Fear in Mediation

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(Joint with J. Diaz, D. Mitsche, N. Rustagi)

Price of Anarchy(POA)

- Social Welfare (SW) = Sum of utilities of all agents
- In most games, SW with selfish players is worse than SW with benevolent dictator
- POA measures that difference

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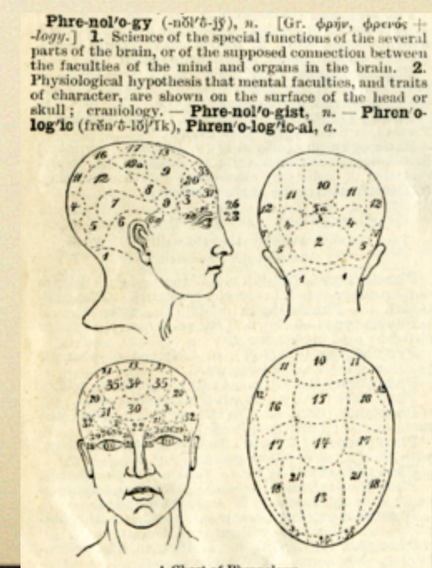

POA (KP '99)

$$POA = \frac{\text{SW in Worst Equilibria}}{\text{SW with Benevolent Dictator}}$$

- Intuitively, gives quantitative measure of the "tragedy of the commons" effect for a game

733 Cites Later

- POA can vary widely from one game to the other
- But there are many, many games with high POA

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- POA can vary widely from one game to the other
- But there are many, many games with high POA
- Problem: Everybody talks about POA, but nobody does anything about it!

Mediator

- Mediator privately suggests an action to each player
- Players can ignore suggestions of mediator; they retain free-will and remain selfish
- Goal: Use mediator to improve SW

Outline

- Multi round game
- Digression on mediators
- Single round mediator
- Single round impossibility result
- Conclusion and open problems

A Bandwidth Game

- n players; 1 channel
- each player decides whether or not to transmit on the channel
- If exactly 1 player transmits, their utility is 1
- Otherwise each player that transmits has utility of α
- Price of Anarchy: $1/\alpha n$

Multi-round BW

- Each player chooses an action
- Utilities are calculated and actions of players are all revealed
- Continue for another round with probability 1-p
- Price of Anarchy: $1/\alpha n$

BW Mediator

- Select a player x randomly; tell x to send on the channel and all other players to not send on channel
- If any player ever disregards advice, from that round on tell all players to send on channel

BW Mediator

- If a player disregards mediators advice expected utility is: $(1 + 1/p)\alpha$
- If player follows mediator advice, expected utility is $1/pn$
- Players will follow mediator if $p \leq 1/(n\alpha) - 1$

Generalization

- Simple strategy: Let H be the configuration with the highest Social Welfare and let L be the configuration with the lowest S.W.
- Mediator tells players to perform actions as in H until some player disregards and then tell all players to follow L
- Works (minimizes p) if all players have same utility in H and also in L

Generalization

- What about for general multiround games or for general classes of multiround games???
- In general want to find a mediator that 1) optimizes S.W. and 2) works for the smallest value p possible
- These are open problems!

Mediator Digression

- Correlated Equilibria: A probability distribution over strategy vectors that ensures no player has incentive to deviate
- Correlated equilibria: players share a global coin; Nash equilibria: private coins only
- A mediator implements a correlated equilibria

Mediator?

- Is there really ever a trusted third party?

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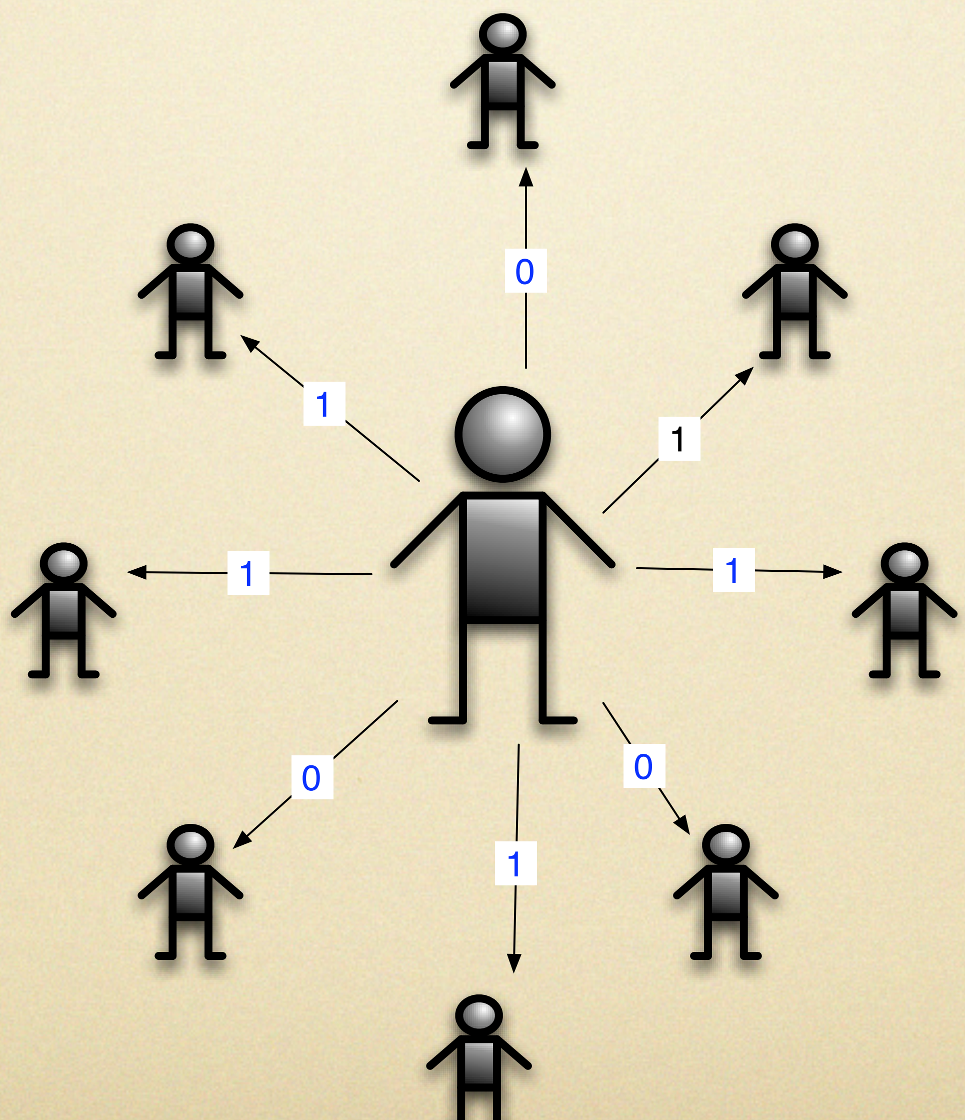
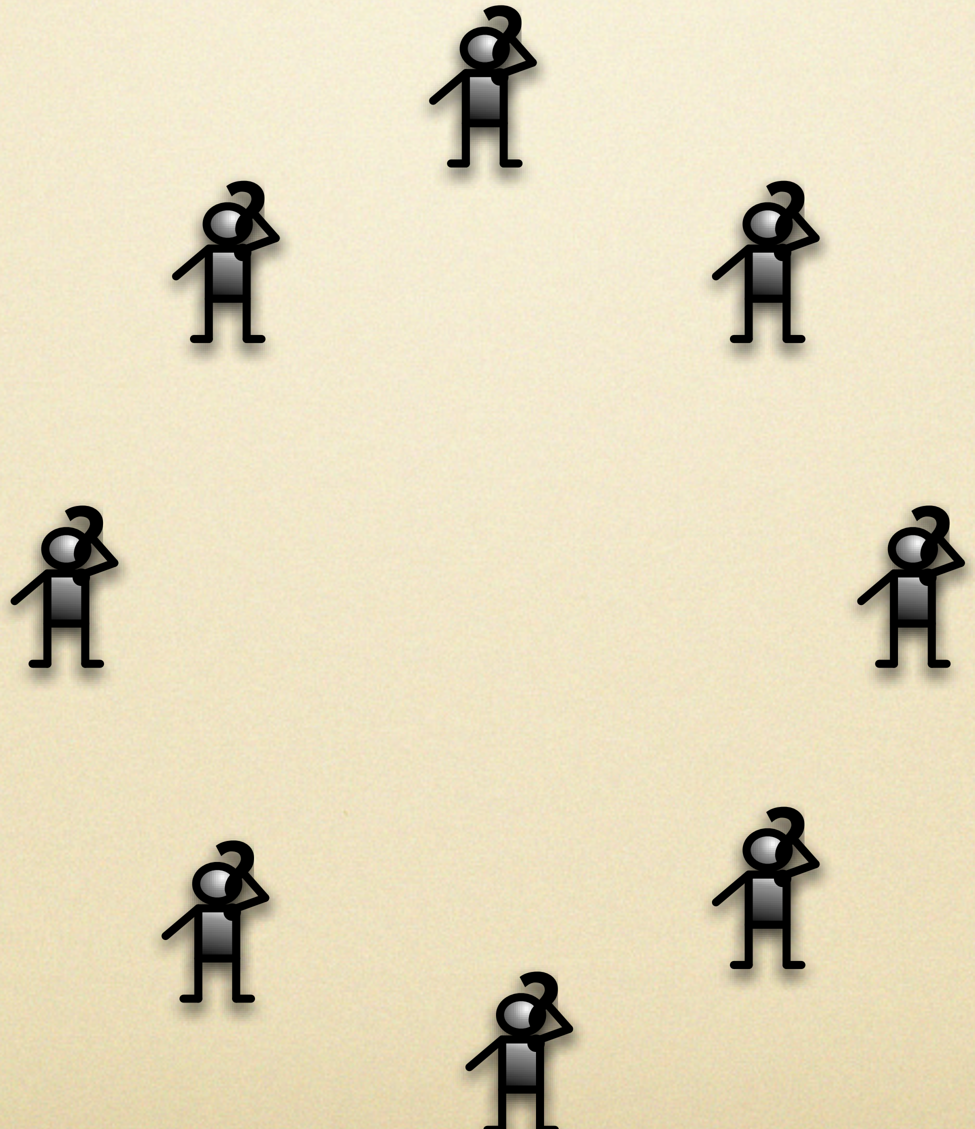
"It is the final proof of God's omnipotence that he need not exist in order to save us." - Peter De Vries

Mediator



No Mediator



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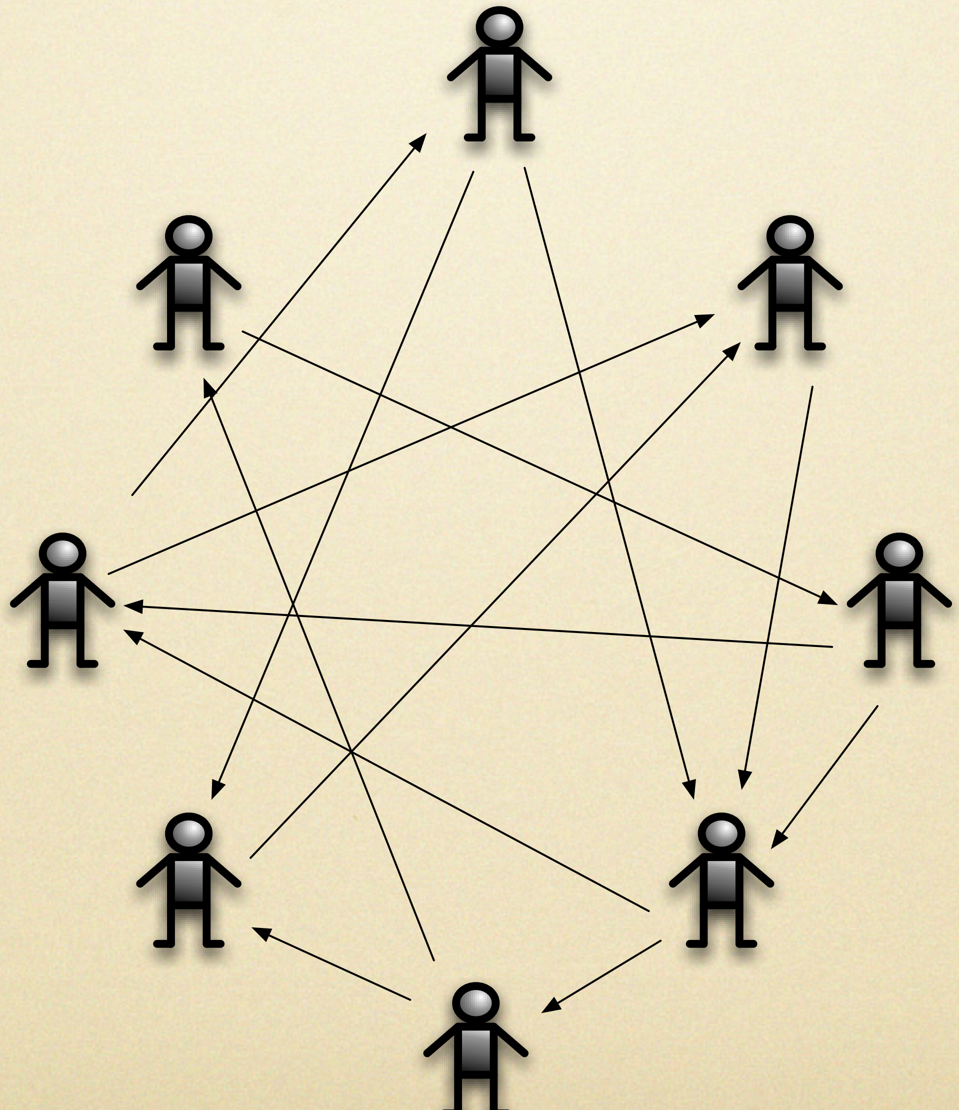

Distributed Mediation

- A mediator can be implemented in a fully distributed manner by the players themselves ("cheap talk") [ADGH '06, ADH '08]
- Similar to cryptographic results on e.g. global coin toss and secure multiparty computation
- This can be done quickly and with reasonable communication overhead [KS '09]

Single Round

- Multi round is fine, but what about single round games???
- Problem: Mediator can no longer react to players choices
- Idea: Exploit “windfall of malice”

Windfall of Malice

- "Windfall of malice": Presence of adversarial players can actually decrease the price of anarchy [MSW '06, BKP '07]
- Selfish players assume adv. players are out to get them
- Idea: Design a mediator that achieves windfall of malice even without Byzantine players

Our Technique

- Two configurations
  - "Fear Inducing": Players who do not follow mediator's advice have low utility
  - "Benevolent": Optimal or near optimal social welfare

Inoculation Game

- Each node of a grid is a player
- Players choose whether or not to inoculate
- Then, a virus infects a random node in the grid; all nodes in the uninoculated connected component of this node are infected
- Inoculation costs $1; infection costs $L

Analysis

- Nash Eq.
  - Component size $\theta(n/L)$
  - SW: $\theta(n)$
- Optimal
  - Component size: $\theta((n/L)^{2/3})$
  - SW: $\theta(n^{2/3}L^{1/3})$

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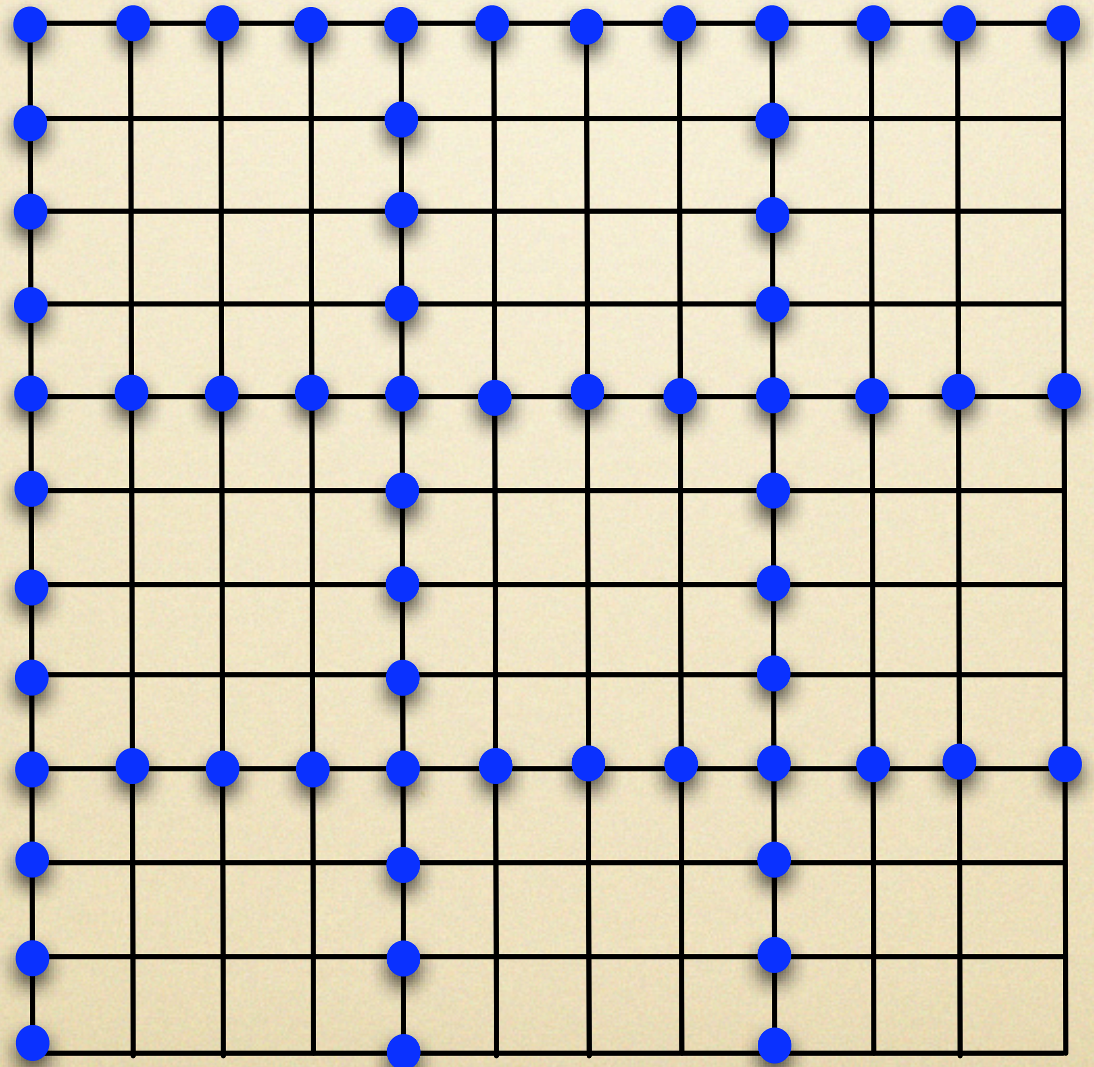
Our Result: Mediator that achieves optimal SW.

Config 1: Optimal



Component Size: $(n/L)^{2/3}$

Config 2: Fear Inducing

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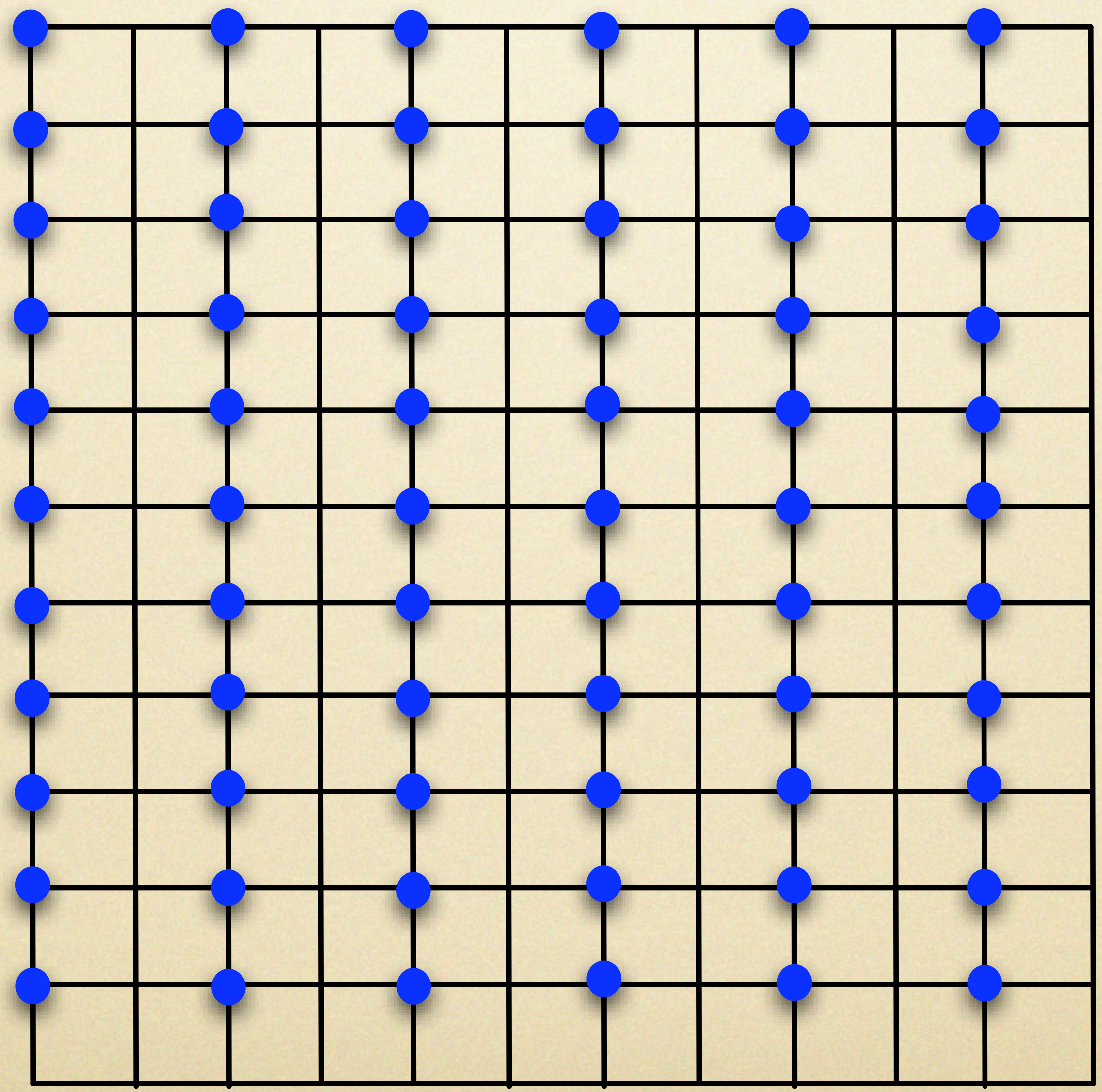
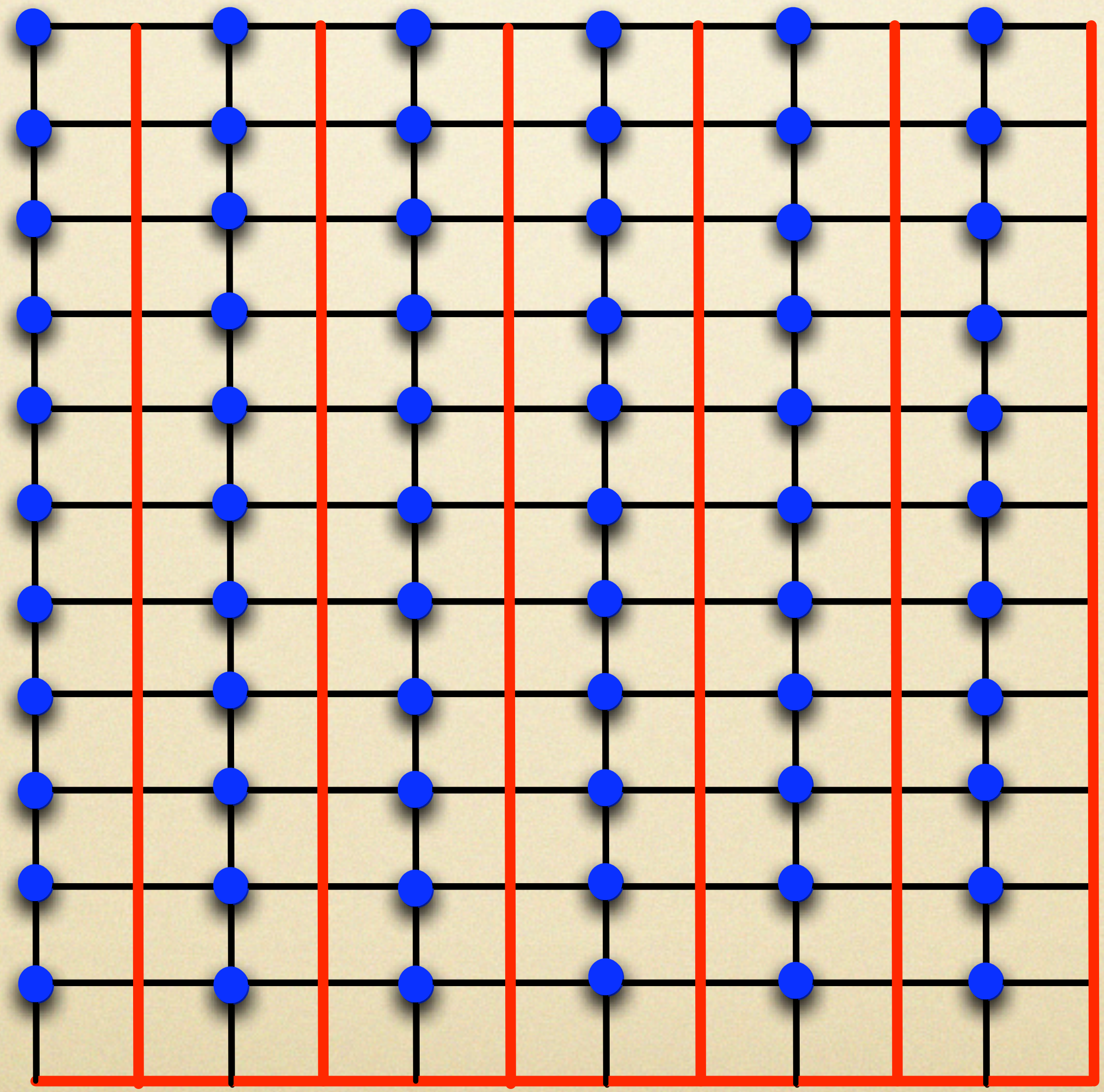


Config 2: Fear Inducing



Mediator

- Mediator chooses config 1 with probability

$$p_1 = \theta(L^{-2/3}n^{-1/3})$$

- Mediator chooses config 2 with probability

$$p_2 = 1 - \theta(L^{-2/3}n^{-1/3})$$

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- If players listen, S.W. is $\theta(n^{2/3}L^{1/3})$

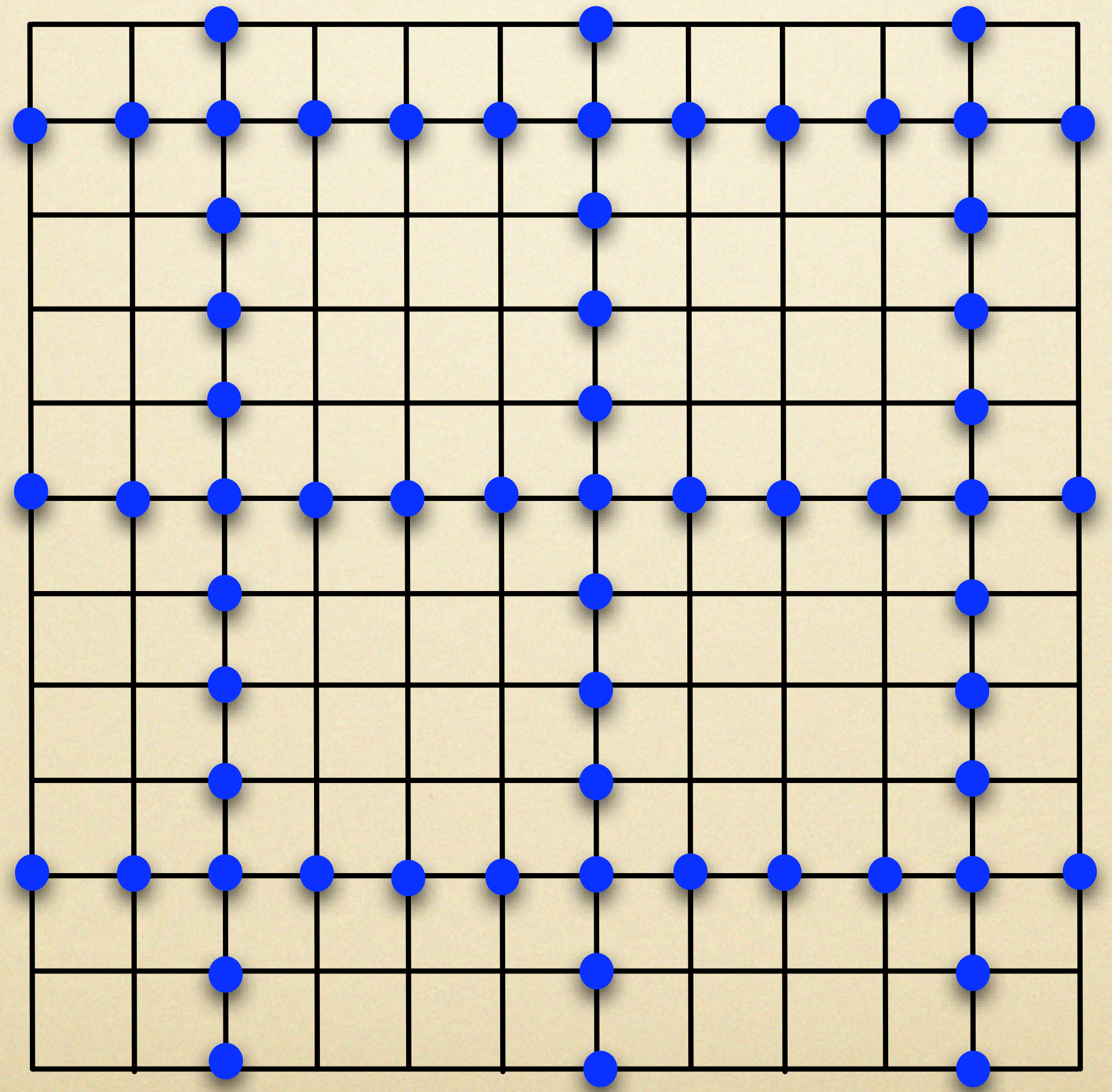
Desired Property

- If a player is advised to inoculate, its estimate of likelihood of being in config 2 increases
- Thus, this player is more likely to follow the advice to inoculate

Problem

- Problem: Players at certain locations can determine the configuration based on advice
- Given this info, they will not follow advice
- Solution: Randomly perturb both configurations so that each player is equally likely to be told to inoculate.

Random Perturbation



Fact: Players Listen

$$\xi_I = \text{told to inoculate}$$
$$\xi_1 = \text{distribution 1 chosen}$$
$$\xi_A = \text{attacked}$$

Pf:

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Pf:

$$\begin{aligned} Pr(\xi_A|\xi_I) &= Pr(\xi_A, \xi_1|\xi_I) + Pr(\xi_A, \bar{\xi_1}|\xi_I) \\ &= Pr(\xi_A|\xi_1, \xi_I)Pr(\xi_1|\xi_I) + Pr(\xi_A|\bar{\xi_1}, \xi_I)Pr(\bar{\xi_1}|\xi_I) \\ \ldots &\geq 1/L \end{aligned}$$

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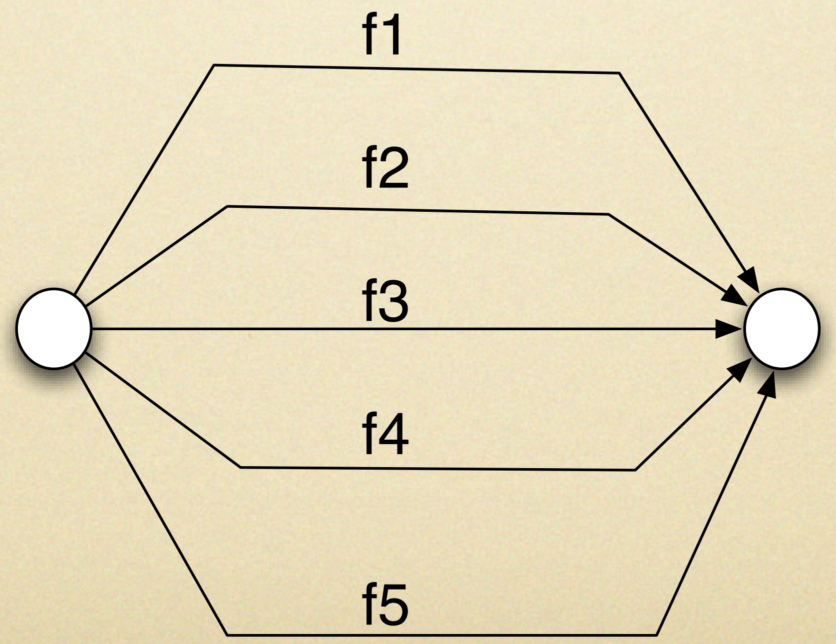
Intuition

- Posterior probability of being in distribution 1 increases significantly if told to inoculate
- Implies nodes that are told to inoculate are more likely to be infected
- Also, nodes told not to inoculate are very likely to be in distribution 2 and thus not to be attacked

Generalization

- Non-atomic, anonymous, congestion games
- Sum of flows from s to t is 1
- Cost of an edge is function of flow over it



Applicability

- Question: Can a mediator always help improve the SW of a game?
- Answer: No!

Impossibility Result

$\mathcal{F}_h(a, x) =$ Max cost of a when x fraction of players choose a

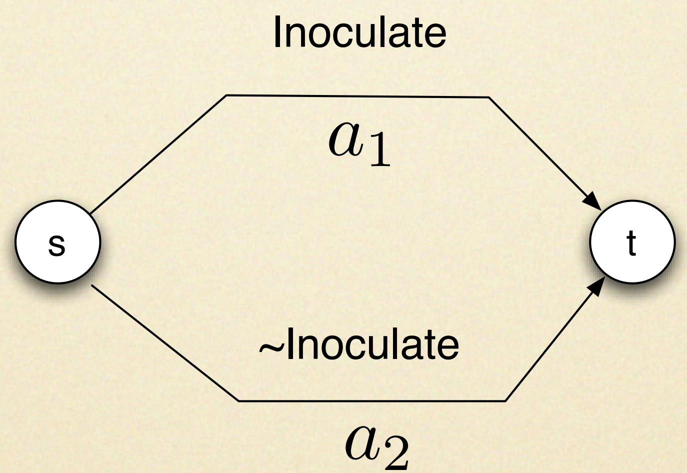
$\mathcal{F}_\ell(a, x) =$ Min cost of a when x fraction of players choose a

Theorem: If for all $a \in A$ and $0 \leq x \leq x' \leq 1$, $\mathcal{F}_h(a, x) \leq \mathcal{F}_\ell(a, x')$ then the smallest cost of a correlated equilibrium is no less than the smallest cost of a Nash equilibrium.

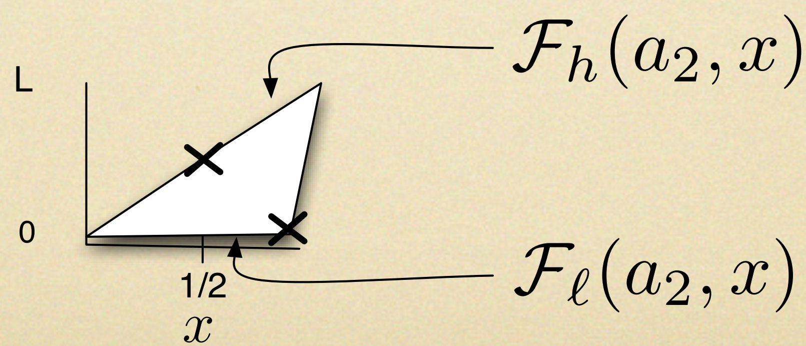
Theorem Intuition

- Cost of some action must decrease as more players choose that action
- Otherwise, a mediator will not help

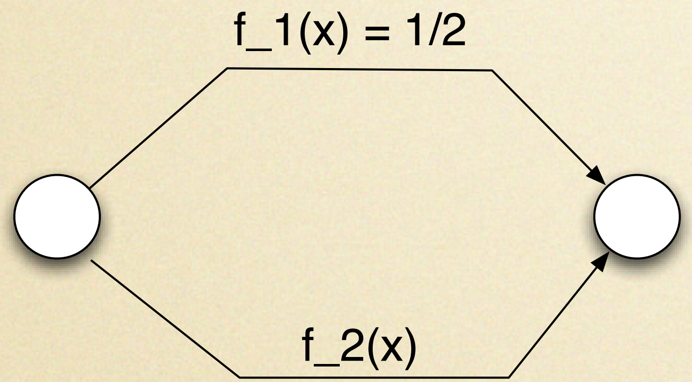
Inoculation



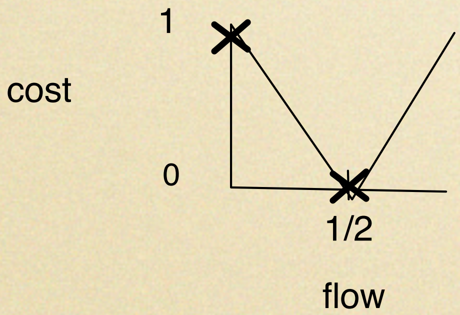
a_2 :



El Farol Var.



f_2(x):



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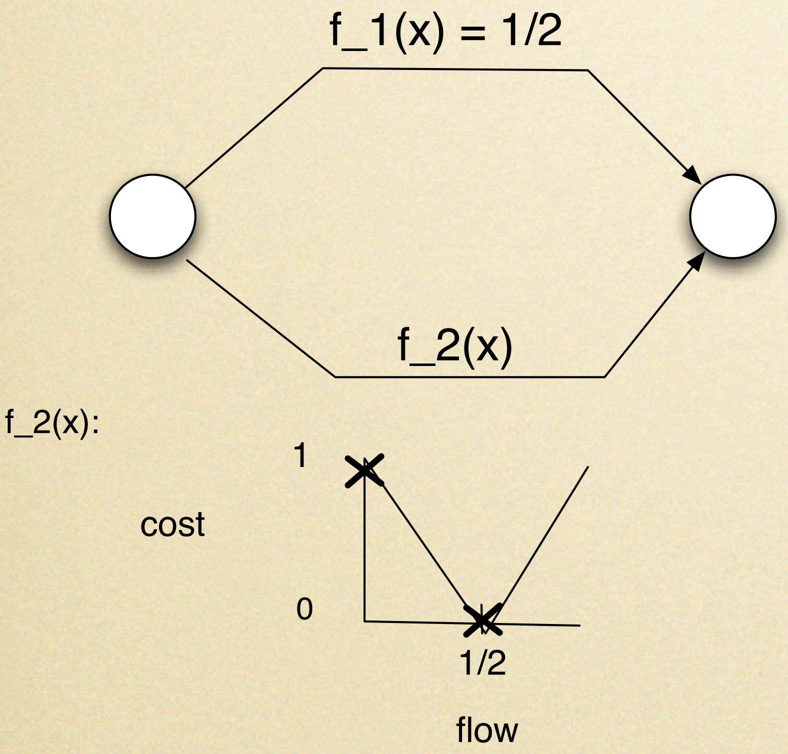
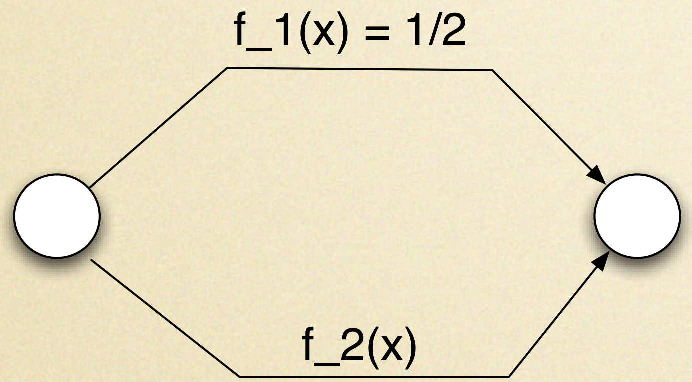
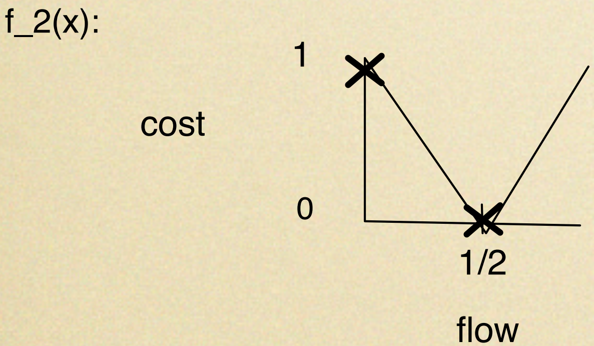


Mediator:

- With probability 1/3, tell all players to go up
- With probability 2/3, tell half the players to go up and half to go down

El Farol Var.





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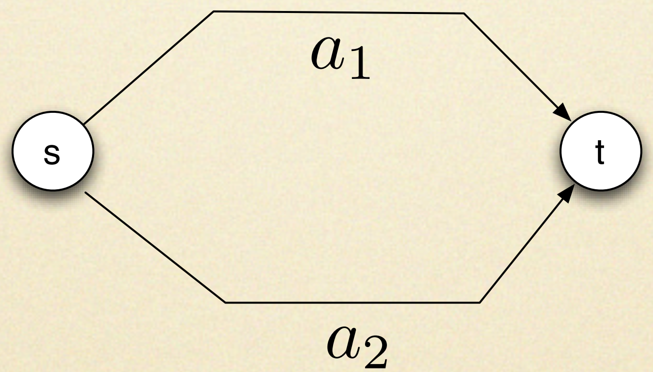
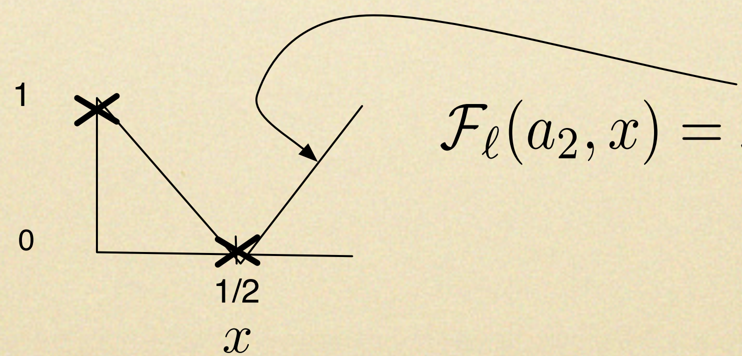
Achieves S.W. of 1/3 vs 1/2 for the Nash

El Farol



a_2 :



$$\mathcal{F}_\ell(a_2, x) = \mathcal{F}_h(a_2, x)$$

Impossibility Proof

$\text{POST}\,(a, a') =$ expected cost of performing action a if action a' is suggested

$\text{PRI}\,(a) =$ expected cost of ignoring mediator and performing action a

Lemma 1: If conditions of theorem hold, then for all actions a, $\text{POST}\,(a, a) \geq \text{PRI}\,(a)$

Lemma 2

Y is cost of a player if follows advice of mediator

X is cost of a player if ignores mediator and always chooses action a minimizing **PRI**(a)

Lemma 2 For any mediator, $E(Y) \leq E(X)$

Main Proof

If for all $a \in A$ and $0 \leq x \leq x' \leq 1$, $\mathcal{F}_h(a, x) \leq \mathcal{F}_\ell(a, x')$

Then

$\textbf{Lemma 1} \rightarrow E(Y) > E(X)$

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Contradicts Lemma 2

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Contradicts Lemma 2

Thus, there can be no non-trivial mediator

Technical Challenge

- Must show that E(Y) > E(X) even when inequality in Lemma 1 is not strict
- Handle this by 1) subtle case analysis in proof of main theorem; and 2) augmenting Lemma 1 to show that in some cases inequality is strict

Conclusion

- Described general technique for designing mediators to improve SW for some games
- Showed for large class of games, no mediator will improve SW

Questions

- Q: Do two configurations suffice to define an optimal mediator for congestion games with just 2 edges?

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- Q: Do two configurations suffice to define an optimal mediator for congestion games with just 2 edges?
- A: In some cases, it's possible to achieve an equilibria with 3 configurations but not with 2. However, when a pair of these distributions can be used to form an equilibria, the S.W. achievable with this pair is at least as good as what is achievable with 3.

Open Problems

- Can we determine necessary and sufficient conditions for a game to allow a non-trivial mediator
  - for general congestion games?
  - for arbitrary anonymous games?
- Can we find necessary and sufficient conditions for non-symmetric multi-round games

Open Problems

- What does mediation say about the power of coalitions in games?
- Note: we have found that for some games, a clever coalition strategy can significantly improve the utility of all members of the coalition (provided the coalition is the right size)

Open Problems

- Consider games where one coalition competes against another
- Many such games are like “chicken” in that a non-responsive strategy works best.
- Q: Can we design a mediator that ensures that the strategy of a coalition is non-responsive?