CS 422/522: Digital Image Processing
Homework 1 (Fall ’12)

1 Theory

1. The joint p.m.f. of two discrete random variables \( X \) and \( Y \) is given below:

\[
\begin{pmatrix}
5/12 & 1/18 & 5/72 \\
1/36 & 1/36 & 5/36 \\
1/12 & 1/6 & 1/72 \\
\end{pmatrix}
\]

Determine whether \( X \) and \( Y \) are statistically independent.

2. The joint p.m.f. of two discrete random variables \( X \) and \( Y \) is given below:

\[
\begin{pmatrix}
0 & 1/9 & 1/3 \\
1/18 & 5/36 & 1/12 \\
1/36 & 1/4 & 0 \\
\end{pmatrix}
\]

Compute and tabulate:

(a) Marginal p.m.f., \( p_X(x_i) \).

(b) Marginal p.m.f., \( p_Y(y_j) \).

3. A p.d.f. for a continuous random variable \( X \) is defined as follows:

\[
f_X(x) = \begin{cases} 
2x/9 & \text{if } 0 < x < 3 \\
0 & \text{otherwise.}
\end{cases}
\]

Find the value of the c.d.f. at 1:

\[
F_X(1) = \int_0^1 f_X(x)dx.
\]

4. Let \( X \) and \( Y \) be continuous random variables where \( f_X(x) = \frac{1}{\tau} e^{-x/\tau} \) and let \( Y = X^2 \). Derive an expression for \( f_Y \).
2 Practice

1. Write a function \textit{cumulative-distribution-function} which takes an image as its argument and returns the discrete cumulative distribution function (c.d.f.) for the image:

\[
F(j) = \frac{255}{nm} \sum_{i=0}^{j} H(i)
\]

where \(n\) is the number of rows, \(m\) is the number of columns, and \(H\) is the grey-level histogram. You may assume that the image contains grey-levels in the range \([0, 255]\). The c.d.f. should be returned as a \textit{vector}. Compute the discrete c.d.f. for the \textit{frog} image and for an image of your choice. Hint: Although not strictly necessary, learning the Scheme \textit{do} macro might help you.

2. Write a function \textit{histogram-equalize} which takes an image as its argument and returns an image which has been histogram equalized using the discrete c.d.f. as a grey-level transformation. Plot the histograms for the \textit{frog} image and for an image of your choice before and after histogram equalization. You should also show both images before and after histogram equalization. Hint: This is easy to do using \textit{image-map}.

3. Write a function \textit{inverse-cumulative-distribution-function} which takes an image as its argument and returns the discrete inverse cumulative distribution function (discrete i.c.d.f.) for the image. The value of the discrete i.c.d.f. \(F^{-1}\) at \(j\) is the minimum value \(k\) such that \(F(k) \geq j\). You may assume that grey-levels are in the range \([0, 255]\). The i.c.d.f. should be returned as a Scheme \textit{vector}.

4. Write a function \textit{histogram-match} which takes two images as its arguments and returns an image which is the result of applying the histogram matching grey-level transformation to the first image so that its histogram is matched to that of the second image. Plot the histograms for the \textit{frog} image and for the \textit{cropped-rad} image after its histogram has been matched to that of the \textit{frog} image. Show the transformed \textit{cropped-rad} image. Repeat the above for two equal sized images of your choice. Plot the histograms and show the images before and after (for the second image) histogram matching. Hint: This is easy to do using \textit{image-map}.