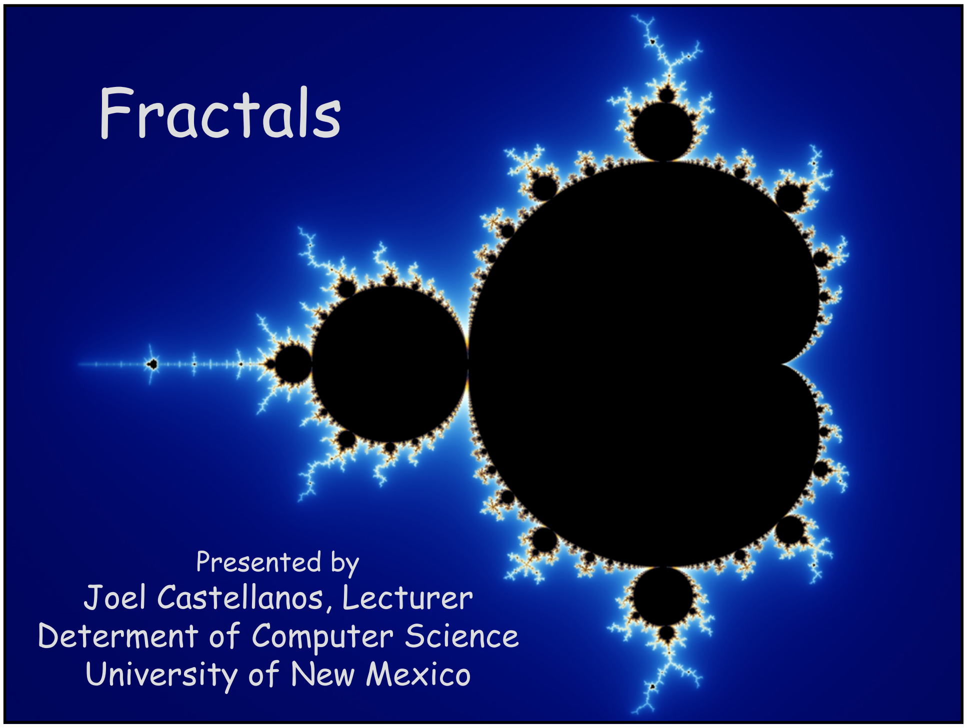
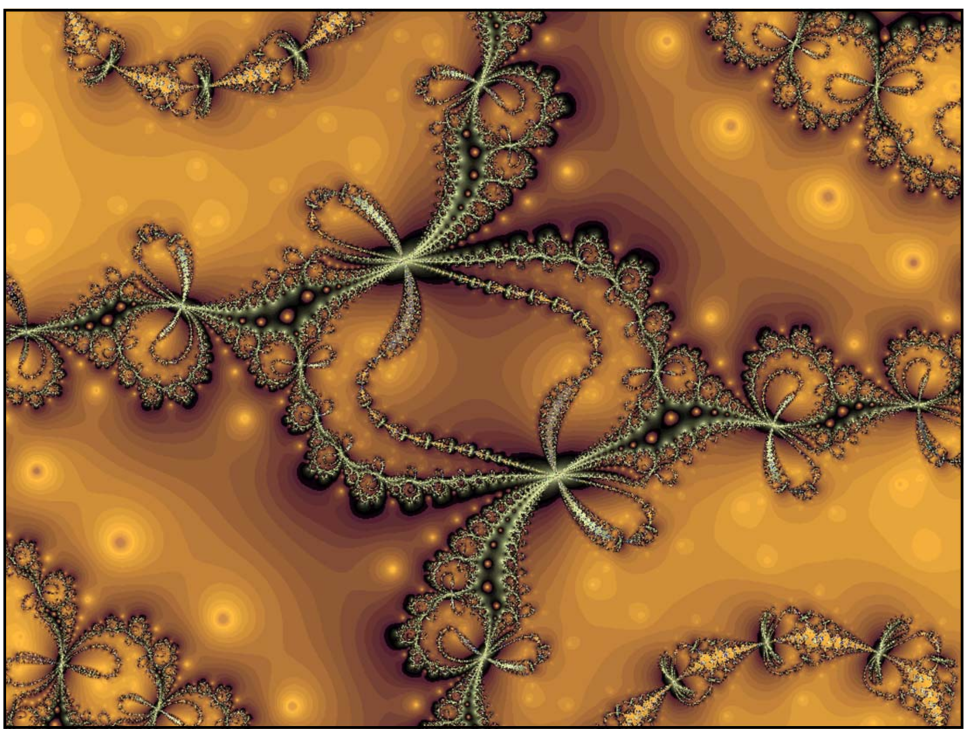
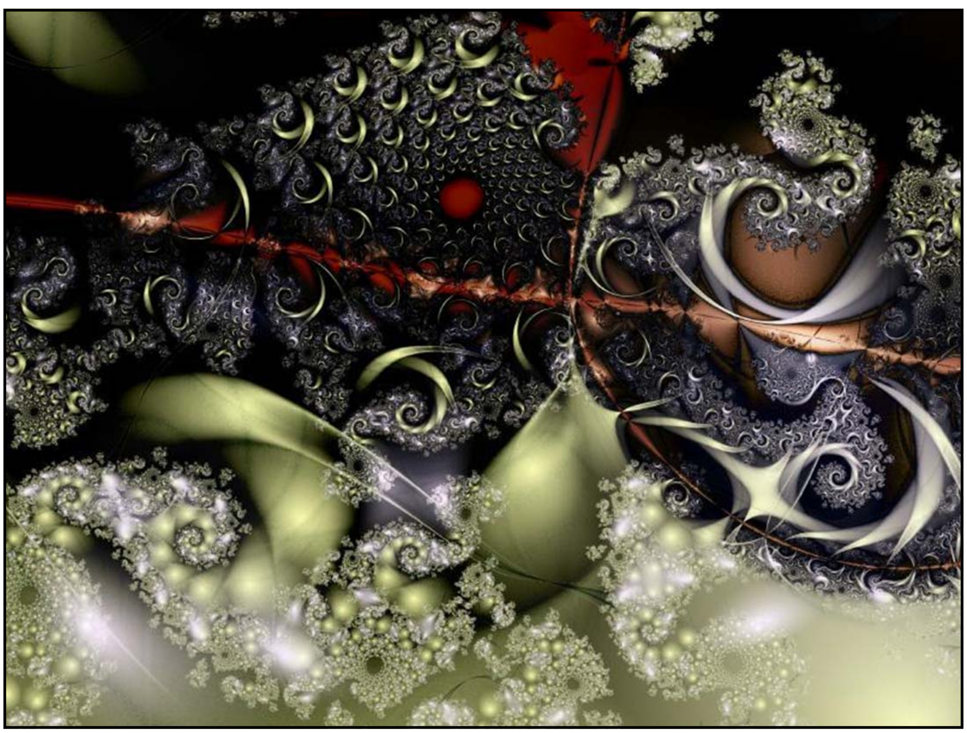
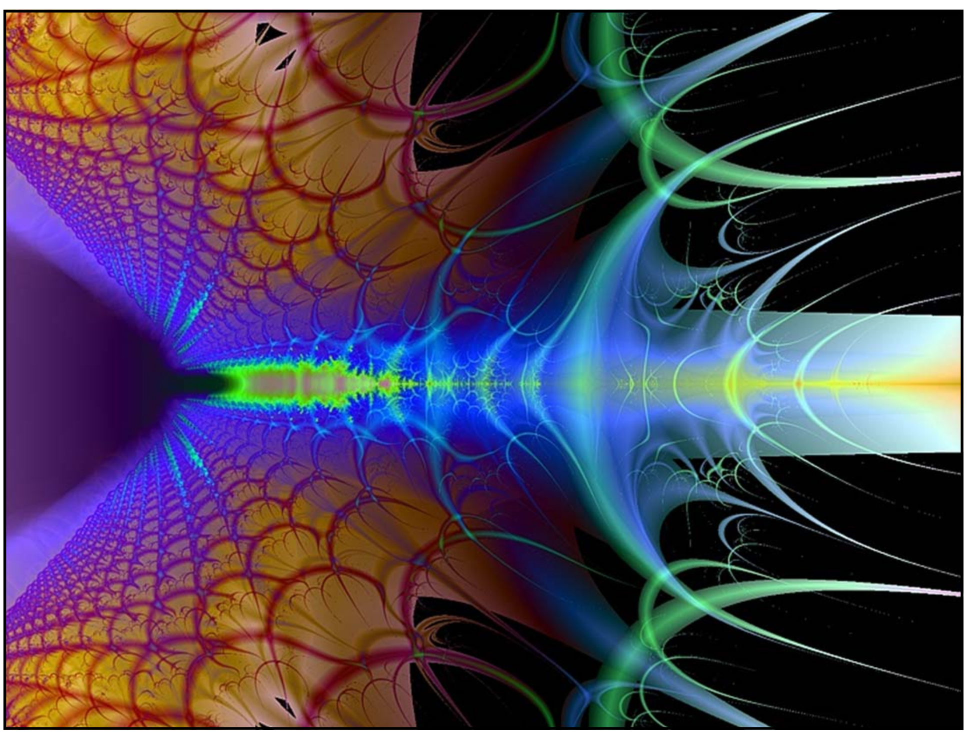
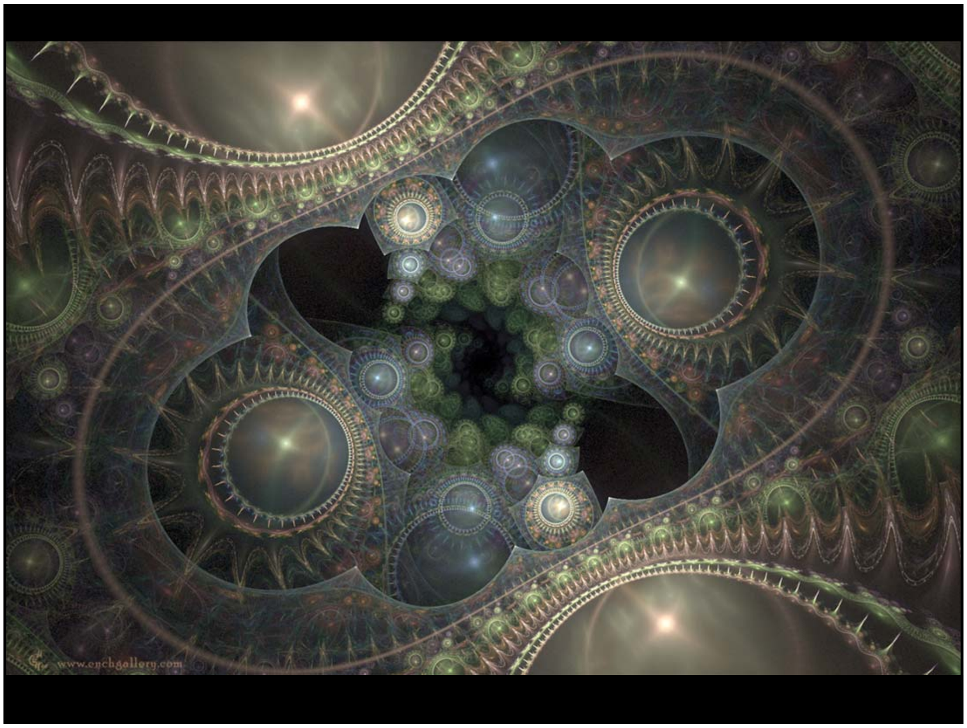

Fractals
Presented by
Joel Castellanos, Lecturer
Determent of Computer Science
University of New Mexico





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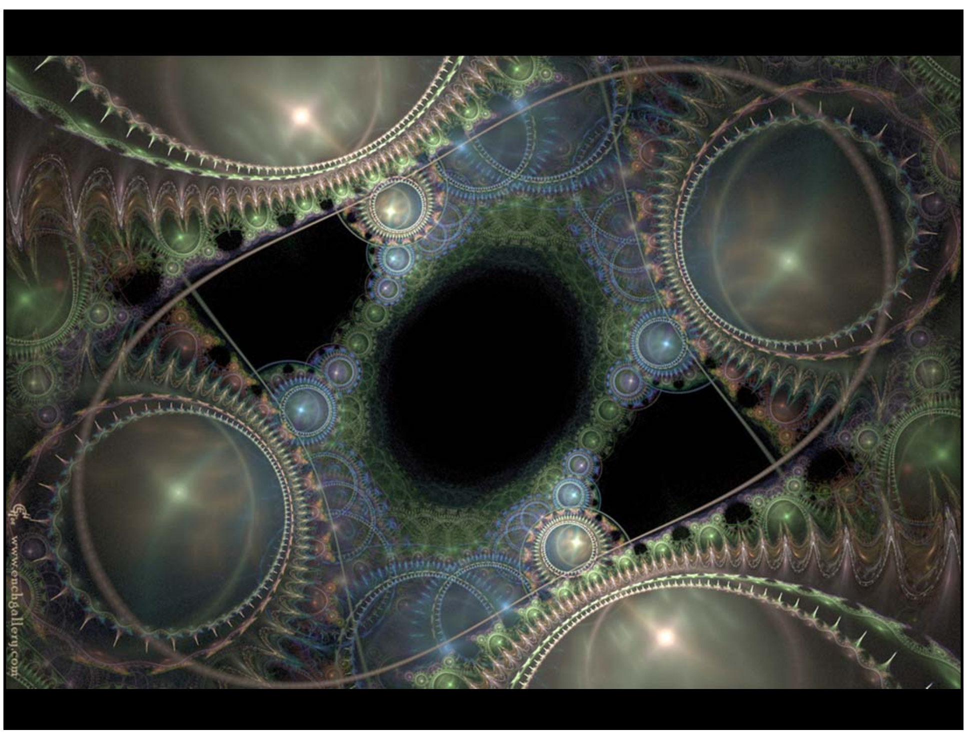
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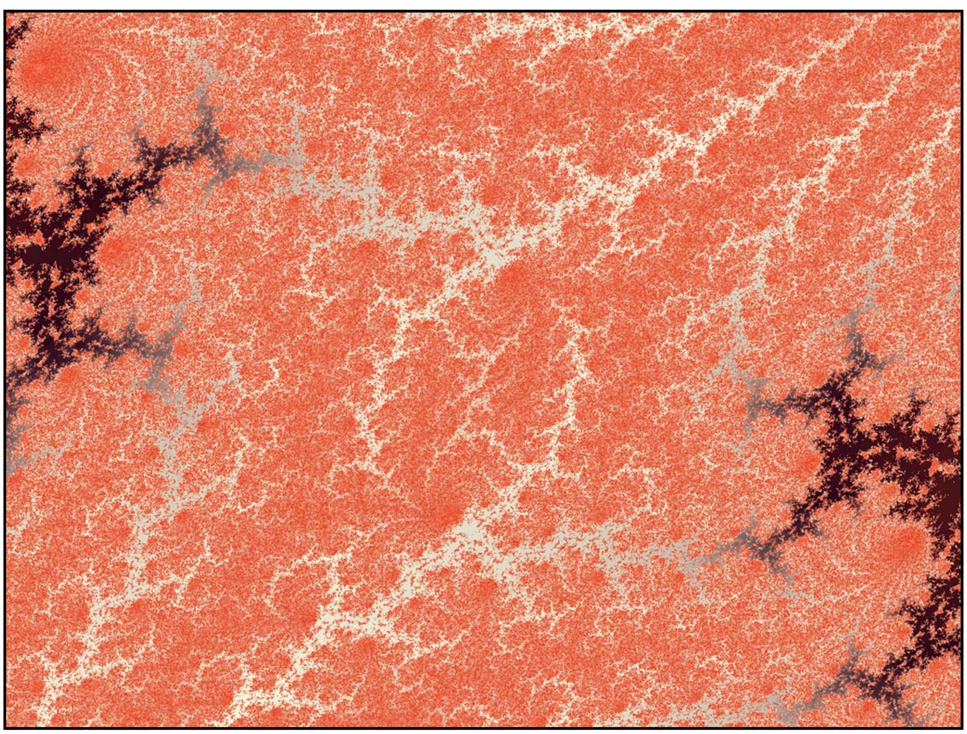
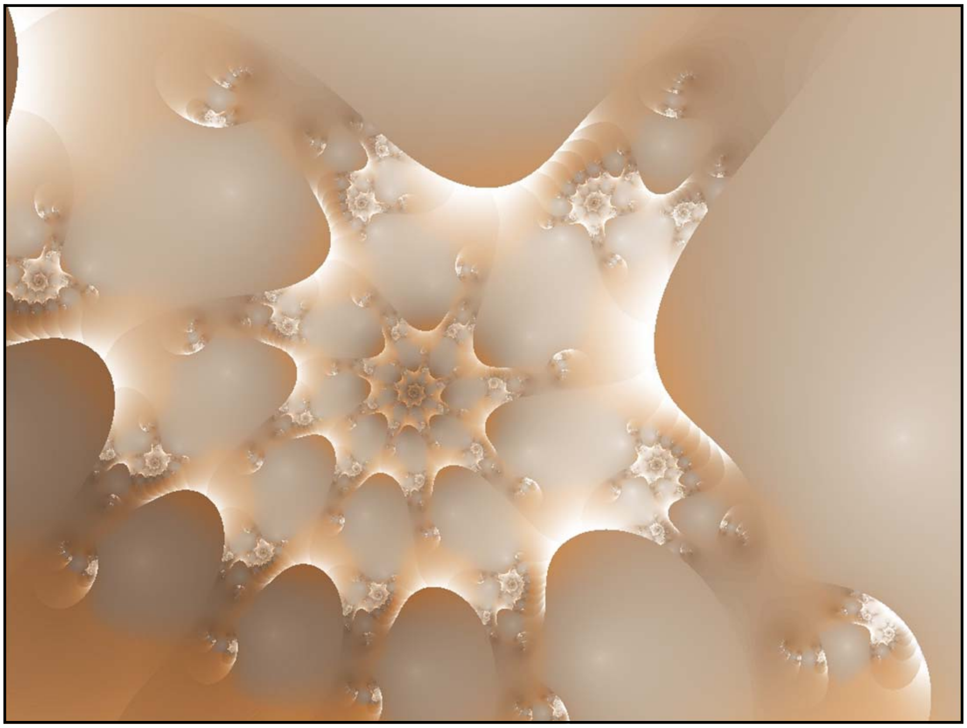
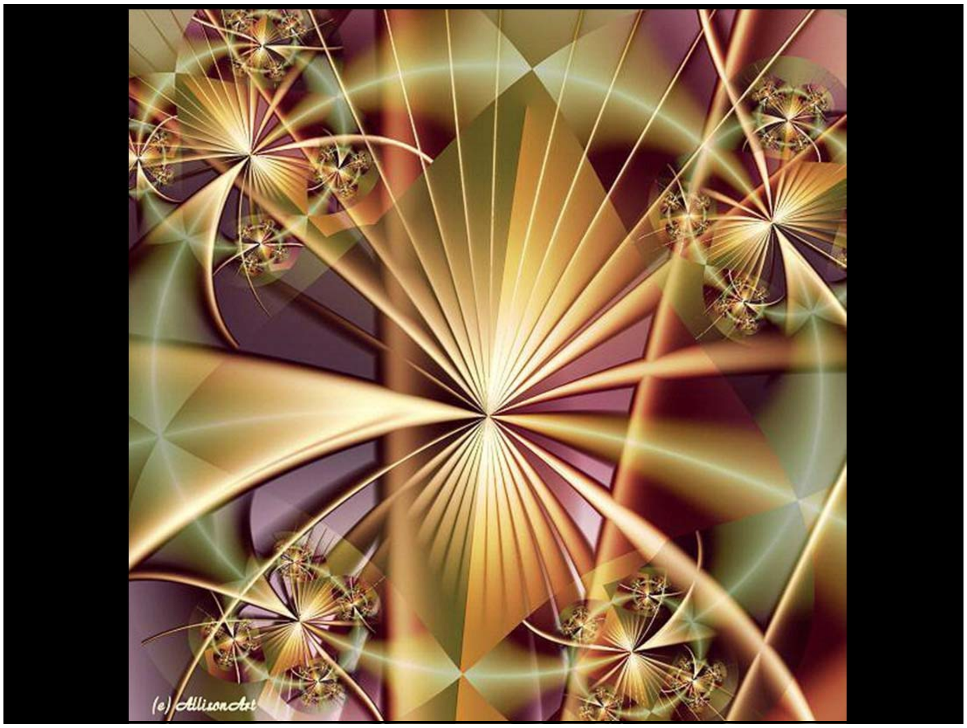


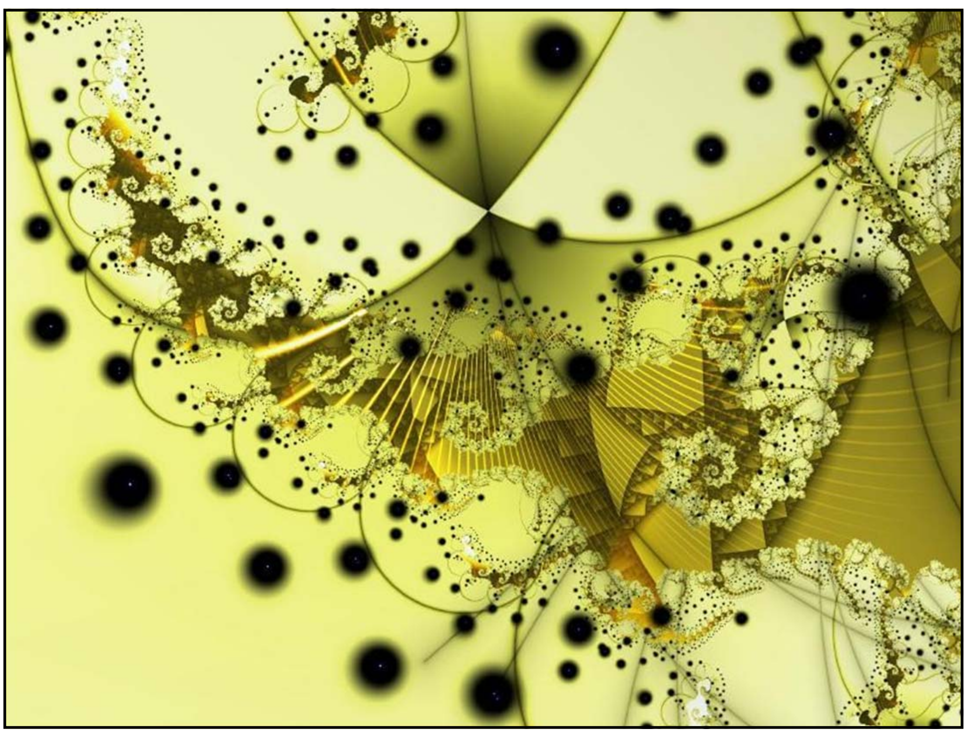



(c) AllisonArt


(c) AllisonArt


(c) AllisonArt


(c) 2005 Linda Allison



What is a Fractal?

A *fractal* is something that is ***Self Similarity on Multiple Scales.***

Something is ***fractal*** when little parts resemble big parts.

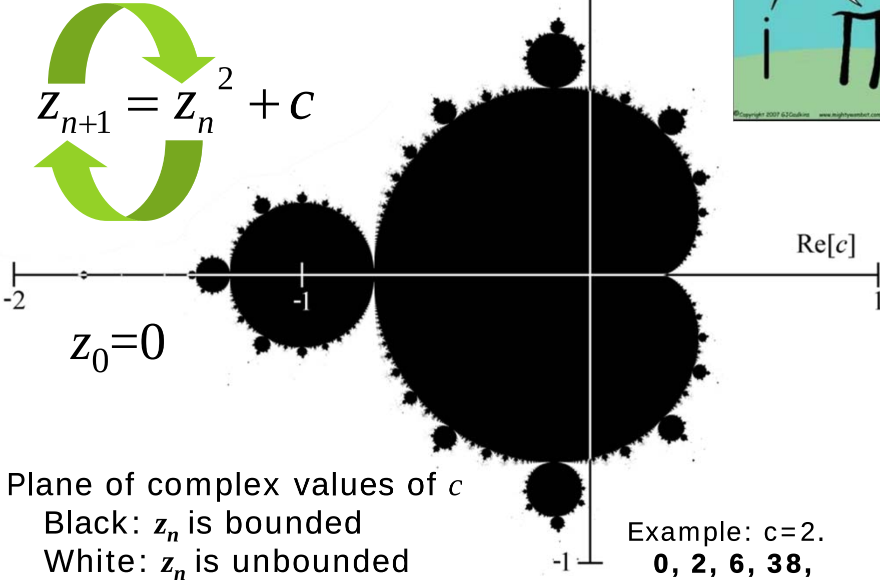
Natural fractals (such as trees and mountains) are self similar on a finite number of scales.

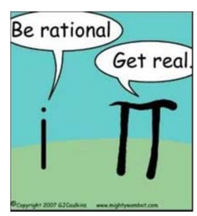
Mathematical fractals are self-similar on endless scales.

Mandelbrot Set

$$z_{n+1} = z_n^{\,2} + c$$

$$z_0 = 0$$





Plane of complex values of c
- Black: z_n is bounded
- White: z_n is unbounded

Example: c=2.
0, 2, 6, 38, 1446, …

Adding Complex Numbers

Real Part | Imaginary Part

$$
\begin{aligned}
c_1 &= 2+4\sqrt{-1} = 2+4i \\
c_2 &= 3+i \\
\\
c_3 &= c_1+c_2 = (2+4i)+(3+i) \\
&= 5+5i
\end{aligned}
$$

Multiplying Complex Numbers (FOIL)

$$
\begin{aligned}
c_1 &= 2+4i \\
c_2 &= 3+i \\
c_3 &= c_1 \times c_2 = (2+4i)(3+i) \\
&= 2(3)+2i+4i(3)+4i(i) \\
&= 6+2i+12i+4\sqrt{-1}\sqrt{-1} \\
&= 6+14i-4 \\
&= 2+14i
\end{aligned}
$$

Squaring Complex Numbers (FOIL)

$$
\begin{aligned}
c &= 0.5+0.5i \\
\\
c^2 &= (0.5+0.5i)(0.5+0.5i) \\
&= 0.5(0.5)+0.5(0.5i)+0.5i(0.5)+(0.5i)(0.5i) \\
&= 0.25+0.25i+0.25i+0.25i^2 \\
&= 0.25+0.5i-0.25 \\
&= 0.5i
\end{aligned}
$$

Squaring Complex Numbers General Formula

$$
\begin{aligned}
c &= a+bi \\
\\
c^2 &= (a+bi)(a+bi) \\
&= a(a)+a(bi)+bi(a)+(bi)(bi) \\
&= a^2+2abi+b^2i^2 \\
&\boxed{= \underbrace{a^2-b^2}_{a'}+\underbrace{2abi}_{b'}}
\end{aligned}
$$

Magnitude of Complex Number

$$c = a + bi$$

$$\begin{aligned} \|c\| &= \sqrt{(a-0)^2 + (b-0)^2} \\ &= \sqrt{a^2 + b^2} \end{aligned}$$

Note: When calculating the Mandelbrot set, it is more efficient to check if the magnitude squared (a^2+b^2) is greater than the cut-off squared.

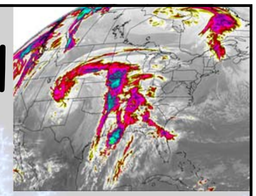
Does a Solution Exist?

What ***laws of nature*** applied to what ***data measurements*** at what ***level of precision*** are required to determine which way the ball will fall?

Humans have enjoyed fantastic success with being able to predict and control physical phenomenon by using ever improving data collection and data processing.

Is every such question that we cannot yet answer simply out of our current reach ***or are some answers unknowable?***

Sensitivity to Initial Conditions



In 1961, Edward Lorenz was using a numerical computer model to rerun a weather prediction, when, as a shortcut on a number in the sequence, he entered the decimal **.506** instead of entering the full **.506127** the computer would hold.

The result was a completely different weather pattern!

Lorenz published his findings in a 1963 paper for the New York Academy of Sciences noting that:

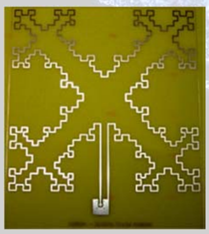
"*One meteorologist remarked that if the theory were correct, one flap of a seagull's wings could change the course of weather forever.*"

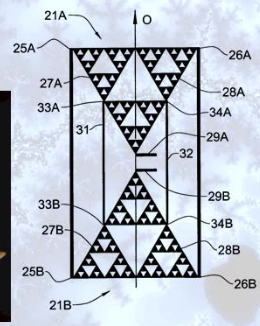
Chaotic Systems and the Butterfly Effect

- A ***chaotic system*** is one in which small differences in the initial condition of a dynamical system may produce large variations in the long term behavior of the system.
- The ***butterfly effect*** is a metaphor that encapsulates the concept of sensitive dependence on initial conditions.
- Although this may appear to be an esoteric and unusual behavior, it is exhibited by very simple systems.
- How *small* are "*small differences*"?
- How is a chaotic system different from any system where we simply need more data, more accurate data, and more accurate theories?

Fractal Antennas in Cell Phones

- About 15 years ago, cell phones all had large antennas that you typically pulled out before making a phone call.
- It is still true that the longer the antenna, the better the reception, but with special fractals called "space filling curves", very long antennas can fit inside very small spaces.

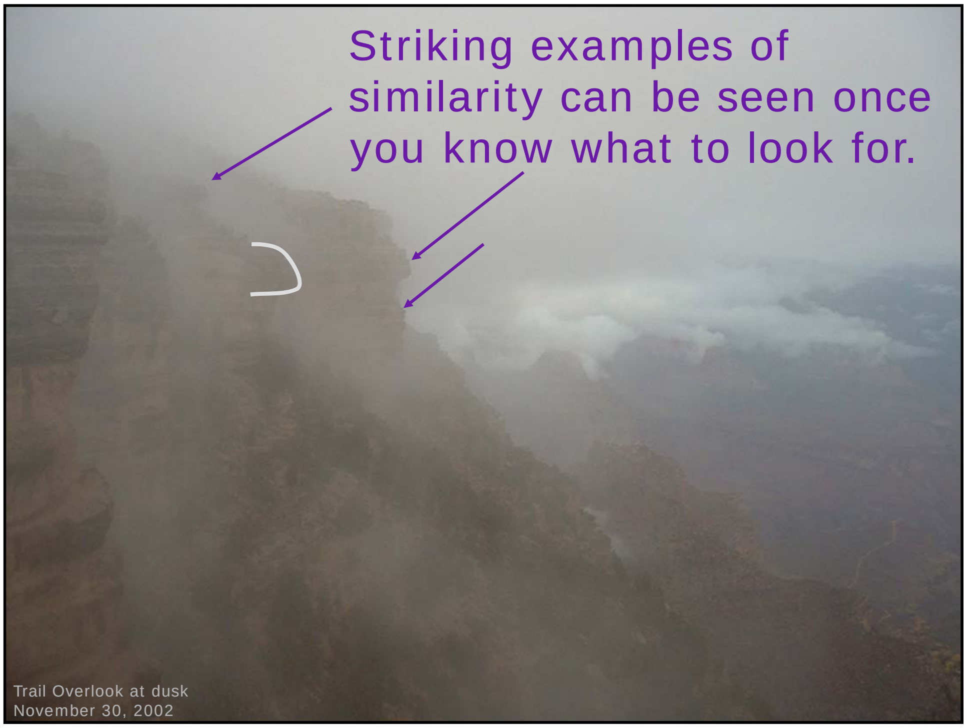


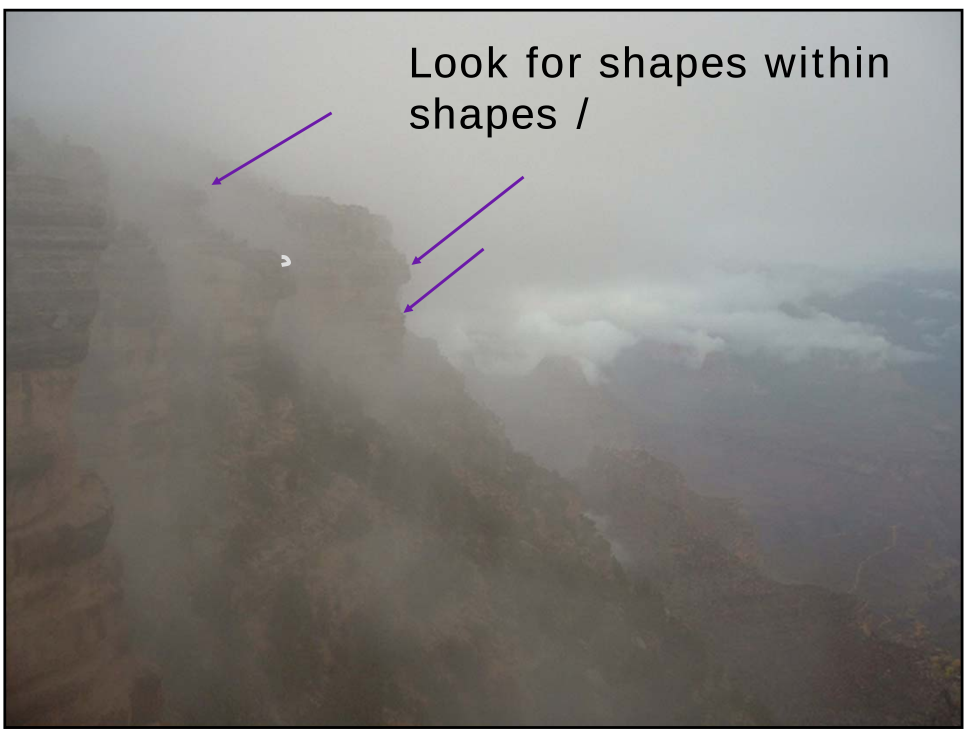


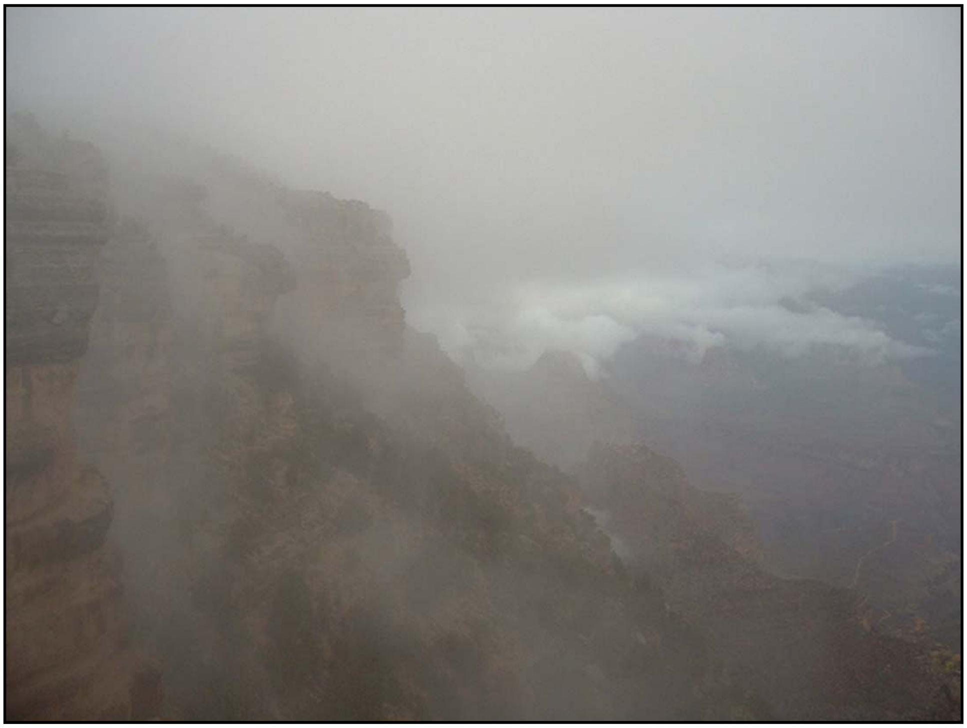
A Fractal antenna can be used to receive a very wide range of frequencies.

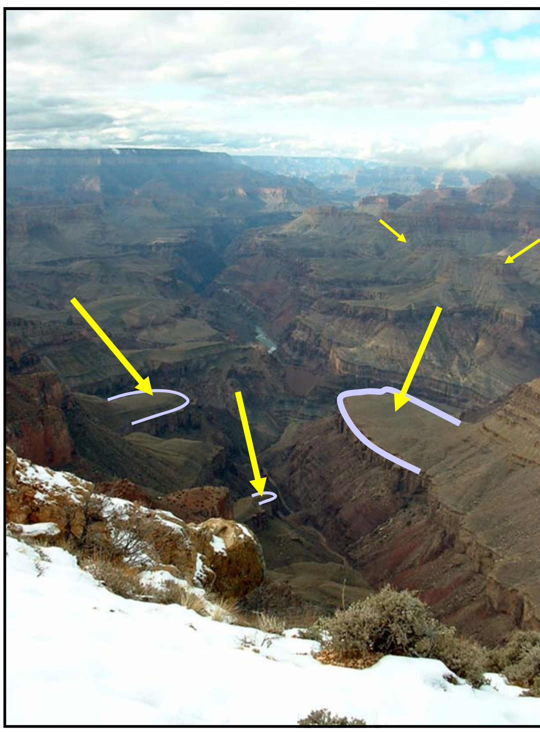
Natural Fractals in the Grand Canyon

Photographs by Paul Bourke


Striking examples of similarity can be seen once you know what to look for.
Trail Overlook at dusk
November 30, 2002

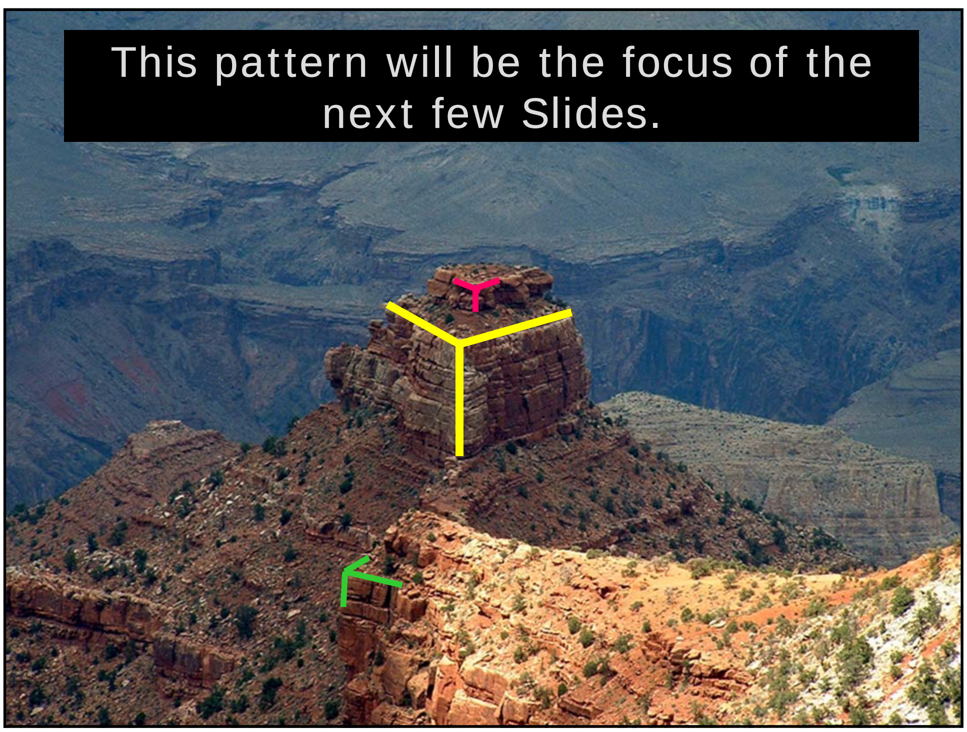

Look for shapes within shapes /

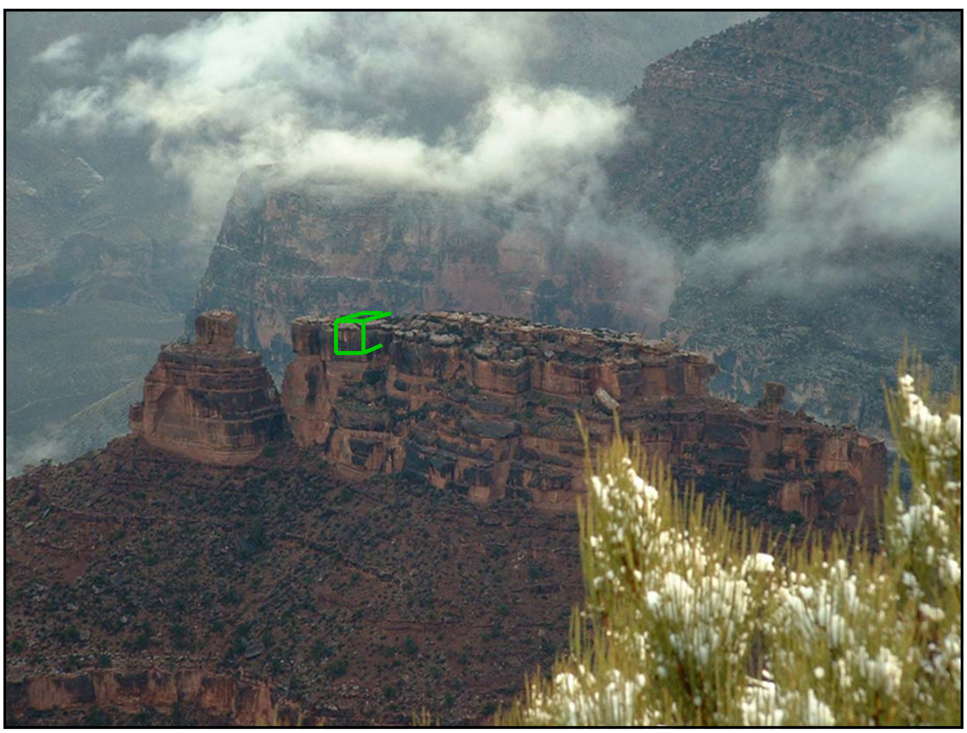


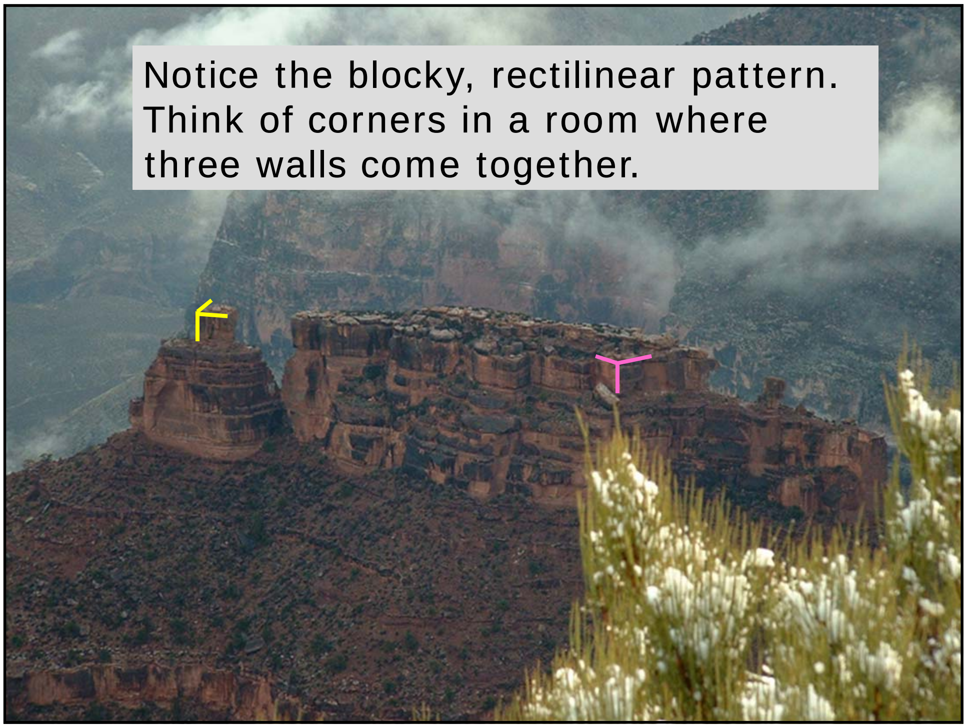


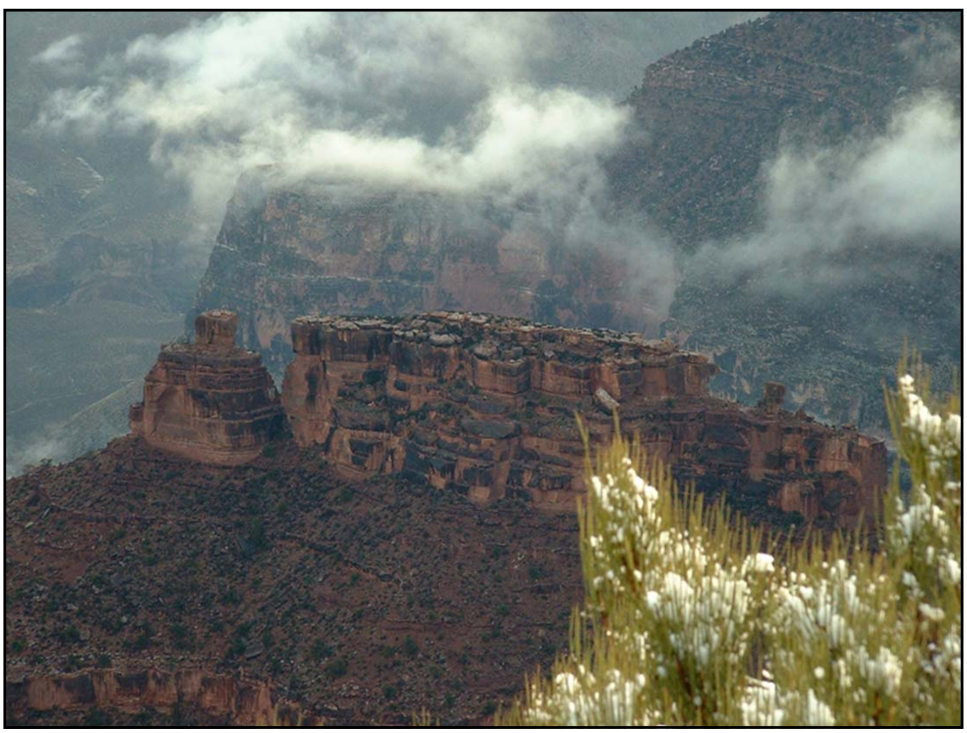
Learn to Recognize Similarity

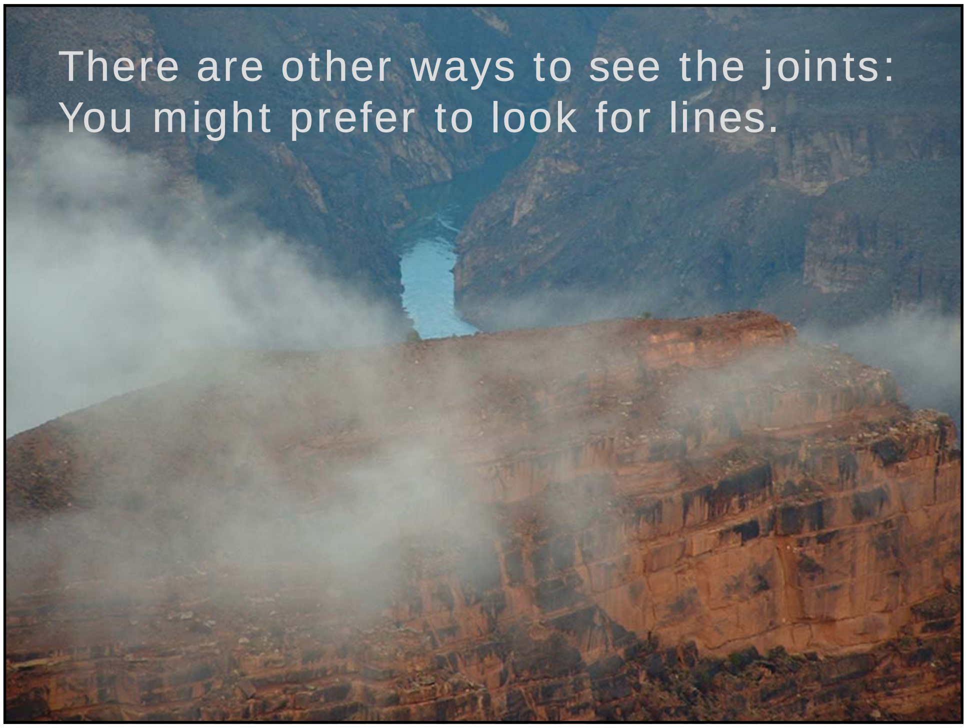
…in the Grand Canyon specifically …in Nature generally.

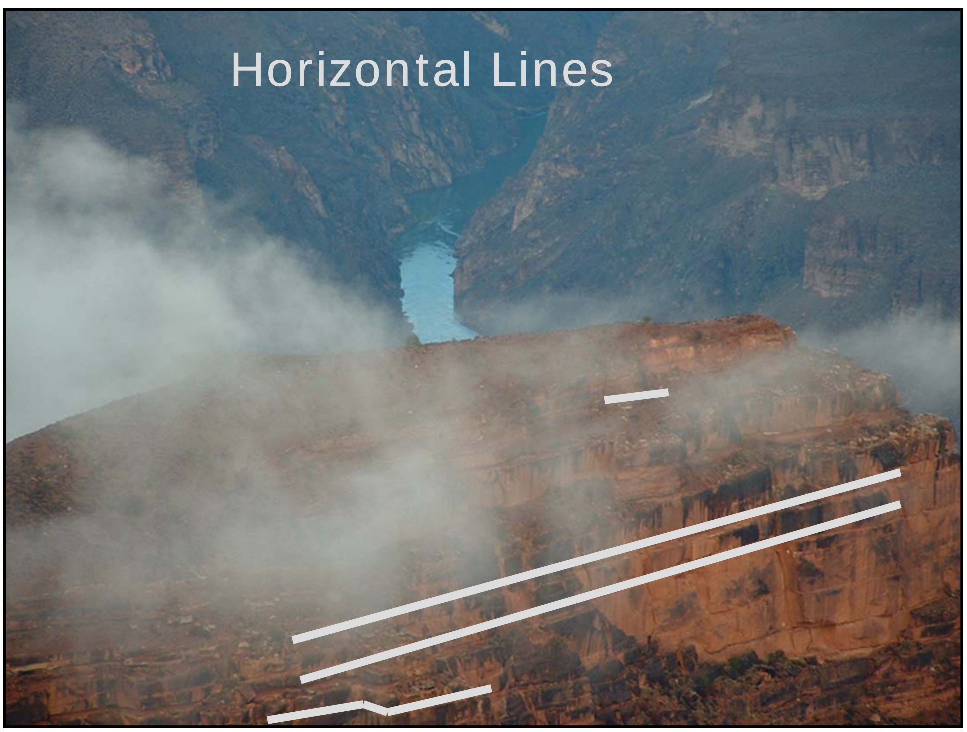




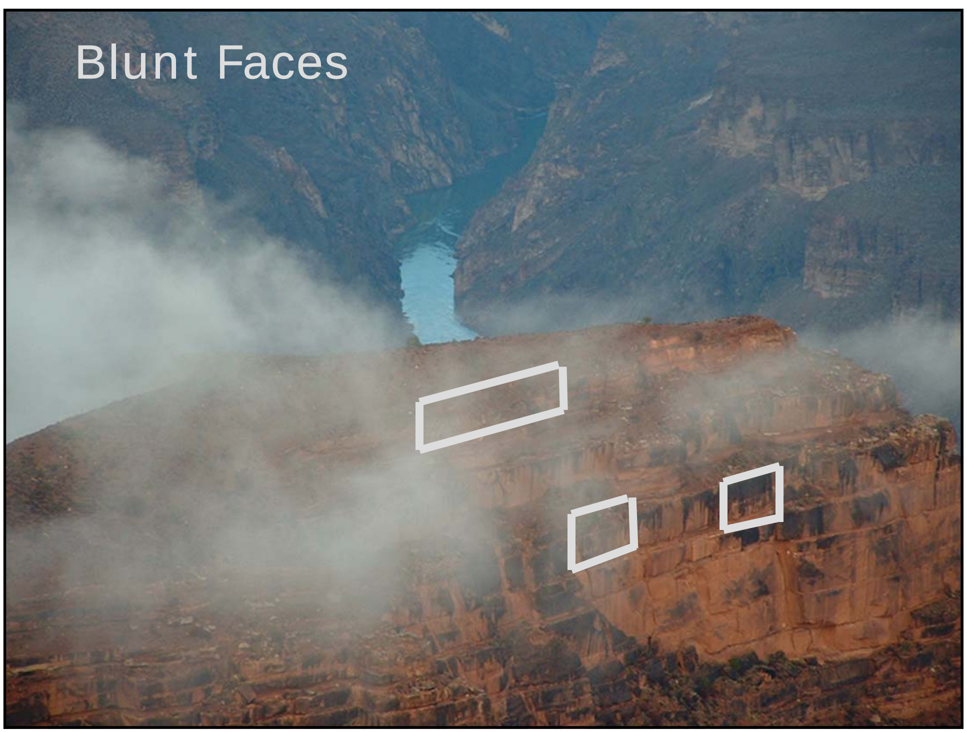

Notice the blocky, rectilinear pattern.
Think of corners in a room where
three walls come together.

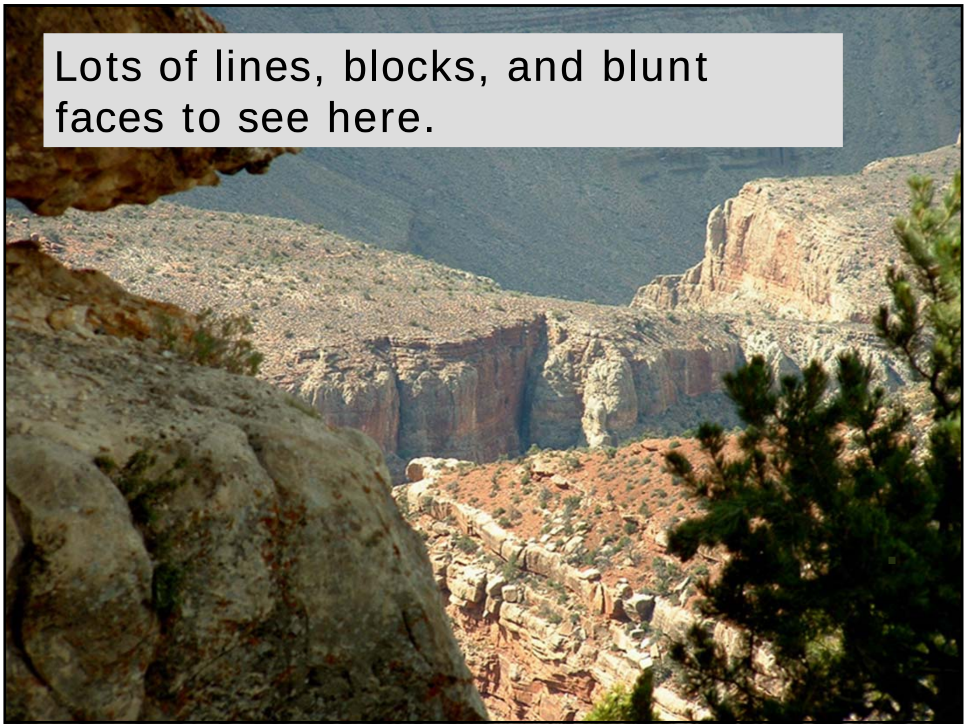


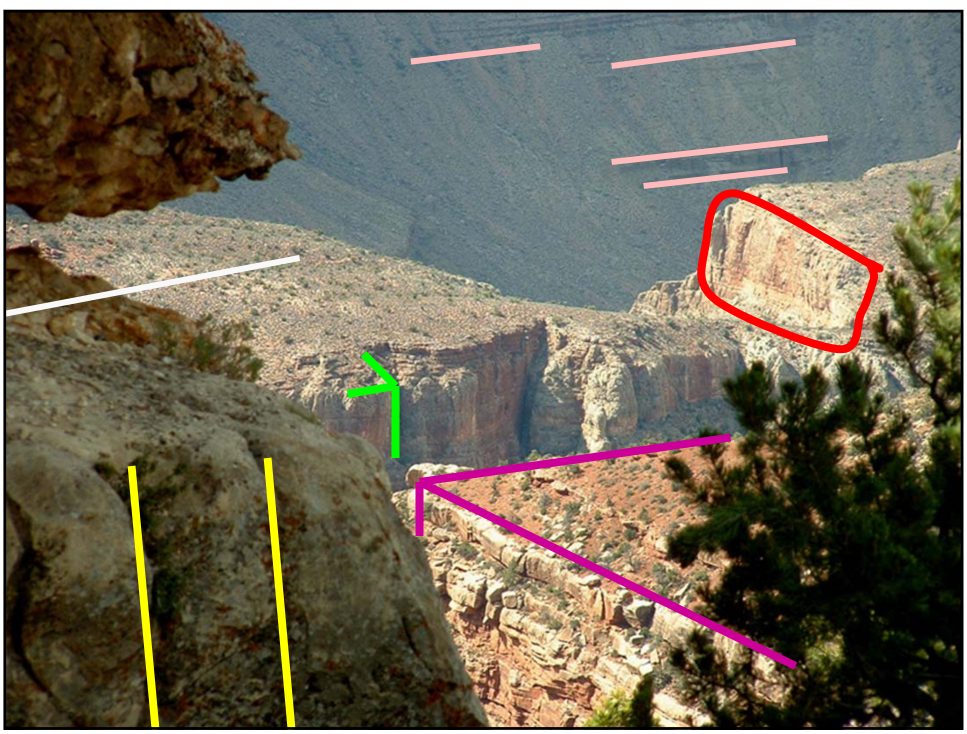

There are other ways to see the joints:
You might prefer to look for lines.

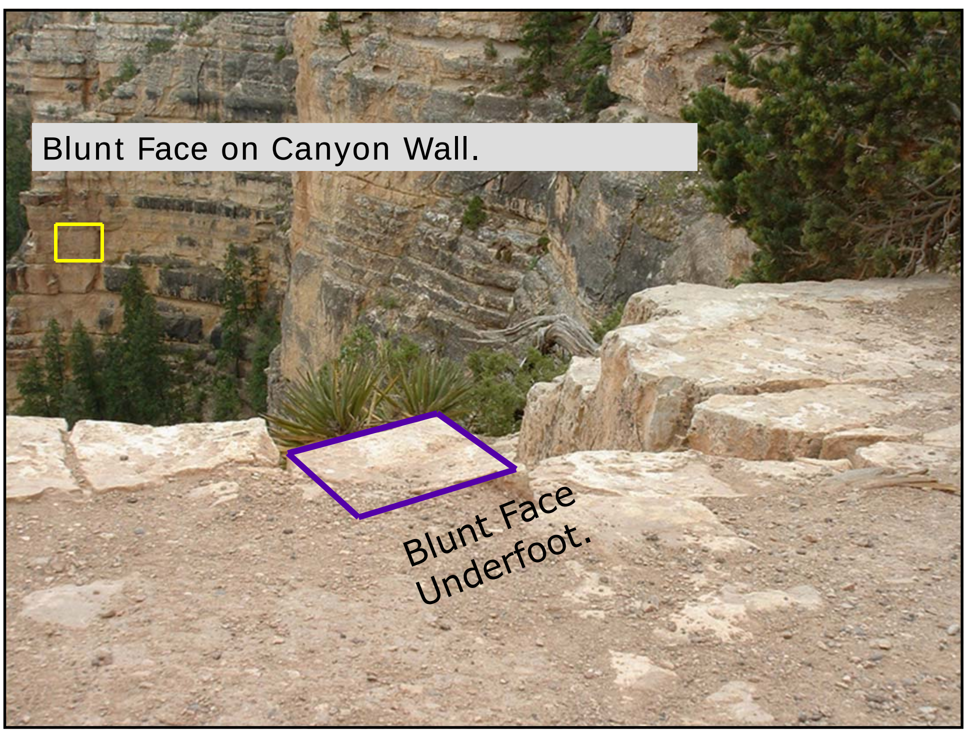

Horizontal Lines

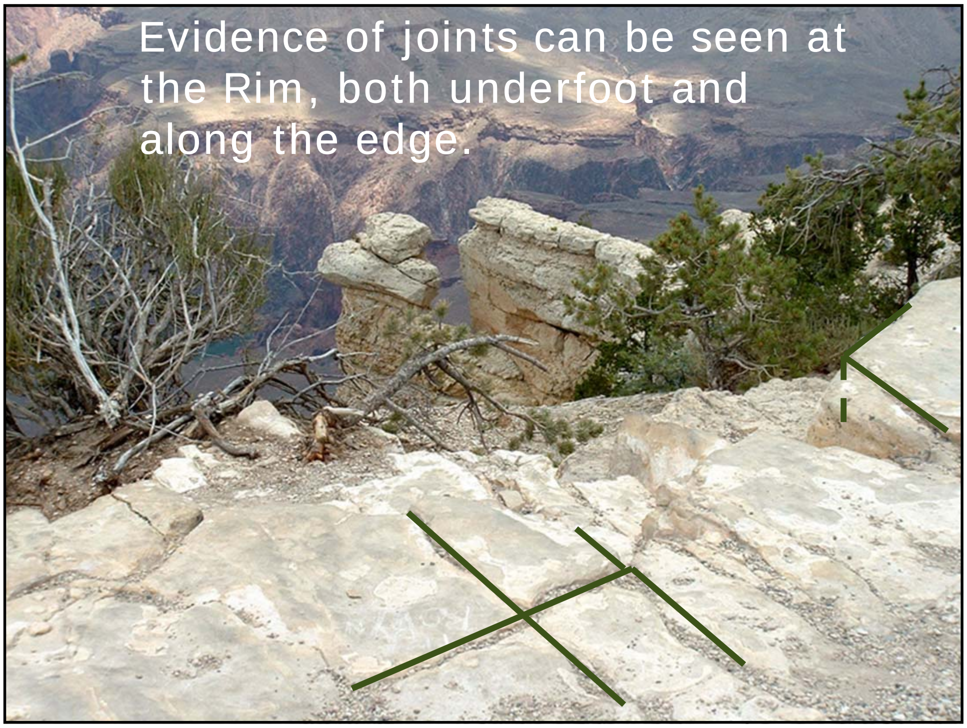

Vertical Lines

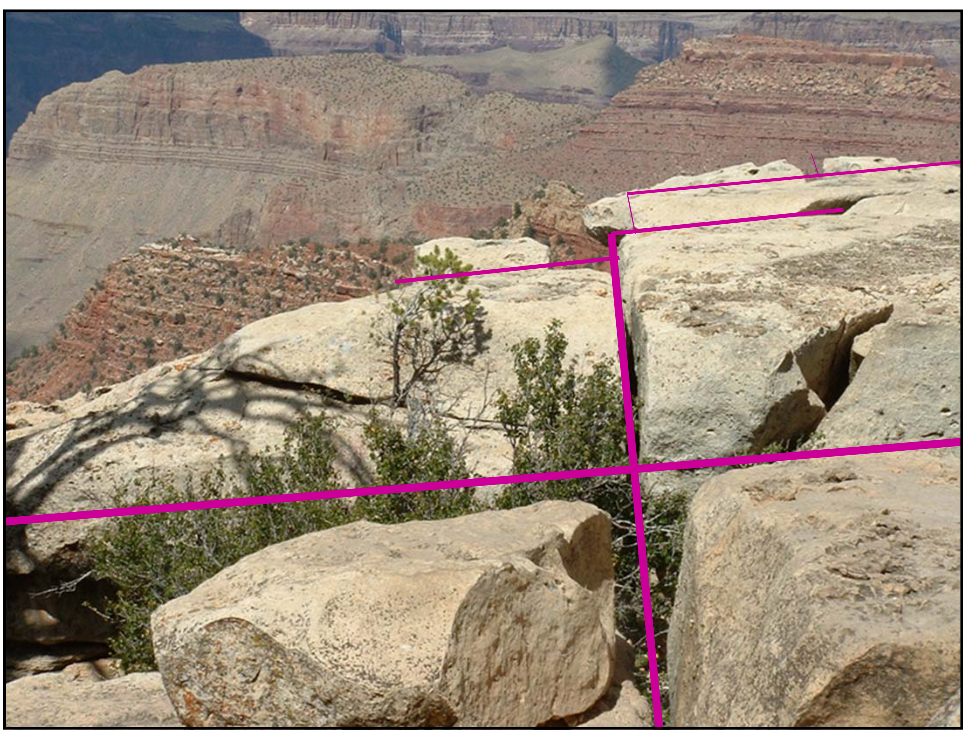

Blunt Faces

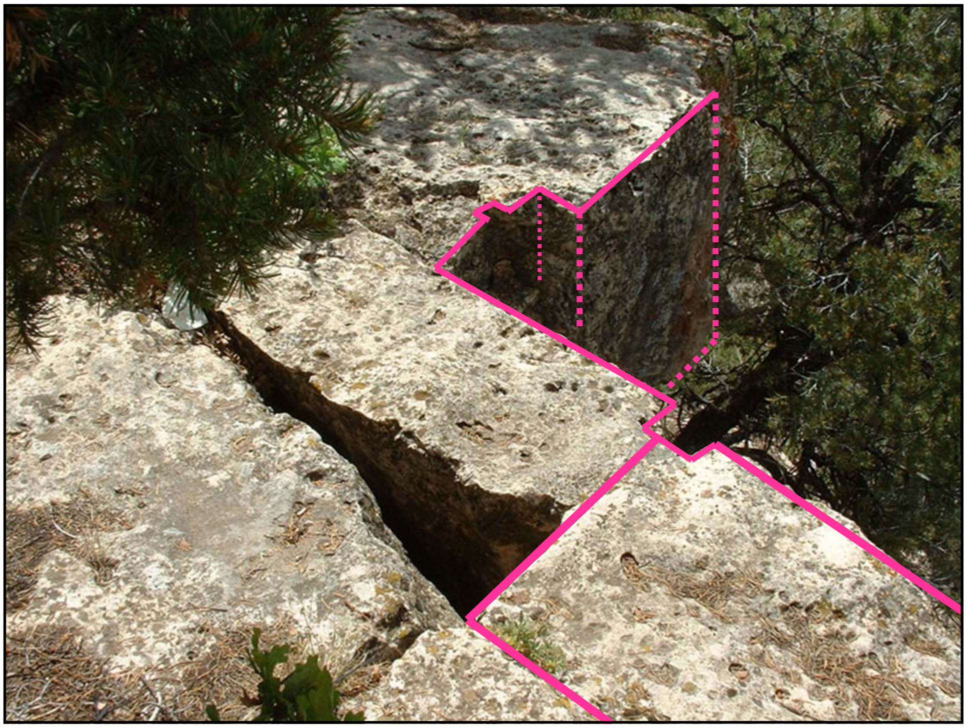

Lots of lines, blocks, and blunt
faces to see here.



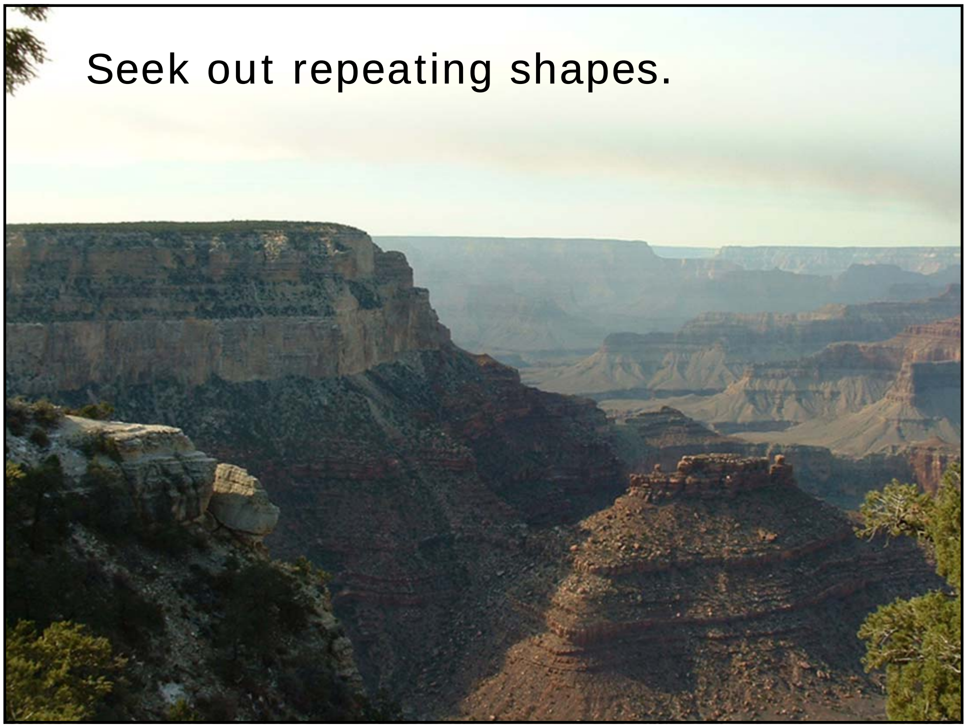

Blunt Face on Canyon Wall.
Blunt Face
Underfoot.

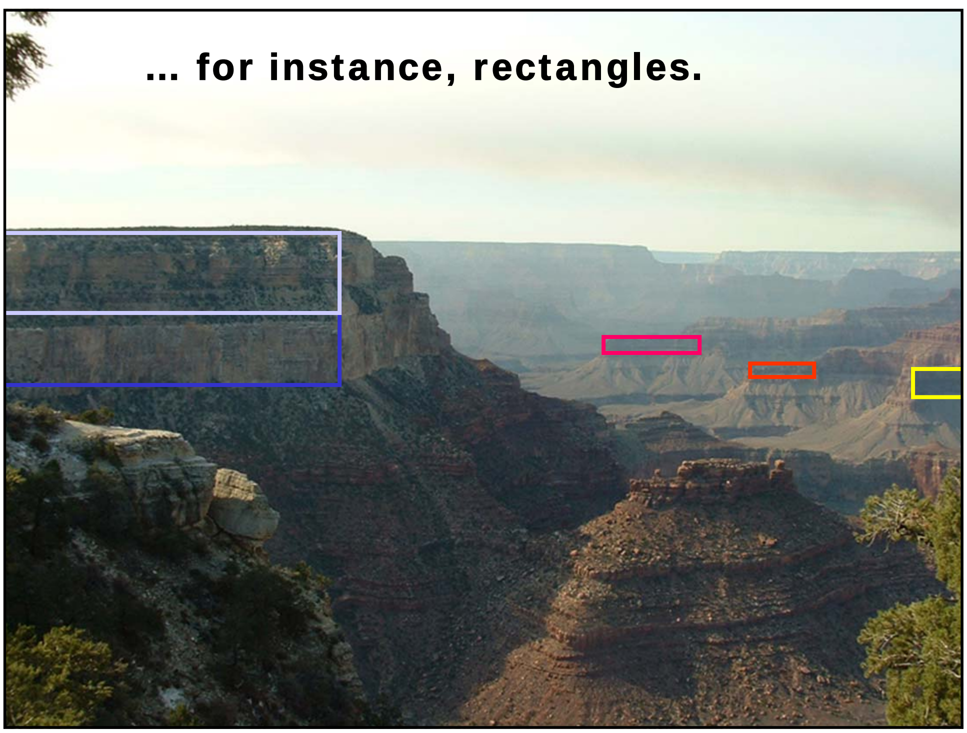

Evidence of joints can be seen at the Rim, both underfoot and along the edge.

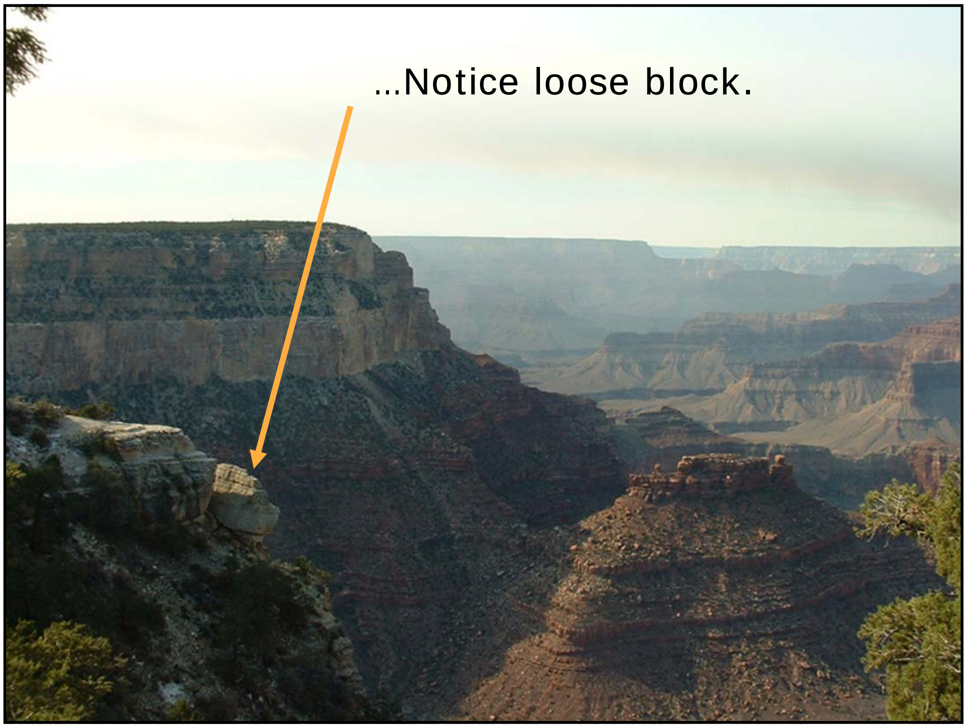


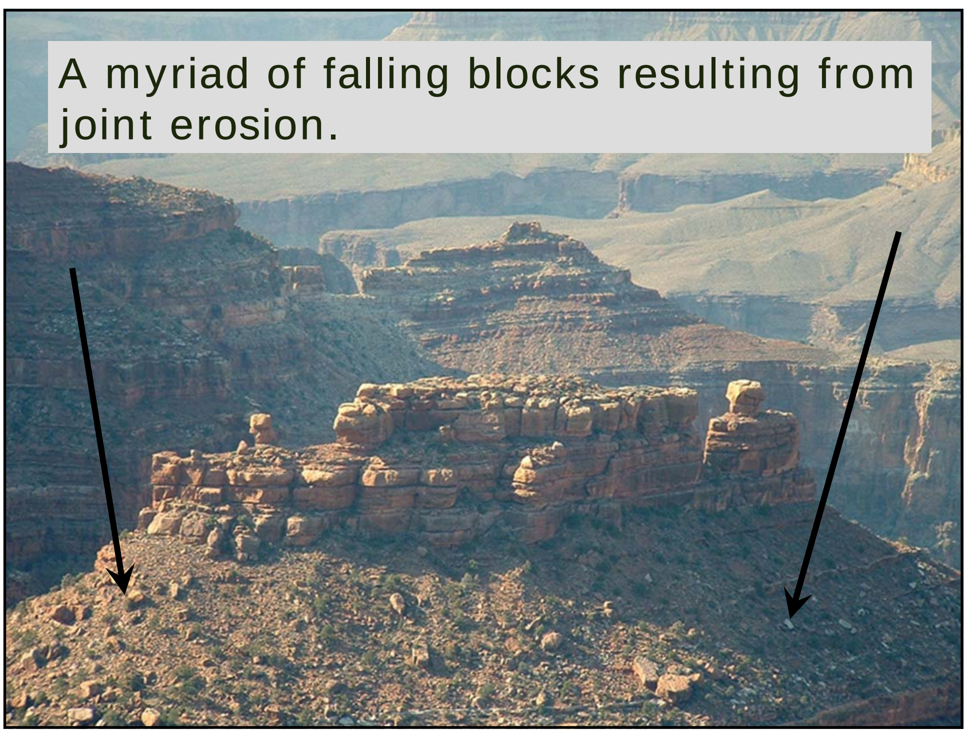


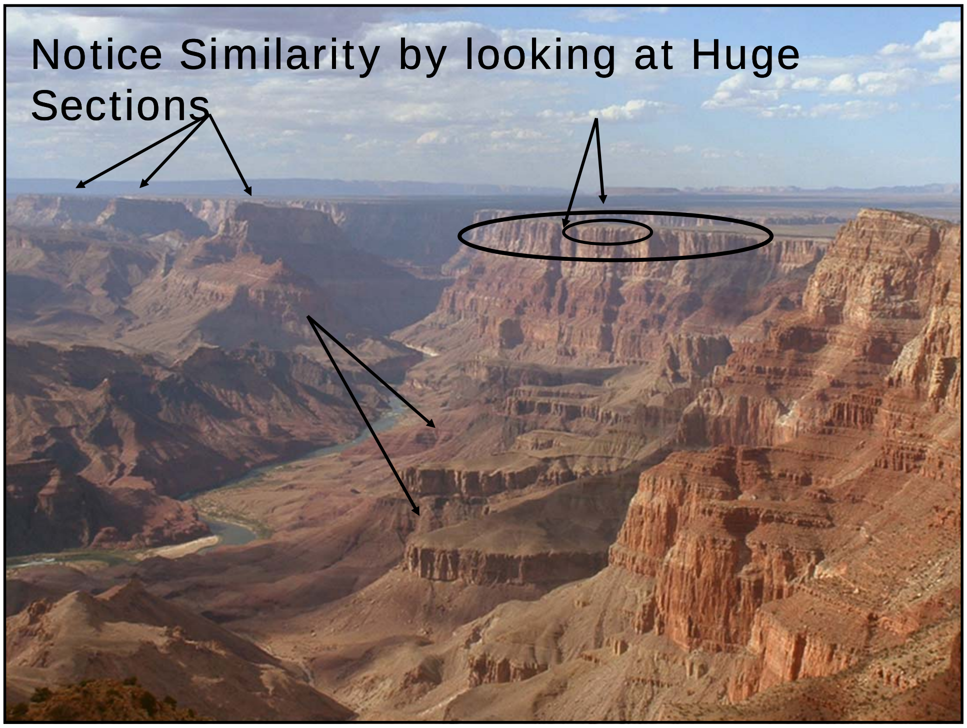

42

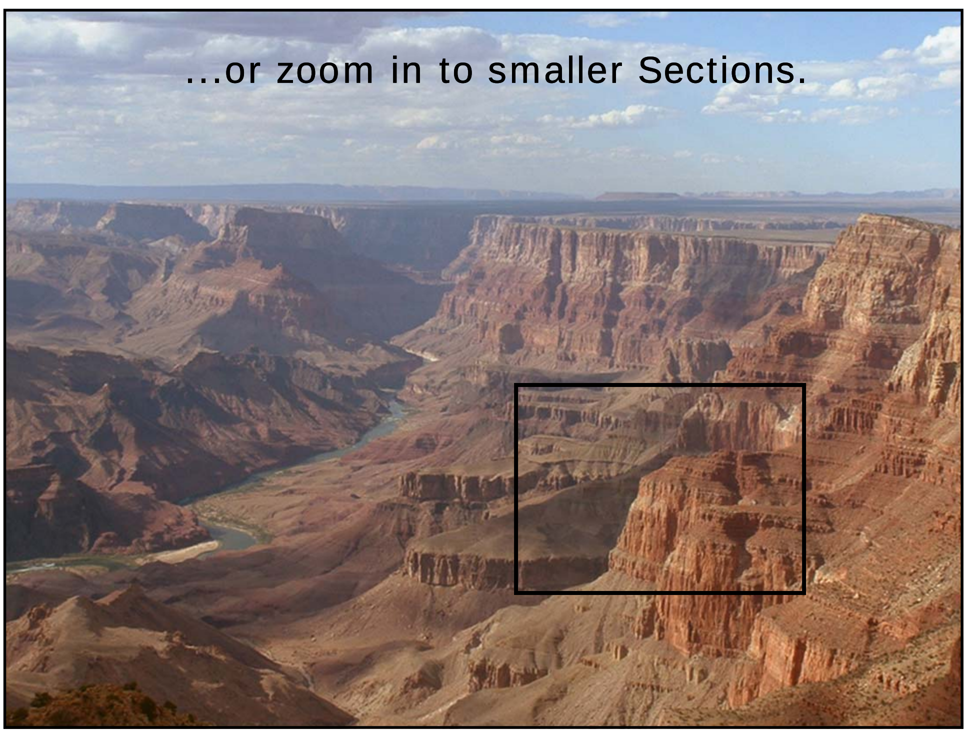

Seek out repeating shapes.

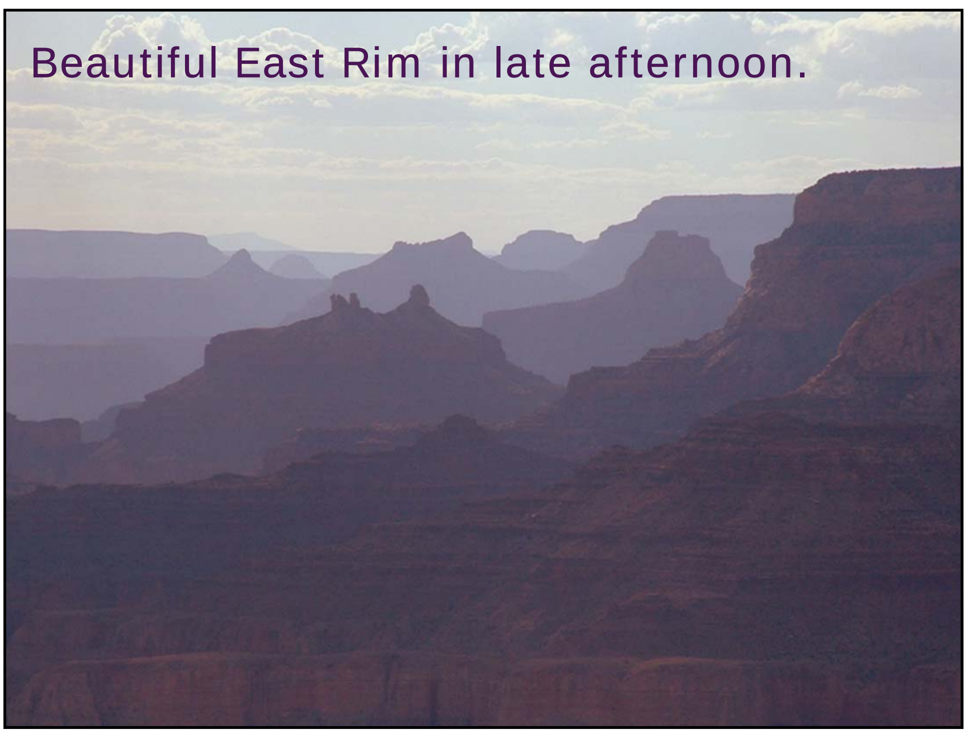

… for instance, rectangles.


...Notice loose block.


A myriad of falling blocks resulting from joint erosion.

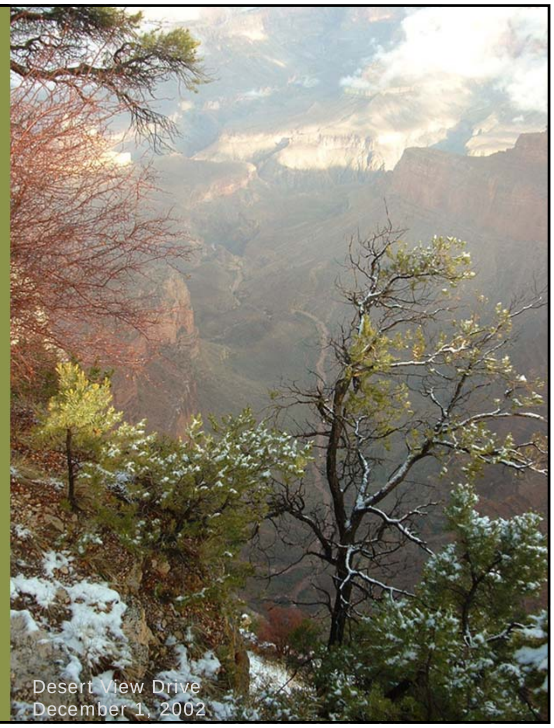

Notice Similarity by looking at Huge Sections


...or zoom in to smaller Sections.

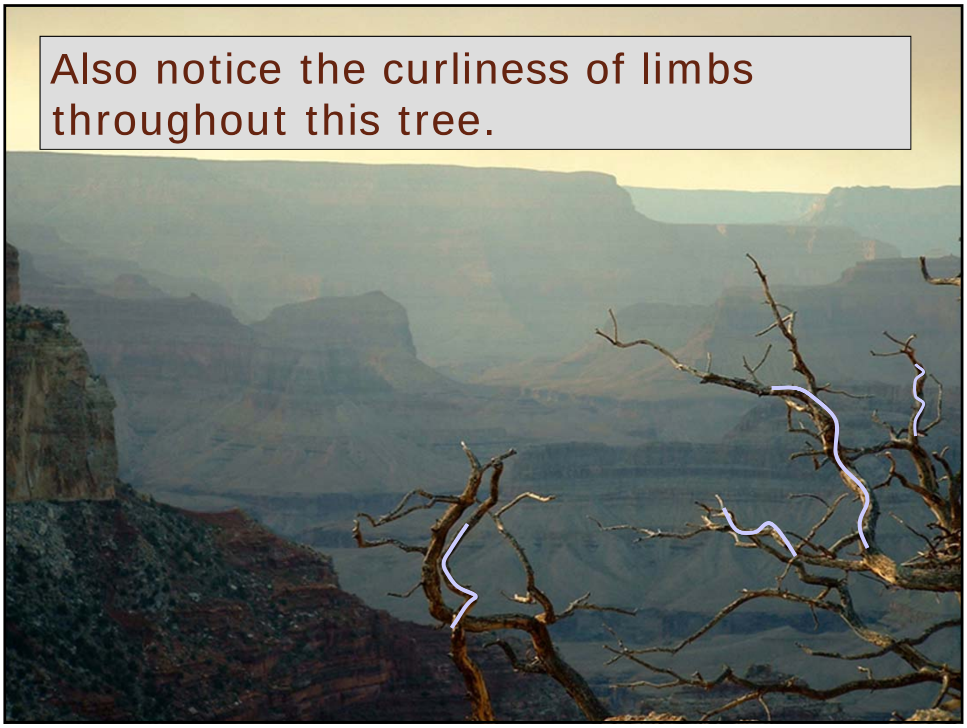

Beautiful East Rim in late afternoon.

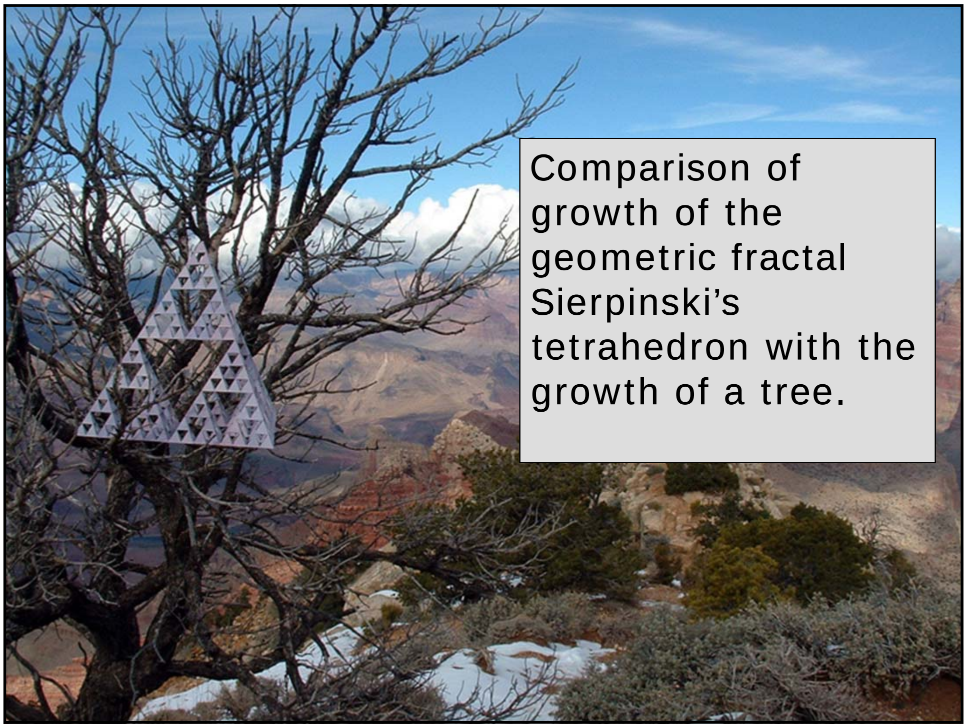
Trees

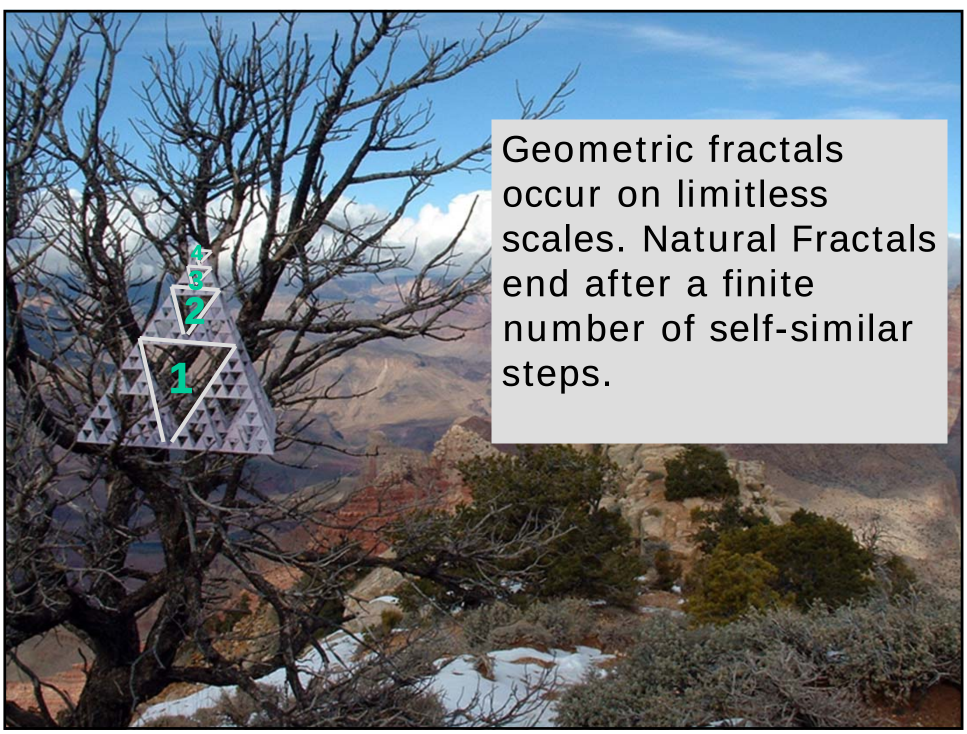
Little Branches Resemble Big Branches.

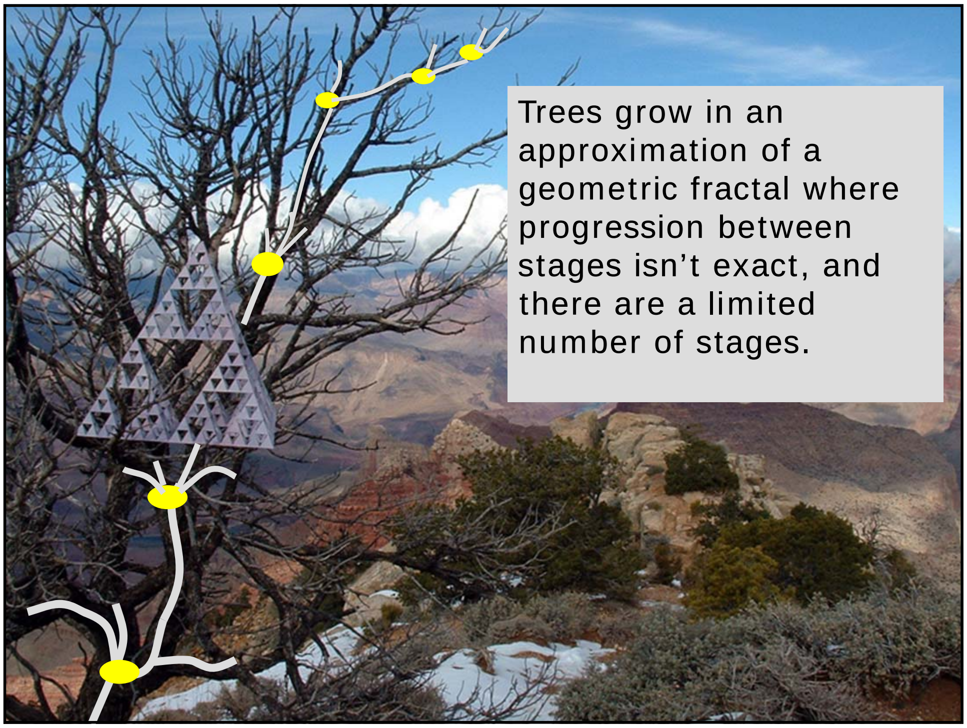


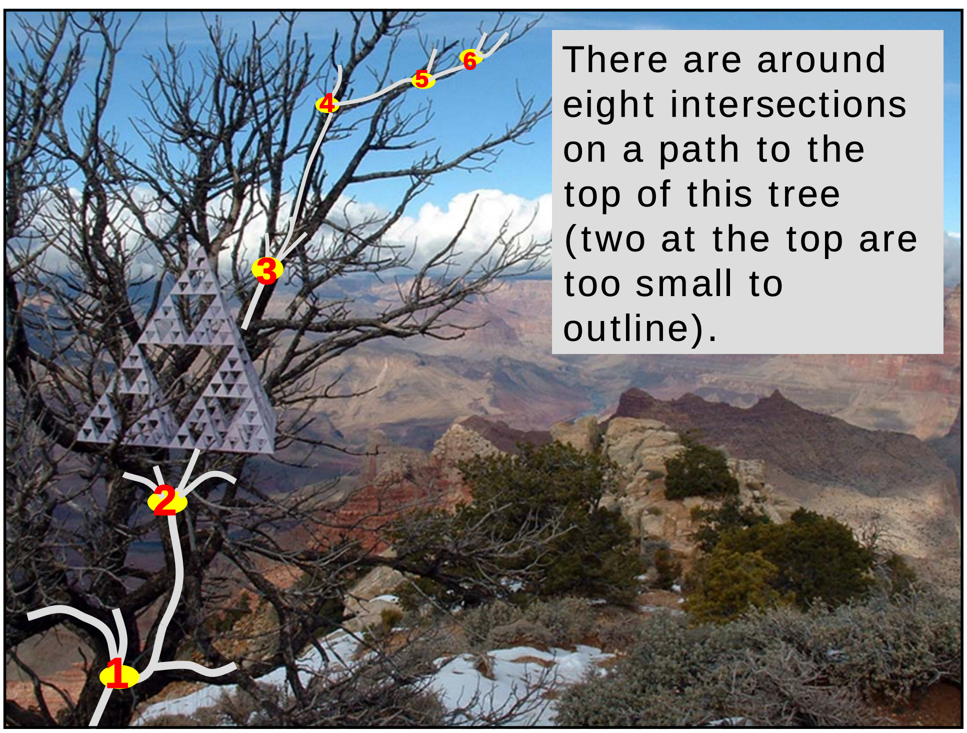
Bare tree limbs are one of the easiest places to observe similarity.

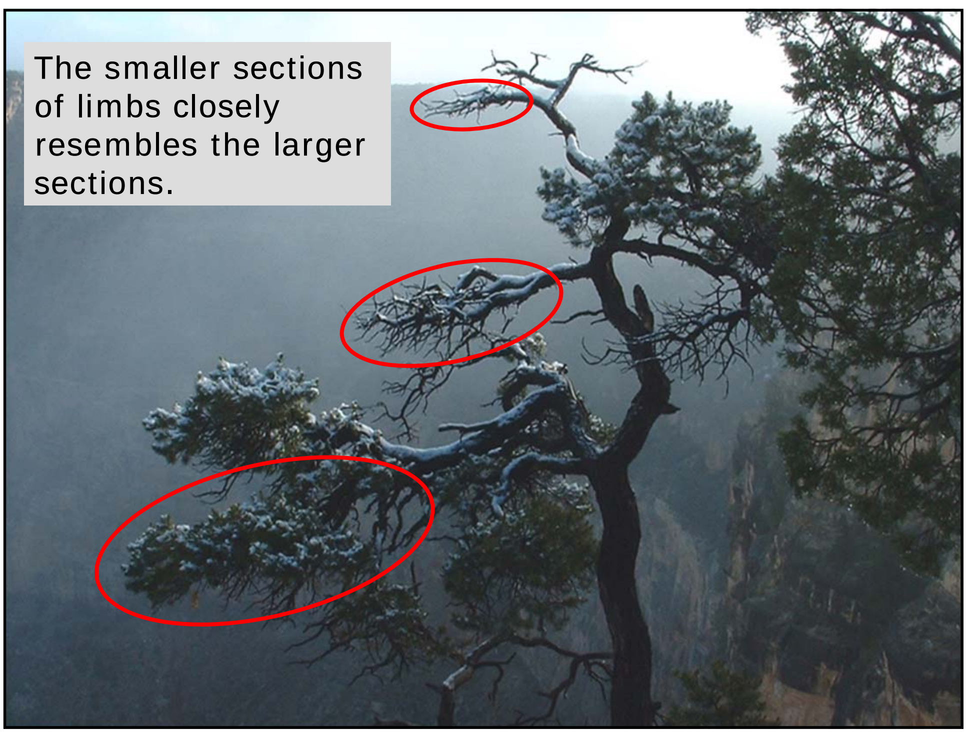

Also notice the curliness of limbs throughout this tree.

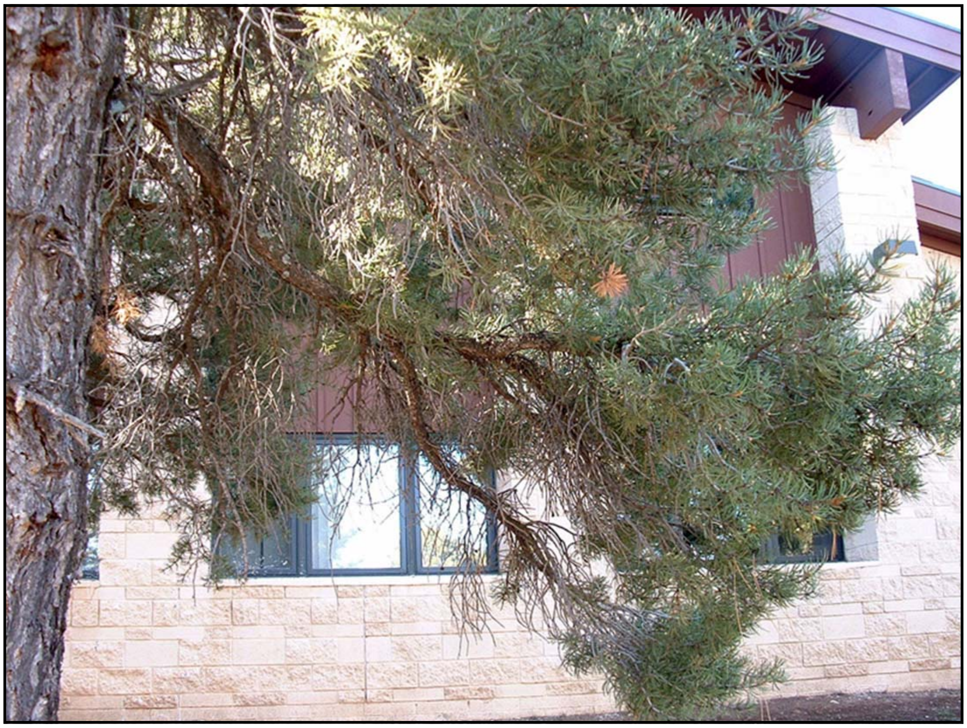

Comparison of growth of the geometric fractal Sierpinski's tetrahedron with the growth of a tree.

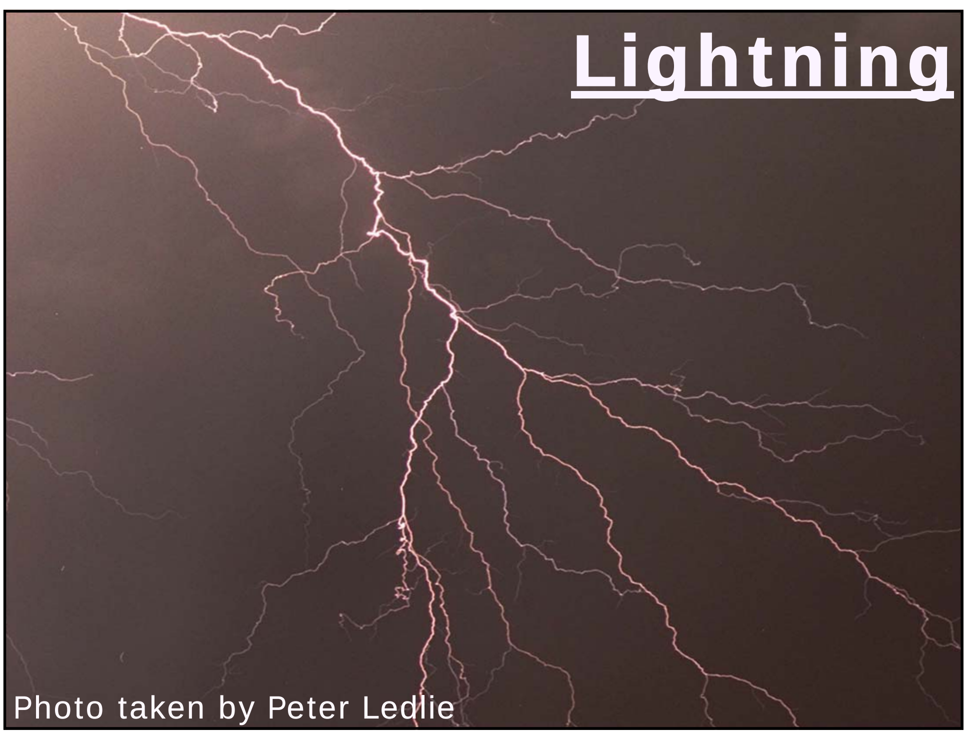

Geometric fractals occur on limitless scales. Natural Fractals end after a finite number of self-similar steps.
4
3
2
1

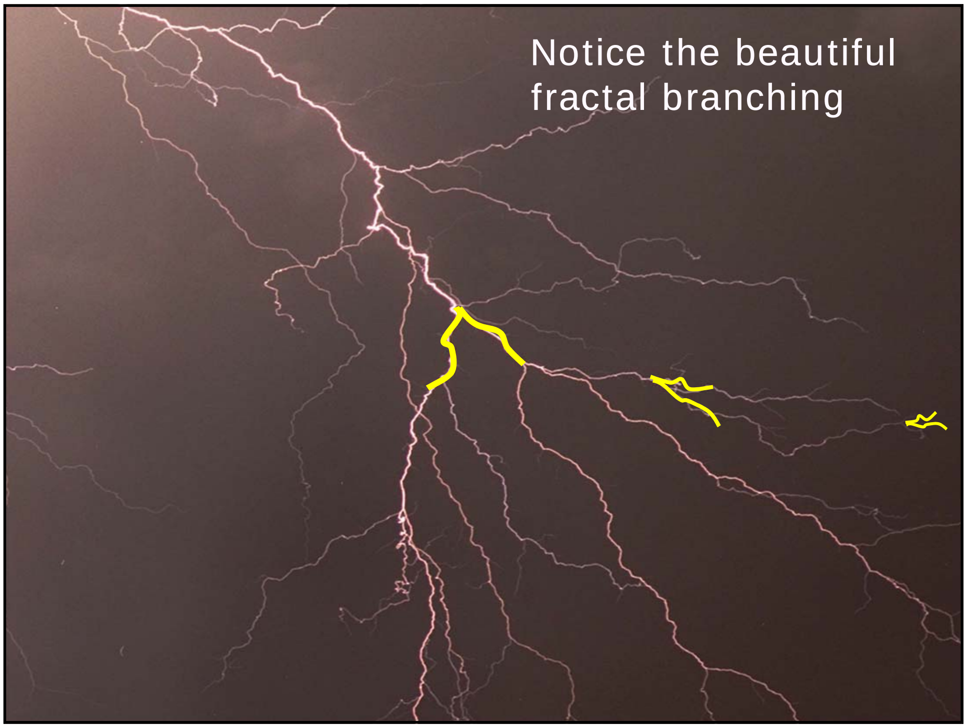

Trees grow in an approximation of a geometric fractal where progression between stages isn't exact, and there are a limited number of stages.

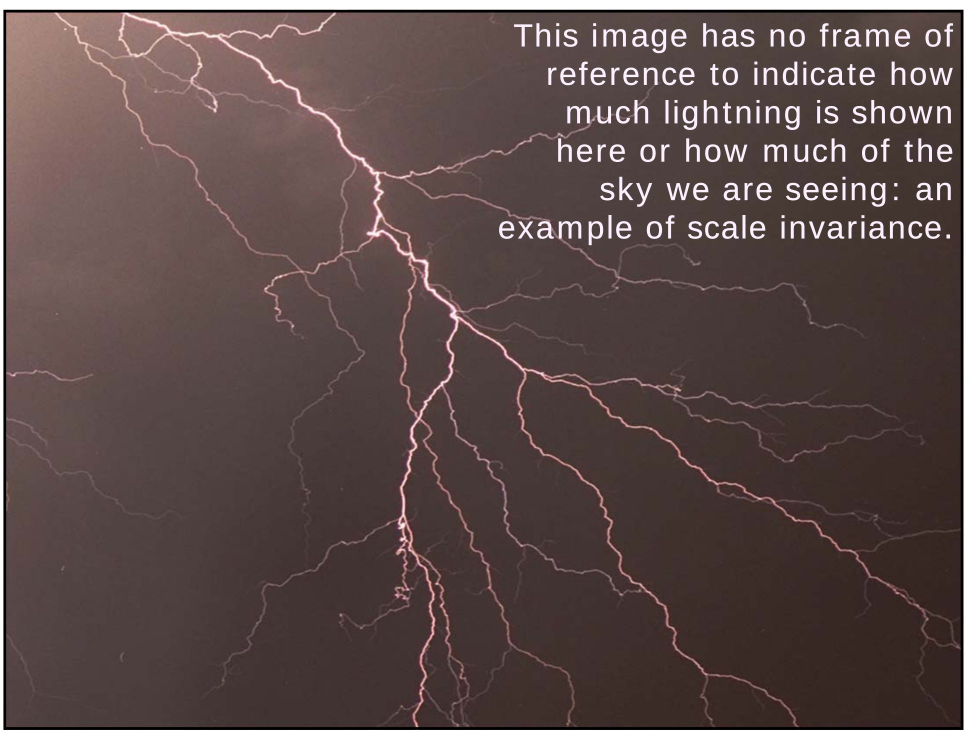

There are around eight intersections on a path to the top of this tree (two at the top are too small to outline).
1
2
3
4
5
6


The smaller sections
of limbs closely
resembles the larger
sections.



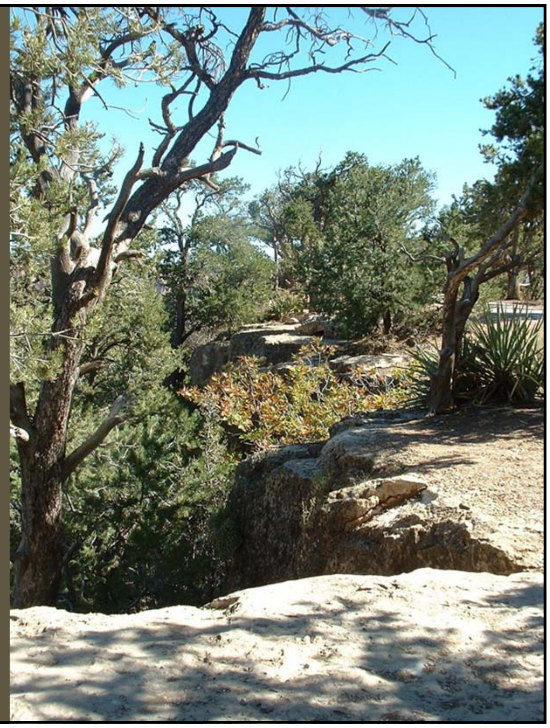

Lightning
Photo taken by Peter Ledlie


Notice the beautiful
fractal branching

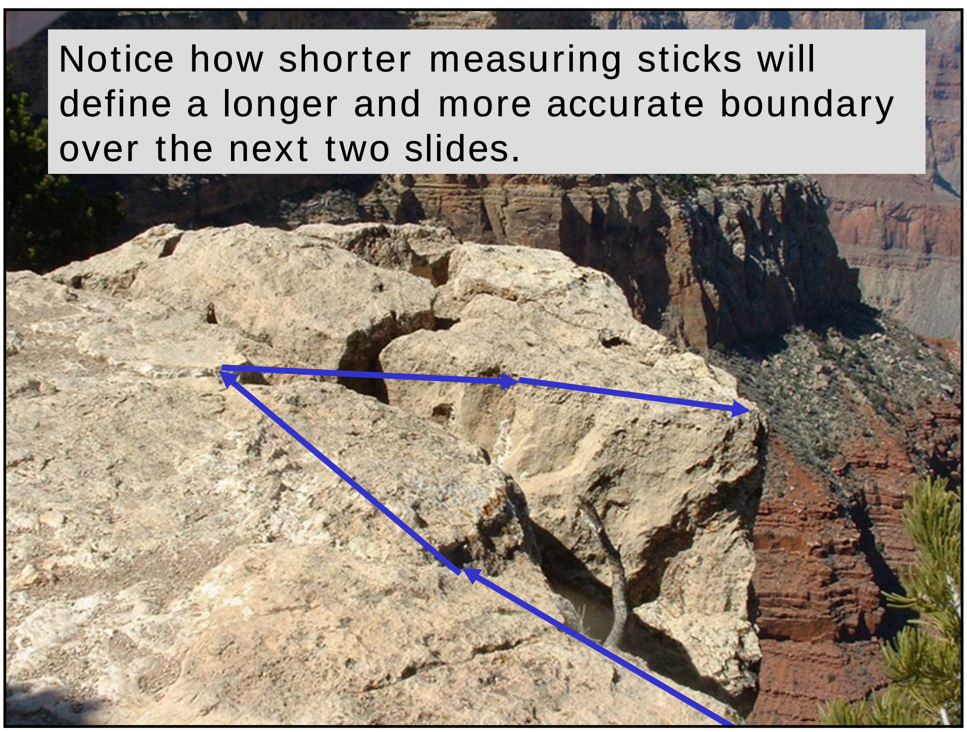

This image has no frame of
reference to indicate how
much lightning is shown
here or how much of the
sky we are seeing: an
example of scale invariance.

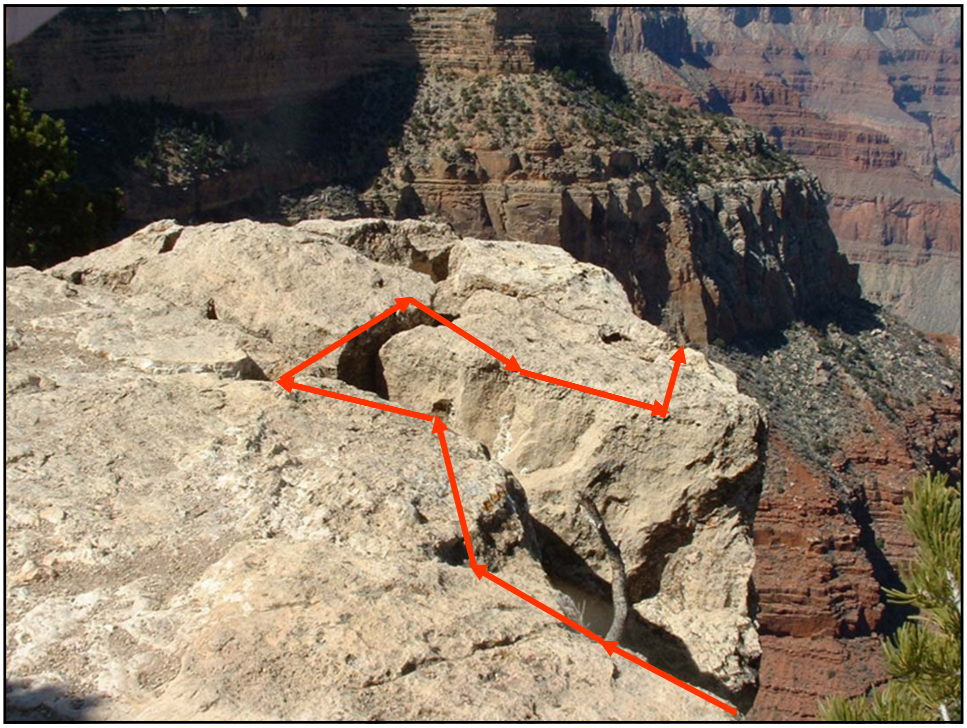
Fractal Boundaries

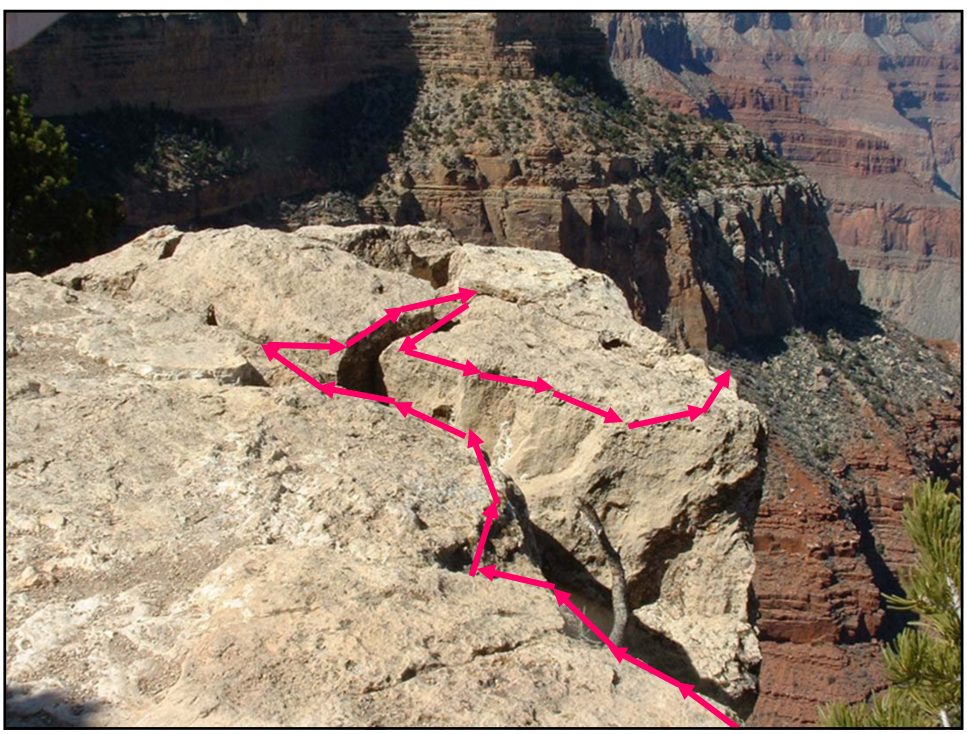
Shorter measuring sticks produce longer boundaries.



Notice how shorter measuring sticks will define a longer and more accurate boundary over the next two slides.







How Long Is the Coast of Britain

Statistical Self-Similarity and Fractional Dimension

Is a paper by Benoît Mandelbrot, first published in Science in 1967

Link on class website

Of course, the number gets larger when a smaller stick is used.

What is surprising is that for many levels of scale:

$$\frac{len(n)}{len(n/2)} \approx \frac{len(n/2)}{len(n/4)} \approx \frac{len(n/4)}{len(n/8)}$$

Fractals Have Crinkly Edges

The word *fractal* comes from *fractional dimension*.

A fractal is an object who's ***topological dimension*** is **less than** its ***box-counting dimension***.

The Koch curve is a bent up line: its topological dimension is 1.

$$\dim_{box} \equiv \lim_{\ell \to 0} \frac{\log N(\ell)}{\log(1/\ell)}$$