Homework set 9: λ-calculus — due Monday 5 March

Total number of points available on this homework is 100. Full credit is equivalent to 100 points.

1. (30 pts.) Show that the following terms have a normal form:

   (a) $(\lambda y.yyy)((\lambda ab.a)\mathbf{I}(\mathbf{SS}))$

   (b) $(\lambda yz.zy)((\lambda x.xxx)(\lambda x.xxx))(\lambda w.\mathbf{I})$

   (c) $\mathbf{SSSSSSS}$

2. (40 pts.) For each of the following λ-expressions either find its normal form or show that it has no normal form:

   (a) $(\lambda x.xx)(\lambda x.x)$

   (b) $(\lambda x.xx)(\lambda x.xx)$

   (c) $\mathbf{Y}$ *(see below)*

   (d) $\mathbf{Y}(\lambda y.y)$

3. (30 pts.) A λ-expression **Fix** with the property that $\mathbf{Fix}\,E = E(\mathbf{Fix}\,E)$ for any E is called a fixed-point operator (or combinator). One well known fixed-point operator is **Y**, defined as $\mathbf{Y} \triangleq \lambda f.(\lambda x.f(xx))(\lambda x.f(xx))$. Show that **Y** is a fixed-point operator.