

3. Graph Theory

Your first job after completing CS461 is at the prestigious consulting company Downsize.com. Downsize.com is making a fortune helping other Internet companies efficiently downsize. Your first task there is to create an algorithm which will automate this process.

An Internet company is represented as an undirected, connected graph, where the employees are the nodes of the graph and there is an edge between two employees if the two employees work together. You want to select a single employee in this graph to lay-off. For moral reasons, it's crucial that even after the layed-off employee is removed from the graph, the graph is still connected.

The problem then is the following. You are given an undirected, connected graph $G = (V, E)$. You must return a single vertex x in V such if x , and all edges incident to x , are removed from G , the remaining graph will still be connected. Please do the following:

- (a) Give an algorithm to solve this problem in $O(|V| + |E|)$ time. Hint1: Use DFS or BFS. Hint2: You can describe the algorithm in 3 steps or less.
- (b) Describe briefly (two or three sentences) why your algorithm works

Solution: The algorithm is the following: 1) Let T be a DFS tree of G 2) Let x be an arbitrary leaf node of T . 3) return x . The reason this works is the following: even after x is removed from G , the remaining nodes in G are still connected by the edges of T .