CS 261, HW1

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Due: Jan 29th

1. Let p and q be the propositions "You are an engineer" and "You love plaid" respectively. Express each of the following compound propositions as an English sentence.

   (a) $\neg p$

   (b) $\neg p \wedge q$

   (c) $\neg q \Rightarrow \neg p$

   (d) $\neg q \vee (\neg p \wedge q)$

2. Let p, q and r be the following propositions: "You are a mathematician"; "You are an engineer"; and "You are a sharp dresser" respectively. Write these propositions using p, q and r and logical connectives.

   (a) You are a mathematician but you are not an engineer

   (b) You are a mathematician, you are an engineer and you are a sharp dresser

   (c) If you are a sharp dresser, you must be a mathematician

   (d) Being a mathematician and an engineer is sufficient to be a sharp dresser

   (e) You are a sharp dresser if and only if you are a mathematician or you are an engineer

3. Excercise 1-17-24: State the converse, contrapositive and inverse of each of these conditional statements

   (a) If it snows tonight, then I will stay home

   (b) I go to the beach whenever it is a sunny day

   (c) When I stay up late, it is necessary that I sleep until noon

4. You are on an island where all people are either *truth tellers*, who always tell the truth, or *liars*, who never tell the truth. A person on this island is accused of a crime, and hires an attorney. The defendant is publicly known to be a truth teller. The following exchange takes place in court:
   Attorney: "If the defendant committed the crime, he had an accomplice."
   Defendant: "That is not true!"
   Did the attorney help his client? Justify your answer.

5. What if the Attorney was publicly known to be a liar and she says "If the defendant committed the crime, he did *not* have an accomplice." Does the attorney help or hurt his client, or neither? Justify your answer. Hint: Let p be the proposition "The defendant committed the crime" and q be the proposition "The defendant had an accomplice". Use De Morgan's rule!

6. Show that $p \Rightarrow q$ and $\neg q \Rightarrow \neg p$ are logically equivalent using either truth tables or rules of logical equivalence (p. 24)

7. Show that $((p \vee q) \wedge (\neg p \vee r)) \Rightarrow (q \vee r)$ is a tautology.

8. How many of the following disjunctions can be made simultaneously true by an assignment of truth values to p, q and r: $p \vee q$, $\neg p \vee r$, $\neg p \vee \neg r$, $\neg p \vee \neg q$, $\neg r \vee p$.

9. Exercise 1.3.6

10. Exercise 1.3.38

11. Exercise 1.3.42

12. You are lounging on the beach on the island of liars and truth tellers with a large group of natives and you hear the following exchange:
    Alice: "We are all liars and Bob is a truth teller"'
    Bob: "We are all liars or Carol is a liar"
    What can you say about Alice, Bob and Carol? Justify your answer. Hint: Let $L(x)$ be the proposition that x is a liar; use quantifiers and De Morgan's laws for quantifiers in your answer. Consider the two cases where Alice is a truth teller or Alice is a liar.

13. *Challenge*: You are investigating a murder on the island of liars and truth tellers. You have assembled a group and you want to know

if the murderer is in that group. You know that the murderer is a liar. However, you don't know which members of the group, if any are truth tellers. Moreover, you are only allowed to ask yes/no questions to the leader of the group and you want to minimize the number of questions you ask. Hint: Your questions may use propositional logic and quantifiers. Let $M(x)$ be the proposition, "person x is a murderer in the group". Let $L(x)$ be the proposition "person x is a liar".

- Show how to determine if the murderer is in the group by asking two yes/no questions to the group leader.
- Show how to determine if the murderer is in the group by asking a single yes or no question to the leader. Hint: For a person x, let $Say - Murderer(x)$ be a proposition that is true if x would say that there is a murderer in the group.