1. The \( f \) operator takes a function, \( f \), as its argument and returns the antiderivative of the function: \( f \xrightarrow{f} \int f(t)dt \). Prove that the \( f \) operator is:
   
   (a) Linear.
   (b) Shift-invariant.

2. Prove that \( \sin(x) = \frac{e^{ix} - e^{-ix}}{2j} \).

3. The impulse response function of a linear, shift-invariant system is:
   \[
   h(t) = \frac{\sin(\pi t)}{\pi t}
   \]
   and its input is:
   \[
   x(t) = \cos(4\pi t) + \cos(\pi t/2).
   \]
   What is its output?

4. The impulse response function of a linear, shift-invariant system is:
   \[
   h(t) = e^{-\frac{\pi t^2}{2}}
   \]
   and its input is:
   \[
   x(t) = e^{j2\pi s_0 t}.
   \]
   What is its output?

5. The sine Gabor function is the product of a sine and a Gaussian, \( f(t) = e^{-\pi t^2} \sin(2\pi s_0 t) \). Give an expression for \( F(s) \), the Fourier transform of \( f(t) \).

6. The function, \( f(t) \), is defined as:
   \[
   f(t) = \begin{cases} 
   1 & \text{if } |at - b| \leq \frac{1}{2} \\
   0 & \text{otherwise}.
   \end{cases}
   \]
   Give an expression for \( F(s) \), the Fourier transform of \( f(t) \).
7. The transfer function of a linear shift invariant system is $H(s) = 1/s$. The impulse response function, $h(t)$, is $\mathcal{F}^{-1}\{H(s)\}$. Give an expression for $g(t)$ where:

$$g(t) = \int_{-\infty}^{\infty} e^{j2\pi s_0 \tau} h(t - \tau) \, d\tau.$$ 

8. Compute the Fourier transform of $f(t) = -2\pi t e^{-\pi t^2} \cos(2\pi s_0 t)$. Hint: What is $\frac{d(e^{-\pi t^2})}{dt}$?