

CS 530: Geometric and Probabilistic Methods in Computer Science Homework 1 (Fall '15)

1. The joint p.m.f. of two discrete random variables X and Y is given below:

$$X \begin{array}{c} Y \\ \left[\begin{array}{ccc} 5/12 & 1/18 & 5/72 \\ 1/36 & 1/36 & 5/36 \\ 1/12 & 1/6 & 1/72 \end{array} \right]. \end{array}$$

Determine whether X and Y are statistically independent.

2. The joint p.m.f. of two discrete random variables X and Y is given below:

$$X \begin{array}{c} Y \\ \left[\begin{array}{ccc} 0 & 1/9 & 1/3 \\ 1/18 & 5/36 & 1/12 \\ 1/36 & 1/4 & 0 \end{array} \right]. \end{array}$$

Compute and tabulate:

- (a) Marginal p.m.f., $p_X(x_i)$.
- (b) Marginal p.m.f., $p_Y(y_j)$.
- (c) Conditional p.m.f., $p_{X|Y}(x_i|y_j)$.
- (d) Conditional p.m.f., $p_{Y|X}(y_j|x_i)$.

3. Prove that

$$\binom{n}{r} = \binom{n}{n-r}.$$

4. After years of driving through a particular intersection, you know that the probability of being stopped by a red light is 0.27. You drive through the intersection twice a day, five days a week. What is the probability of being stopped by a red light five or more times in a single week?

5. Let X be a discrete random variable with the following p.m.f.:

$$p_X(x_i) = \begin{cases} c x_i^2 & x_i \in \{1, 2, 3, 4, 5\} \\ 0 & \text{otherwise.} \end{cases}$$

- Find c .
- Find the expected value and variance of X .

6. The continuous random variable X has p.d.f.:

$$f_X(x) = \begin{cases} cx^2 & \text{if } 1 \leq x \leq 5 \\ 0 & \text{otherwise.} \end{cases}$$

- Find c .
- Find the expected value and variance of X .
- Find the probability that X exceeds 2.0.

7. A p.d.f. is defined as follows:

$$f_X(x) = \begin{cases} 2x/9 & \text{if } 0 \leq x \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

Find the value of $F_X(1)$, i.e., the value of the c.d.f. at 1.

8. Let X and Y be continuous r.v.'s where

$$f_X(x) = \begin{cases} \frac{1}{\tau} e^{-x/\tau} & \text{if } t > 0 \\ 0 & \text{otherwise.} \end{cases}$$

and let $Y = X^2$. Derive an expression for f_Y .

9. During World War II, German propaganda claimed that they could target their flying bombs, and most of the British public believed this to be true. To investigate this claim, we divide the area of London into 576 equal areas of $1/4$ square kilometers each. The table shows the number of areas receiving different numbers of hits. First, calculate the average number of hits per area. Second, on the assumption that the bombs cannot be aimed with any accuracy, calculate the number of areas receiving $k = 0, 1, 2, 3, 4,$ and 5 hits.

| k | 0 | 1 | 2 | 3 | 4 | 5 |
|--------------------------|-----|-----|----|----|---|---|
| # of areas with k hits | 229 | 211 | 93 | 35 | 7 | 1 |

10. Two fair dice are rolled, one of which is red and the other is green. Let W be the difference (red number less green number). Compute and plot $p_W(w)$.

11. Write a MATLAB function which computes the 2D joint histogram, G_{XY} , of a pair of images, X and Y , of equal size. Test it on the red and green components of the Queen Butterfly image found on the class homepage. Display the joint histogram, G_{XY} , as a grey level image. [Note: Use the online documentation to find out how to generate hardcopies of images in MATLAB (or Octave).]
12. Write a MATLAB function which, given a joint histogram, G_{XY} , returns the marginal histograms, G_X and G_Y . Using the Cactus image, verify that the marginal histograms you compute are the same as those computed using the 1D histogram function found on the class homepage. [Note: Use the online documentation to find out how to plot and generate hardcopies of histograms in MATLAB (or Octave).]