

# Learning Swing-free Trajectories for UAVs with a Suspended Load

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# Motivation

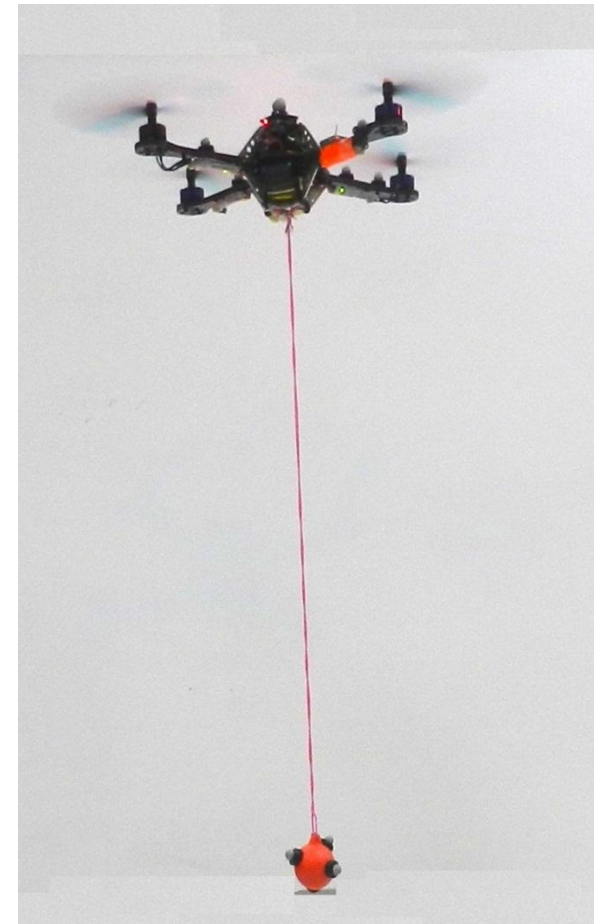
- Aerial transportation applications
  - Humans unable or unwilling
  - Search and rescue missions
  - Supply delivery
  - Patient transportation
  - Wildfires



Photo by UN Photo/Logan Abassi

# Problem Formulation

- Holonomic cargo-bearing UAV
  - Bring the suspended load to the destination
  - Minimal residual load oscillations at arrival
- Challenges
  - Non-linear, unknown dynamics
  - Hardware safety
- Dynamical systems balancing constraints problem
  - Quality vs. quantity
  - Anti-lock brakes, traction control

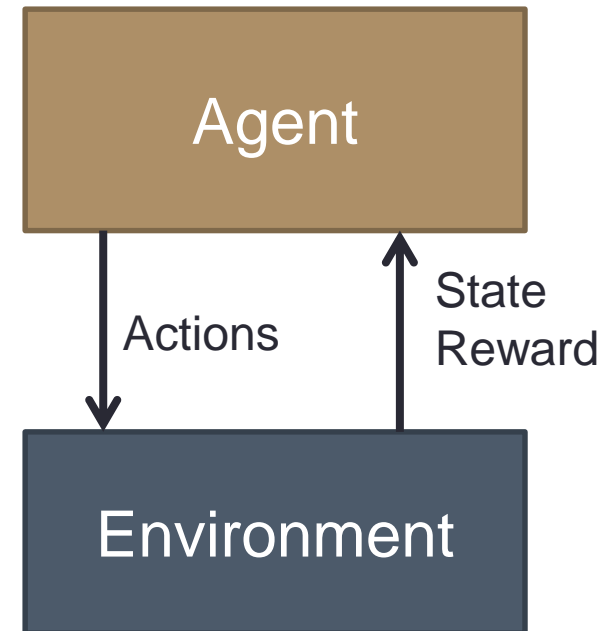


# Expert Attempt



# Reinforcement Learning (RL)

- Markov decision process MDP (S,A,P,R)
- Value  $V:S \rightarrow R$ 
  - Cost to go, potential for accumulated reward
- Induces policy  $\pi:S \rightarrow A$ 
  - Action sequence that maximizes the value
  - $\pi(s) = \operatorname{argmax}_a V(s')$
  - $s'$  resulting state when  $a$  is applied to state  $s$



# Approximate Value Iteration (AVI)

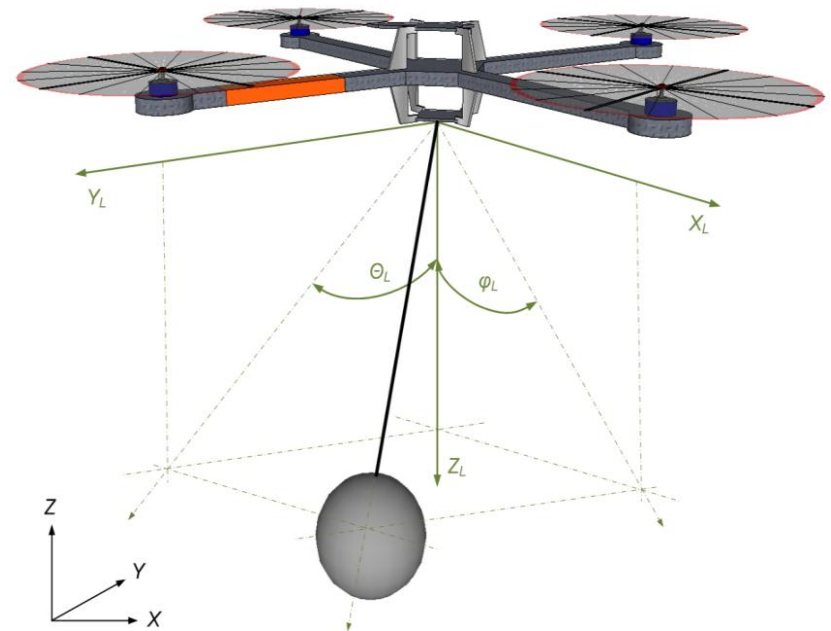
Ernst et al. 2005

- Offline
- Continuous state, discrete action space
- Iteratively finds value function approximation as
  - $V(s) = \psi^T F(s)$
- Algorithm
  - Sample  $(s_i, r_i)$
  - $v_i = r_i + \gamma \max_a \psi_n^T F(s_i')$
  - $\psi_{n+1} = \operatorname{argmin}_{\psi} (v_i - \psi^T F(s_i))^2$



# AVI Implementation

- MDP Setup
  - $s$ : vehicle and load position and velocities
  - Discretized acceleration vector
  - Reward structure
  - Generic holonomic vehicle with suspended load
- Goal state in equilibrium
- Learning sampling domain
- Feature vector  $F(s)$ , squared:
  - Distance from the goal
  - Vehicle's velocity magnitude
  - Load displacement
  - Load's velocity magnitude



# AVI Learning Convergence

- Randomized algorithm
- No convergence guarantees
- Monte Carlo selection
  - Repeat learning over several trials
  - Select best policy
- Can we be sure the policy takes the UAV to the goal?



# Trajectory Generation

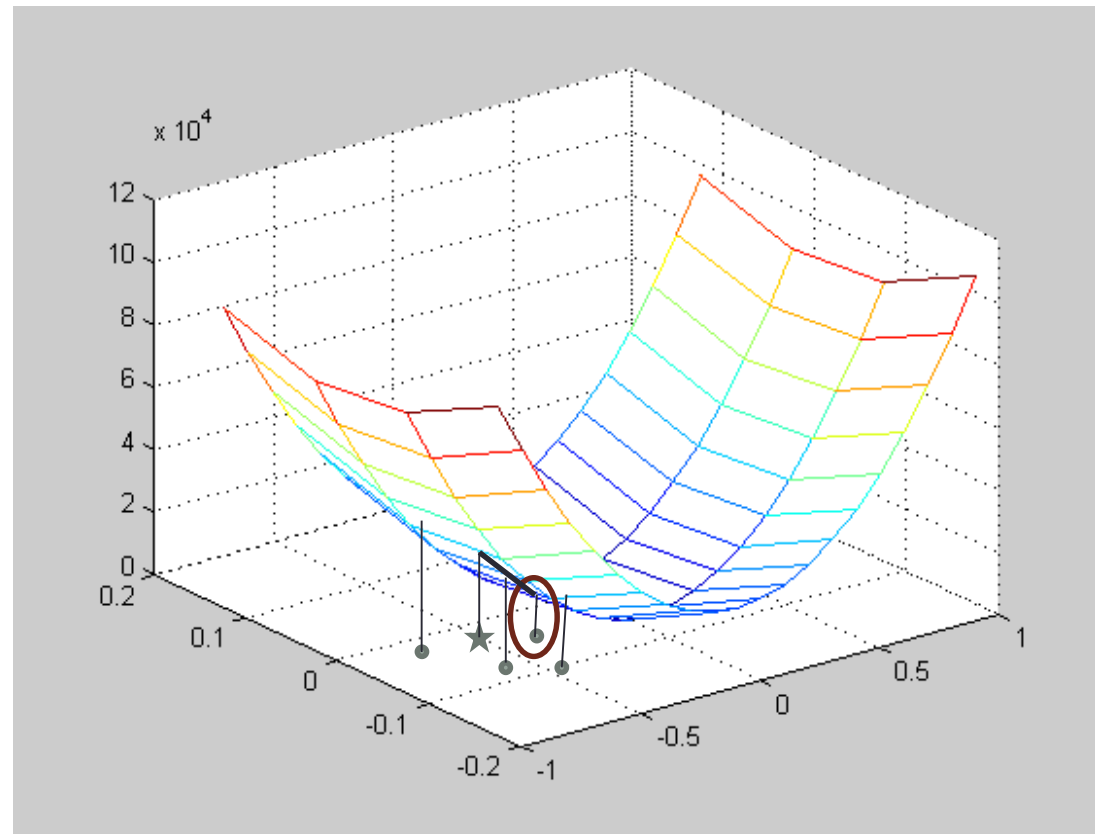
- Lyapunov stability analysis
  - Given start state and control function where will the system end up?
- Control function
  - Policy
  - Transitional probabilities / generative model
  - Action space

# Lyapunov Stability Analysis

- $W(s) = -V(s) = -\psi^T F(s)$  is Lyapunov function if:
  - $\psi^i < 0$
  - $\Delta W(s) < 0$
- System is asymptotically stable
  - If action space discretization allows transitions to a higher-valued states for all states
- State space
  - Learning vs. problem domain
  - Lyapunov criteria holds on the problem domain
  - *Learn in small area, and the policy viable starting at arbitrary position (asymptotically stable)*

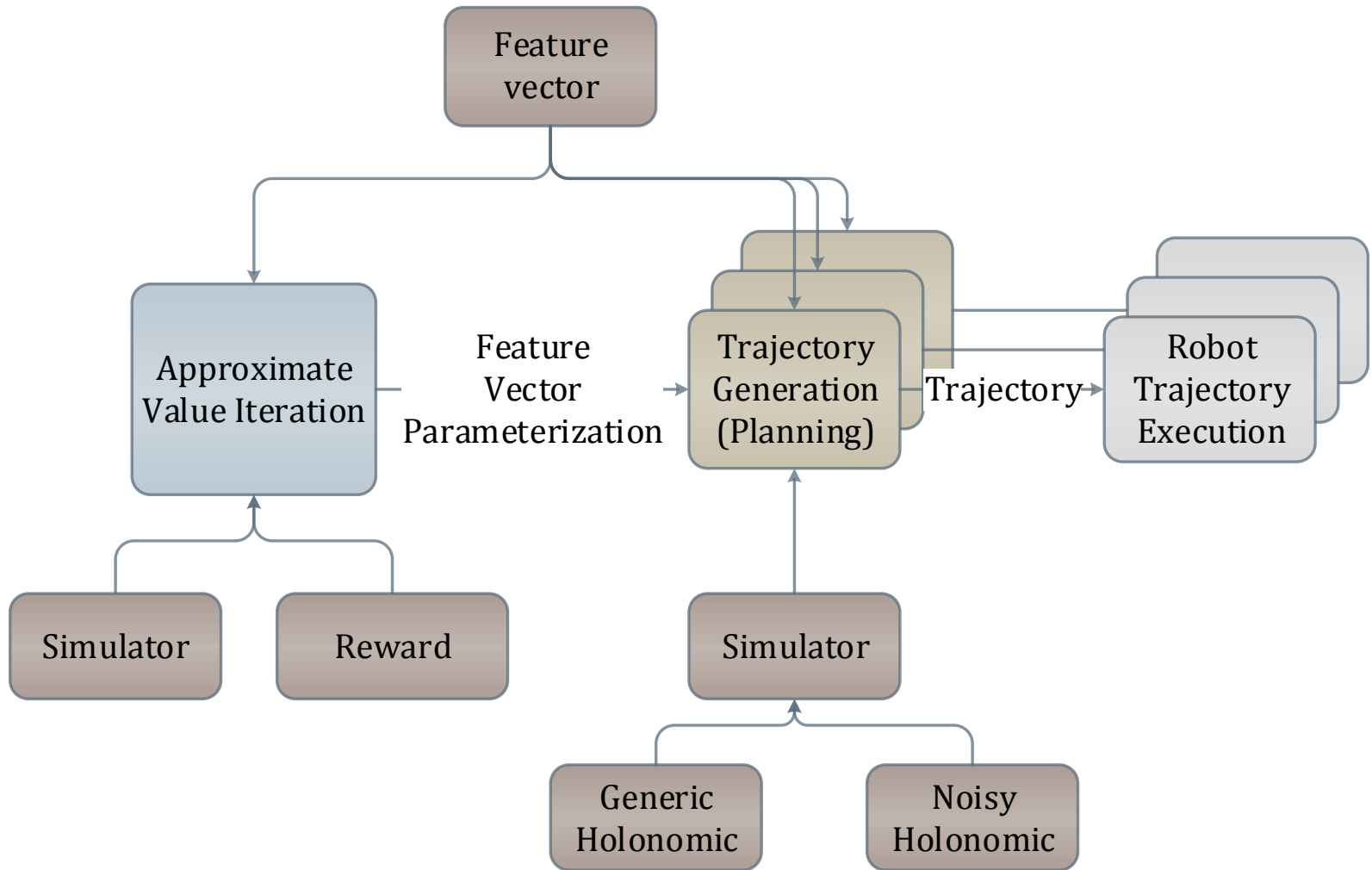
# Can we change generative model and action space between learning and planning?

- Generative model and action space
  - Determine state space connectivity
  - Discretize value / Lyapunov function
  - Determine the difference condition of the Lyapunov function



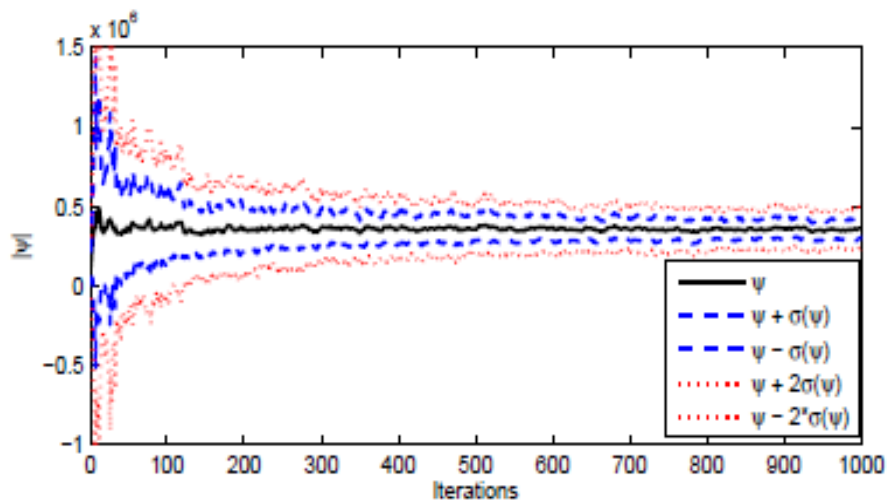
*Yes! - as long as we preserve ability to transition to a higher valued / lower cost state*

# Approach



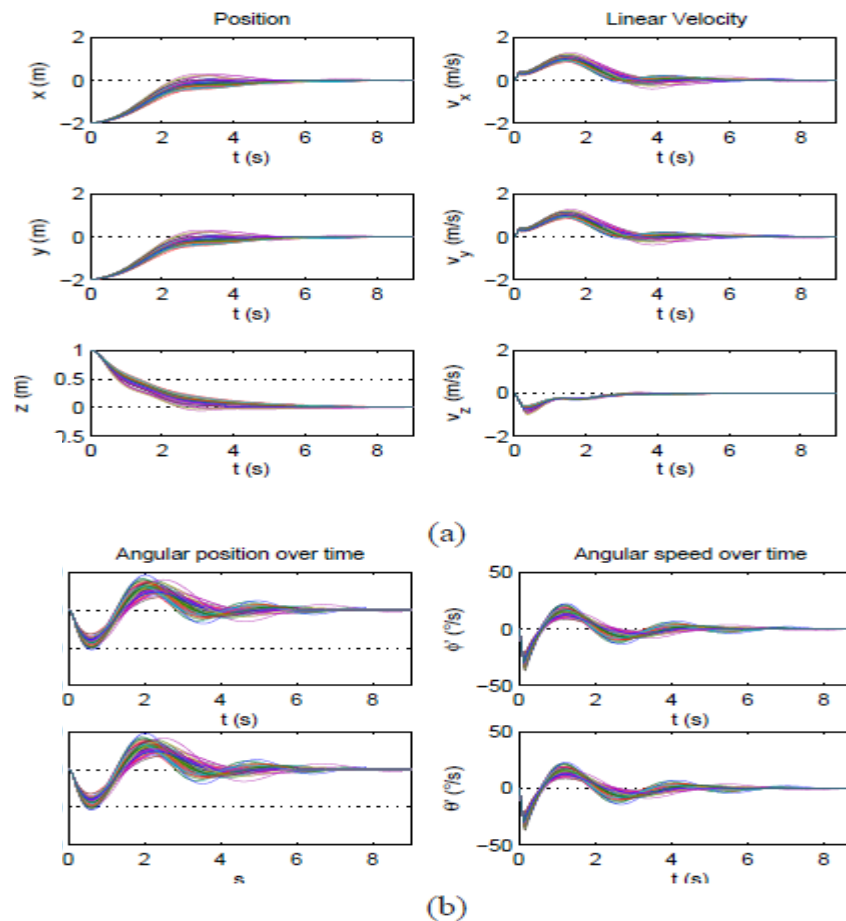
# Learning convergence

- 1000 iterations
- 100 trials
- Fixed start point



Parameter vector norm over 1000 iterations

*Learning converges*



Trajectories over 100 trials from a fixed start point

*Policies generate consistent trajectories*

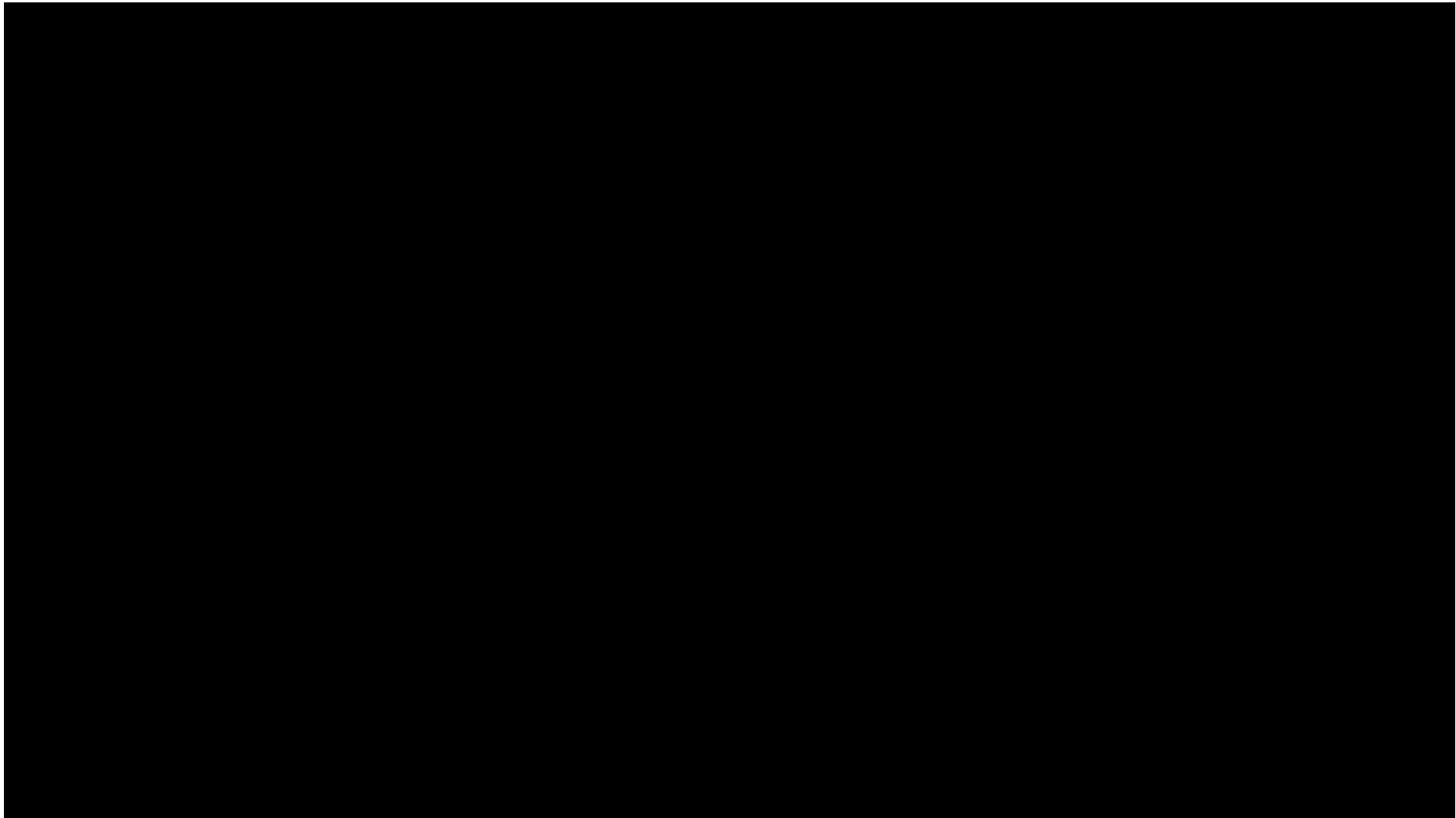
# Variable Start Trajectory Generation

- Trained with coarse-grain 3D action set
- Planned with fine-grain 3D action set

| State                  |                   | Goal reached | t (s) |          | $\ p\ $ (m) |          | $\ \eta\ $ (°) |          | max $\ \eta\ $ (°) |          |
|------------------------|-------------------|--------------|-------|----------|-------------|----------|----------------|----------|--------------------|----------|
| Location               | Simulator         | (%)          | $\mu$ | $\sigma$ | $\mu$       | $\sigma$ | $\mu$          | $\sigma$ | $\mu$              | $\sigma$ |
| (-2,-2,1)              | Generic Holonomic | 100          | 6.13  | 0.82     | 0.03        | 0.01     | 0.54           | 0.28     | 12.19              | 1.16     |
|                        | Noisy Holonomic   | 100          | 6.39  | 0.98     | 0.04        | 0.01     | 0.55           | 0.30     | 12.66              | 1.89     |
| (-20,-20,15)           | Generic Holonomic | 99           | 10.94 | 1.15     | 0.04        | 0.01     | 0.49           | 0.33     | 46.28              | 3.90     |
|                        | Noisy Holonomic   | 89           | 12.04 | 1.91     | 0.08        | 0.22     | 0.47           | 0.45     | 44.39              | 7.22     |
| ((4,5),(4,5),(4,5))    | Generic Holonomic | 100          | 7.89  | 0.87     | 0.04        | 0.01     | 0.36           | 0.31     | 26.51              | 2.84     |
|                        | Noisy Holonomic   | 100          | 7.96  | 1.11     | 0.04        | 0.01     | 0.44           | 0.29     | 27.70              | 3.94     |
| ((-1,1),(-1,1),(-1,1)) | Generic Holonomic | 100          | 4.55  | 0.89     | 0.04        | 0.01     | 0.33           | 0.30     | 3.36               | 1.39     |
|                        | Noisy Holonomic   | 100          | 4.55  | 1.03     | 0.04        | 0.01     | 0.38           | 0.29     | 3.46               | 1.52     |

Trajectory characteristics for various start positions: duration, ending distance from the goal, ending load swing, and maximum swing encountered.

*Trajectory end characteristics are consistent regardless of the start state.*





# Questions

- Thank you!
- Website:
  - <https://www.cs.unm.edu/amprg/Research/Quadrotor/>
  - Code, movie, etc...
- Acknowledgements
  - Thanks to Peter Ruymgaart for piloting quadrotor

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