CS 261, HW3

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Due: Feb. 19th

This homework covers material from Chapter 2.2 up to and including Chapter 3.1 in the textbook.

1. Prove that if A and B are sets then $(A \cap B) \cup (A \cap \bar{B}) = A$. Hint: Show two things (1) if x is in $(A \cap B) \cup (A \cap \bar{B})$, then x is in A; and (2) if x is in A then x is in $(A \cap B) \cup (A \cap \bar{B})$.

2. Give an example of a function from the integers, to the integers that is

   - One-to-one but not onto
   - Onto but not one-to-one
   - Both onto and one-to-one, but NOT the identity function
   - Neither one-to-one or onto

3. Determine if the following functions are bijections from the Reals to the Reals.

   - $f(x) = -2x + 4$
   - $f(x) = x^2 + 9$
   - $f(x) = x/(x + 1)$
   - $f(x) = x^7 + 2$

4. What are the values of the following sums

   - $\sum_{i=1}^{10}(i + 1)$
   - $\sum_{i=1}^{10}(-2)^i$
   - $\sum_{i=1}^{10} 4$

5. What are the values of the following products

- $\prod_{i=0}^{5} i$
- $\prod_{i=4}^{7} i$
- $\prod_{i=4}^{7} 2$
- $\prod_{i=1}^{20} (-1)^i$

6. Use Bubble-Sort to sort the list $6, 4, 5, 1, 3, 2$, showing the list obtained at each step.

7. Consider the set of all tuples of the form (x, y) where x and y are positive integers. Prove that this set is countable.

8. Consider the set of all strings of DNA. Prove that this set is not countable. A DNA string is a sequence made up of the four letter A,C,T and G of any length - possibly infinite. *Hint: Review the proof that the real numbers are not countable*

9. Imagine we change the game of Chomp so that a player can't choose a cookie in a given row until all cookies in the rows below have been eaten. Otherwise the rules are exactly the same as in the book. Does the first player still have a winning strategy? Justify your answer (with a proof).

10. Prove that the problem of determining for a given program and given input whether the program will print out a 1 when run on the input is unsolvable.