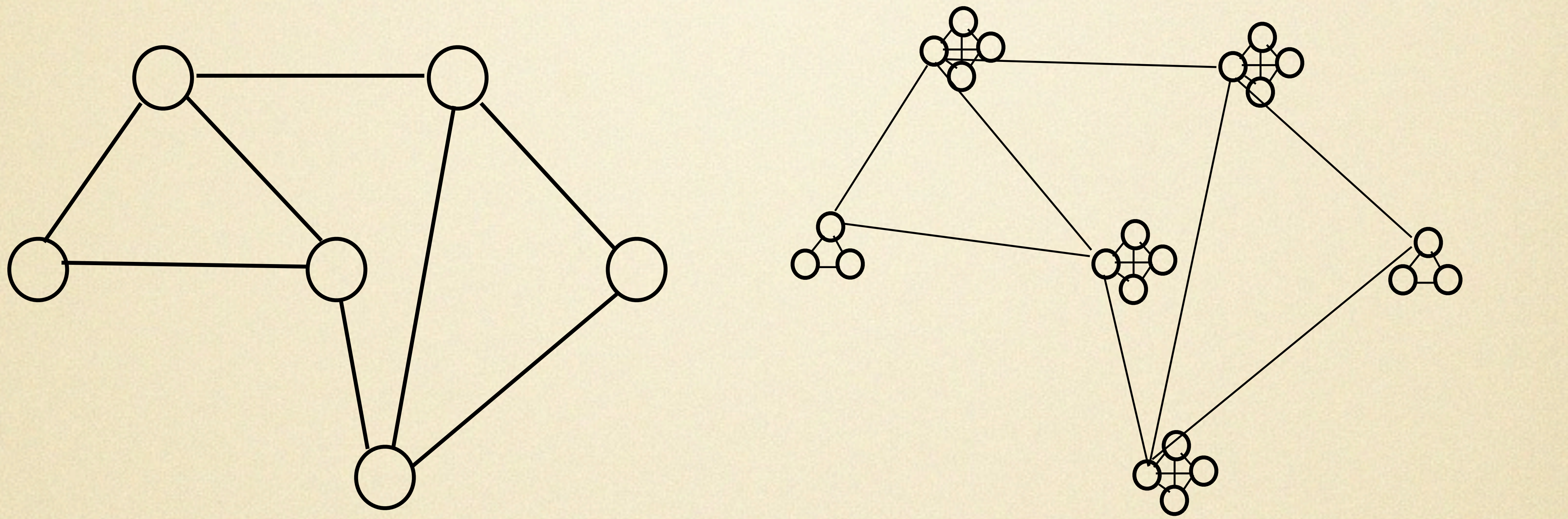

The algorithm colors any graph with $\Delta$+1 colors, where $\Delta$ is the maximum degree of a node

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Note: $\Delta$ is not necessarily the minimum number of colors needed!

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Question: How does the structure of the graph (small world, preferential attachment) effect the difficulty of graph coloring in practice?

Answer: We don't know!

# Conclusion

- Many problems can be solved efficiently over large networks
- Randomness is a powerful tool, but need to get the distributions right!
- Interaction between **Form** (topology) and **Function** (computation) is critical
- Still much work needed to understand this interaction