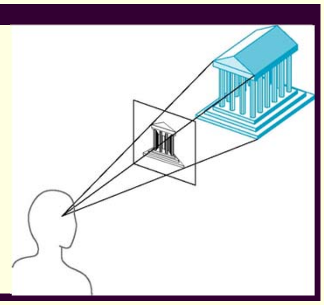
CS 351:
Perspective Viewing

Instructor: **Joel Castellanos**
e-mail: joel@unm.edu
Web: http://cs.unm.edu/~joel/



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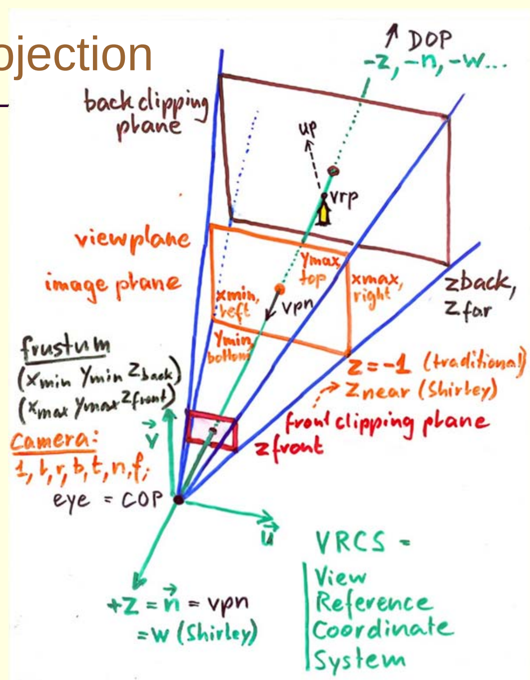
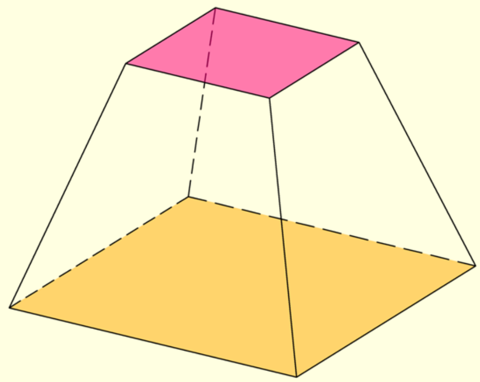
Perspective Projection



Frustum

In computer graphics, the viewing frustum is the three-dimensional region which is visible on the screen which is formed by a clipped pyramid.



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Properties of Perspective Projections

Property 1: The perspective projection of an object becomes smaller as the object gets farther away form the center of projection.

What are some non-obvious, and interesting effects of this?

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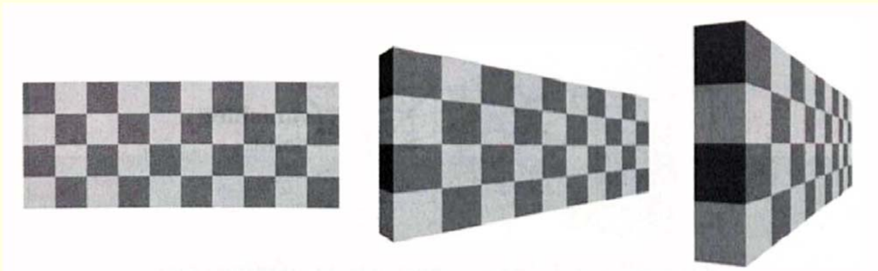
Properties of Perspective Projections

Property 2: As an object is rotated, its projected width becomes smaller. This is known as ***foreshortening***.



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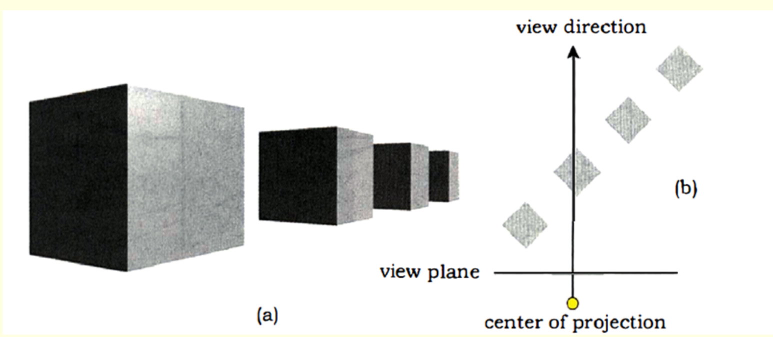
Properties of Perspective Projections

Property 3: Perspective projections preserve straight lines.



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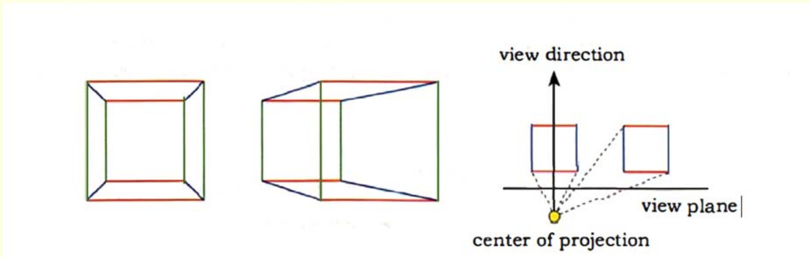
Properties of Perspective Projections

Property 4: Sets of parallel lines that are parallel to the view plane remain parallel when projected onto the view plane.

Property 5: Sets of parallel lines that are not parallel to the view plane converge to a vanishing point on the view plane.



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Equation of a Checkerboard

```
//Checkerboard in x-z plane with y = 0.
public static Color
  getCheckerboardGroundColor(
        double x, double y, double z)
  {
    if ( Math.abs((Math.floor(x))) % 2
      == Math.abs((Math.floor(z))) % 2)
         return Color.BLACK;
    return Color.WHITE;
  }
```

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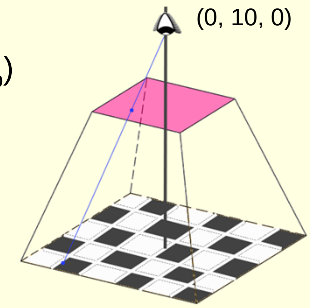
Axis-Aligned Perspective Projection Straight down Y-Axis

- Eye: (0, 10, 0)
- One Point on View Plane: $(x_{vp}, 5, z_{vp})$



$$\overrightarrow{ray} = \overrightarrow{eye} + d(\overrightarrow{ViewPlanePt} - \overrightarrow{eye})$$

$$\begin{pmatrix} x_{hit} \\ 0 \\ z_{hit} \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ 0 \end{pmatrix} + d \begin{pmatrix} x_{vp} & - & 0 \\ 5 & - & 10 \\ z_{vp} & - & 0 \end{pmatrix}$$

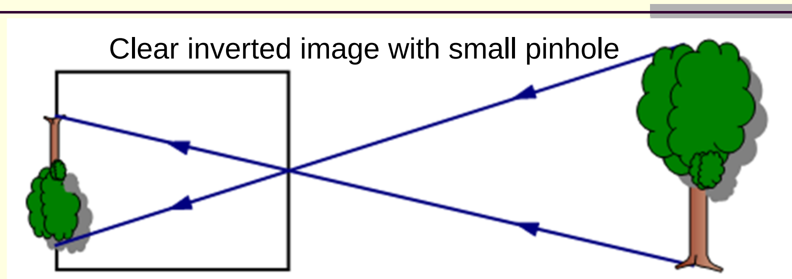
$$0 = 10 + d(5 - 10)$$
$$0 = y_{eye} + d(y_{vp} - y_{eye})$$
$$d = \frac{y_{eye}}{(y_{eye} - y_{vp})}$$

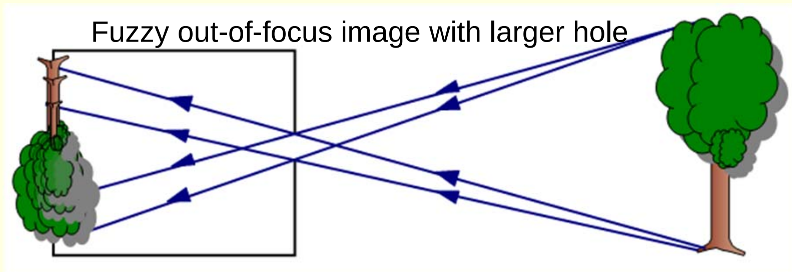
$$x_{hit} = x_{eye} + d(x_{vp} - x_{eye})$$
$$z_{hit} = z_{eye} + d(z_{vp} - z_{eye})$$

A Practical Viewing System

- The ***virtual pinhole camera*** implements perspective viewing with the following features:
- An arbitrary eye point.
- An arbitrary view direction (The view plane is defined as being perpendicular to the view direction and centered on the ray from the eye point).
- An arbitrary orientation about the view direction.
- An arbitrary distance between the eye point and the view plane.

Physical Pinhole Camera





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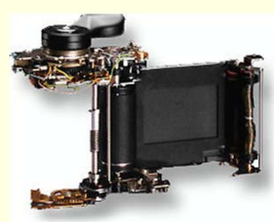
Lens Aperture

Aperture ≠ Shutter

Aperture



Shutter

Why color reflections?

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Large Glass is Expensive

Canon, Prime 50mm f/1.8 USM Lens:	$125.00
Canon, Prime 50 mm f/1.4 USM Lens:	$399.00
Canon, Prime 50 mm f/1.2 USM Lens:	$1,549.00

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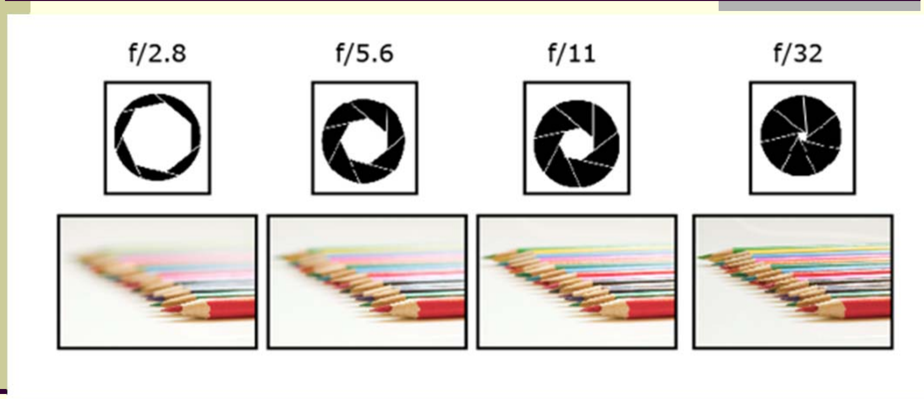
Depth of Field



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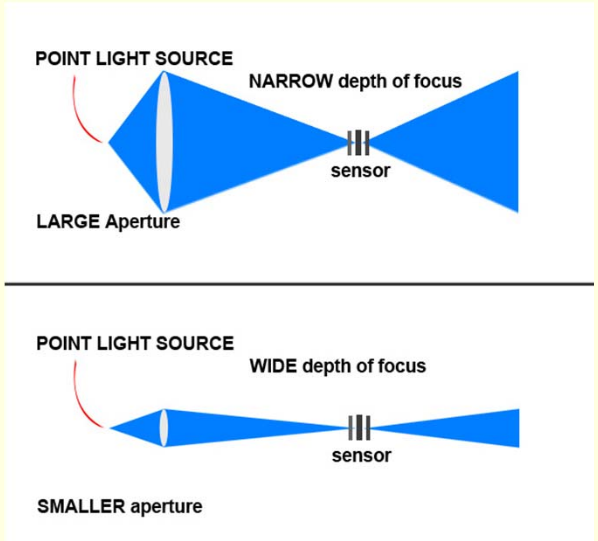
Circle of Confusion

Real lenses do not focus all rays perfectly.

Thus, at best focus, a point is imaged as a spot rather than a point.

The smallest such spot that a lens can produce is often referred to as the *circle of least confusion*.



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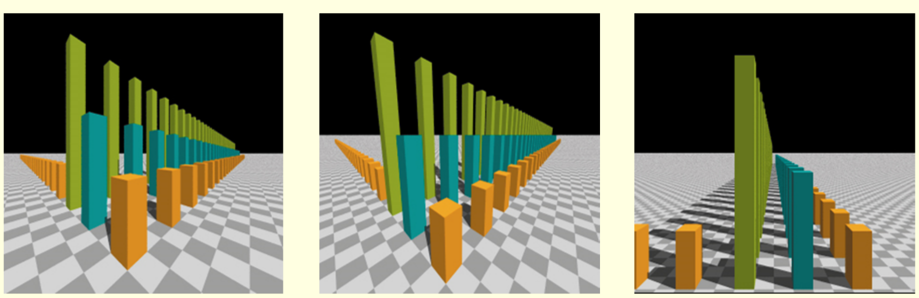
Perspective Views of Boxes

- How many vanishing points are there in each image?
- In each image, does the view direction point up or down or is it horizontal? How can you tell when it's horizontal?



a b c

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Quiz

In the equation: $\vec{a} = \vec{b} \times \vec{c} / \left\| \vec{b} \times \vec{c} \right\|$

$\vec{a}$, $\vec{b}$ and $\vec{c}$ are vectors. This means they have both magnitude and direction. What can be said about the magnitude and direction of $\vec{a}$?

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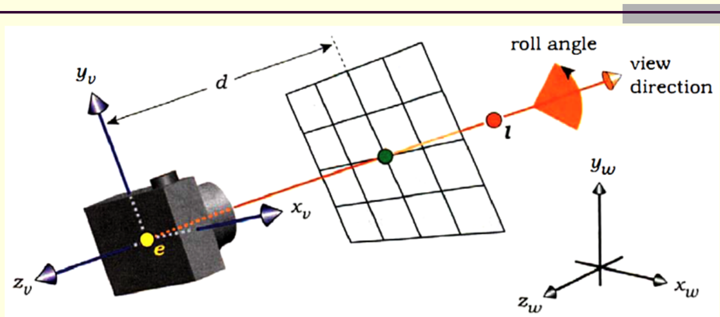
Virtual Pinhole-Camera Viewing System



User Input:

- The eye point, *e*.
- The look-at point, *l*.
- The up vector, *up*.
- The view-plane distance *d*.

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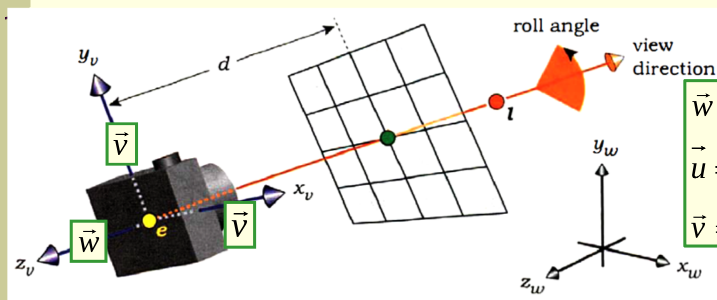
Primary-Ray Calculation



$$\vec{w} = (e-l)/\|e-l\|$$

$$\vec{u} = \overrightarrow{up} \times \vec{w} / \|\overrightarrow{up} \times \vec{w}\|$$

$$\vec{v} = \vec{w} \times \vec{u}$$

The (x_v, y_v) coordinates of a sample point p on the pixel in row r and column c are:

$$x_v = s(c - h_{res}/2 + p_x)$$

$$y_v = s(r - v_{res}/2 + p_y)$$

The primary-ray direction d is: $\vec{d} = x_v\vec{u} + y_v\vec{v} - d\vec{w}$

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3D Translation

A translation is a geometric transformation that moves every point of a figure or a space by the same amount in a given direction.

A translation transformation does not change the object's shape or size.

$$\begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} + \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_x + x \\ d_y + y \\ d_z + z \end{bmatrix}$$

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3D Scaling

Uniform scaling is a linear transformation that enlarges (increases) or shrinks (diminishes) objects by a scale factor that is the same in all directions.

The result of uniform scaling is ***similar*** (same shape different size) to the original.

$$\begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} rx \\ ry \\ rz \end{bmatrix}$$

Uniform scaling

$$\begin{bmatrix} r_x & 0 & 0 \\ 0 & r_y & 0 \\ 0 & 0 & r_z \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r_x x \\ r_y y \\ r_z z \end{bmatrix}$$

Non-uniform scaling

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3D Rotation about the x-axis

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1x + 0y + 0z \\ 0x + (\cos\alpha)y - (\sin\alpha)z \\ 0x + (\sin\alpha)y + (\cos\alpha)z \end{bmatrix} = \begin{bmatrix} x \\ (\cos\alpha)y - (\sin\alpha)z \\ (\sin\alpha)y + (\cos\alpha)z \end{bmatrix}$$

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3D Rotation about the y-axis and z-axis

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$