## CS-150L Computing for Business Students <br> Future Value of a Retirement Annuity

## Instructor:

Matthew Barrick
e-mail: barrick@cs.unm.edu www.cs.unm.edu/~barrick
Office: Farris Engineering Center (FEC) room 106


## Lab 8

- Calculating Future Value of a Retirement Annuity
- Calculating Retirement Income
- Using the Social Security Administration's benefit calculator.
- Excel FV(rate, nper, pmt), Future Value function.
- Using an annuity accrual table to vary the payment amount.


## Retirement Annuity

Accrual Phase: payments are made into the annuity account. During this phase, the value of the annuity increases by:

- Contributions: Deposits may be made weekly, monthly, quarterly, or yearly. An ordinary annuity is an annuity whose payments are made at the end of each period.
- Interest or Return on Current Investment.

Pension Phase: This begins upon retirement.

- Each period, payments are made out of the annuity.
- Interest on the annuity balance continues to add to the value of the annuity.


## Excel FV(rate, nper, -pmt)

- Excel built-in function for calculating the future value of an annuity.

$$
F V_{\text {annuity }}=p m t \times\left(\frac{(1+\text { rate })^{n p e r}-1}{\text { rate }}\right)
$$

- rate is the periodic interest rate (APR divided by the number of periods per year),
- nper is the total number of periods of the annuity,
- pmt is the amount contributed to the annuity each period. In order to use this equation, the payment must be the same every period.


## FV(rate, nper, -pmt) example

|  | A | B | C | D |
| :--- | :--- | :--- | ---: | ---: |
| 1 |  | Named <br> Constant |  |  |
| 2 | Periodic Rate | rate | $12.000 \%$ |  |
| 3 | Number of Periods | nper | 30 |  |
| 4 | Periodic Contribution | pmt | $\$ 150.00$ |  |
| 5 |  |  |  |  |
| 6 | Future Value | FV | =FV(rate,nper,_-pmt) |  |
| 7 |  |  | $\$ 36,199.90$ |  |

- All of the values are given in periods.
- Thus, it does not need to be stated whether the period is days, weeks, months, years, or Mars years.
- Judging by the values, make a guess at the period.


## FV - The Magic of Compound Interest

|  | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  | Named Reference | Value | Equation |
| 2 | $\sum_{i=1}^{\infty}$ | APR |  | 12.000\% |  |
| 3 |  | Number of Years |  | 40 |  |
| 4 |  | Number of Periods per Year |  | 12 |  |
| 5 |  | Monthly Contribution | pmt | \$150.00 |  |
| 6 |  | Periodic Rate | rate | 1.000\% | D2/D4 |
| 7 |  | Number of Periods | nper | 480 | D3*D4 |
| 8 | Future Value |  |  | =FV(rate, nper, -pmt) |  |
| 9 |  |  |  | \$1,764,715.88 |  |

- 1.7 million is a not bad "nest egg".
- Are these numbers realistic?


## What if? (Active Excel Worksheet)

| APR |  | $4.000 \%$ |
| :--- | :--- | ---: |
| Number of Years |  | 40 |
| Periodic Rate | rate | $0.333 \%$ |
| Number of Periods | nper | 480 |
| Monthly Contribution | pmt | $\$ 150.00$ |
| Future Value |  | FV |

What if Monthly Contribution is changed? $(\$ 150 \rightarrow \$ 300)$ What if APR is changed?

$$
(12.0 \% \rightarrow 6.0 \% \quad \text { or } \rightarrow 10.0 \% \quad \text { or } \rightarrow 18.0 \%)
$$

## Future Value \& Compounding Periods

| A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Name | Value | Equation |
| $\underset{\sim}{\stackrel{C}{0}}$ | APR | APR | 12.000\% |  |
|  | Number of Years | years | 40 |  |
|  | Annual Contribution | pmt | \$1,800 |  |
| FV (compounded annually) |  |  | \$1,380,765 | FV(APR, years, -pmt) |
| FV (compounded monthly) |  |  | \$1,764,716 | FV(APR/12, years*12, -pmt/12) |
| FV (compounded daily) |  |  | \$1,851,882 | FV(APR/365, years*365, -pmt/365) |

$$
F V_{\text {annuity }}=p m t \times\left(\frac{(1+\text { rate })^{n p e r}-1}{\text { rate }}\right)
$$

## Quiz: Future Value

An annuity is created in which $\$ 50.00$ is invested every month for 25 years. The annuity offers a guaranteed 2.25\% APR.

An Excel worksheet is set up with named references, APR, YEARS, and PMT. What is the Future Value?
a) $=F V(A P R$, YEARS, $-P M T)$

|  | A | B |
| :--- | :--- | ---: |
| 1 | APR | $2.250 \%$ |
| 2 | YEARS | 25 |
| 3 | PMT | $\$ 50.00$ |

b) $=\mathrm{FV}(\mathrm{APR} / 12$, YEARS, -PMT$)$
c) $=F V(A P R / 12$, YEARS/12, -PMT $)$
d) $=\mathrm{FV}(A P R / 12$, YEARS*12, -PMT)
$e)=F V(A P R / 12$, YEARS, $-P M T / 12)$

## Quiz: Future Value

An annuity is created in which $\$ 50.00$ is invested every month for 25 years. The APR of the annuity is $2.25 \%$.
One person uses the equation:
=FV(APR/12,YEARS*12,-PMT)

Another person uses the equation:
=FV(APR,YEARS,-PMT*12)


Which equation is correct?
a) They are both correct.
b) The 1st. The 2nd gives an answer that is too large.
c) The 1st. The 2nd gives an answer that is too small.
d) The 2nd. The 1st gives an answer that is too large.
e) The 2nd. The 2st gives an answer that is too small.

## Research

■ Choose a person: Name \& Retirement Date

- Choose a Career.
- Choose a Geographic Location.
- Starting Salary in 2009.

■ Senior Salary: (salary at retirement in 2009 dollars).

- Contribution Rate (percentage of salary)
- Accrual Rate of Return
- Pension Rate of Return

■ Years of Life After Retirement.

## FV Worksheet: Researched Values

|  | B | C |
| :---: | :--- | ---: |
| 5 | Annual Salary | $\$ 10,000.00$ |
| 6 | Pre-tax Employee Contribution | $8.00 \%$ |
| 7 | After-tax Employee Contribution | $0.00 \%$ |
| 8 | Employer Contribution | $5.00 \%$ |
| 9 | Total Dollar Amount of Annual Contributions | $\$ 1,300.00$ |
| 10 | Annual Investment Return Rate | $20.00 \%$ |
| 11 | Years Contributing | 48 |

Employer Contribution: $\quad=\mathrm{MIN}(\mathrm{C} 6,5 \%)$
Total Annual Contributions: ?

$$
=\mathrm{C} 5^{*} \mathrm{C} 6+\mathrm{C} 5^{*} \mathrm{C} 7+\mathrm{C} 5^{*} \mathrm{C} 8
$$

## Quiz: Total Annual Contributions

|  | A | B |
| :--- | :--- | ---: |
| 1 | Annual Salary | $\$ 15,000.00$ |
| 2 | Pre-tax Employee Contribution | $4.00 \%$ |
| 3 | After-tax Employee Contribution | $0.00 \%$ |
| 4 | Employer Contribution | $4.00 \%$ |
| 5 | Total Dollar Amount of Annual Contributions | $\$ 1,200.00$ |
| 6 | Annual Investment Return Rate | $20.00 \%$ |
| 7 | Years Contributing | 48 |

Which equation gives the Total Annual Contributions in B5?
a) $=\mathrm{B} 2^{*} \mathrm{~B} 1+\mathrm{B}^{*} \mathrm{~B} 1+\mathrm{B} 4^{*} \mathrm{~B} 1$
b) $=\mathrm{B} 2^{*} \mathrm{~B} 6+\mathrm{B} 3^{*} \mathrm{~B} 6+\mathrm{B} 4^{*} \mathrm{~B} 6$
c) $=\mathrm{B} 6$ * $(\mathrm{B} 1+\mathrm{B} 2+\mathrm{B} 3+\mathrm{B} 3)$
d) $=\mathrm{B} 6$ * $(\mathrm{B} 2+\mathrm{B} 3+\mathrm{B} 3)$
e) $=B 2^{*} B 7+B 6 * B 7$

## FV Worksheet: Scenarios

| 12 | Scenarios: | End of Year <br> Contribution | End of Quarter <br> Contribution | End of Month <br> Contribution |
| :--- | :--- | ---: | ---: | ---: |
| 13 | Contributions Per Year | 1 | 4 | 12 |
| 14 | Periodic Interest Rate | $20.00 \%$ | $5.00 \%$ | $1.67 \%$ |
| 15 | Amount Contributed each Period | $\$ 1,300.00$ | $\$ 325.00$ | $\$ 108.33$ |
| 16 | Periods Contributing | 48 | 192 | 576 |
| 17 | Total Contributions | $\$ 62,400.00$ | $\$ 62,400.00$ | $\$ 62,400.00$ |
| 18 | Future Value FV function | $\$ 41,071,866.65$ | $\$ 76,071,445.88$ | $\$ 88,663,784.49$ |
| 19 | Future Value Equation | $\$ 41,071,866.65$ | $\$ 76,071,445.88$ | $\$ 88,663,784.49$ |

- Why is the Future Value so much larger in the third column than in the first column?

■ FV function: =FV(rate, nper, -pmt)
■ FV Equation: $\quad F V=p m t \times\left(\frac{(1+\text { rate })^{n p e r}-1}{\text { rate }}\right)$

## Pension Phase

| 21 | The Size of my "nest egg" | $\$ 88,663,784$ |
| :--- | :--- | ---: |
| 22 | Yearly Interest Rate | $6.99 \%$ |
| 23 | Years of Retirement | 25 |
| 24 | Monthly Pension | $\$ 626,092$ |
| 25 | Monthly Social Security Benefit | $\$ 1,460$ |
| 26 | Monthly Retirement Income: |  |

- Use PMT(rate, nper, $p v$ ) function to determine the amount you will withdraw from your pension each month. At the end of nper, the value will be zero.
- Note: rate is not APR and nper is not years.


## Error in Lab Manual: Lab 8

■ On Page 81,

- 8.2.3: f) Total Dollar Amount of Annual Contribution


## Should be

- 8.2.3: f) Total Dollar Amount of All Contributions this Period


## Accrual table (Active Excel Worksheet)

| ease: | $1.48 \%$ |  |  |  |
| ---: | ---: | ---: | ---: | :--- |
| Pre-tax <br> Employee <br> Contribution | Employer <br> Contribution | Total Dollar <br> Amount of All <br> Contributions this <br> Period | Monthly <br> Investment <br> Return |  |
|  |  |  |  | Value |
| $8.00 \%$ | $5.00 \%$ | $\$$ | 108 | $1.67 \%$ |
| $8.00 \%$ | $5.00 \%$ | $\$$ | 110 | $1.67 \%$ |
| 8.0 | - |  |  |  |
| $8.00 \%$ | $5.00 \%$ | $\$$ | 112 | $1.67 \%$ |
| $8.00 \%$ | $5.00 \%$ | $\$$ | 113 | $1.67 \%$ |
| $8.00 \%$ | $5.00 \%$ | $\$$ | 115 | $1.67 \%$ |
| $8.00 \%$ | $5.00 \%$ | $\$$ | 117 | $1.67 \%$ |

## Total Dollar Amount of Contribution

|  | A |  | B | C | D | E | FTotal DollarAmount ofAnnualContribution |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | Year | Annual <br> Salary |  | Pre-tax Employee Contribution | Employer Contribution | After-tax Employee Contribution |  |  |
| 4 | 0 |  |  |  |  |  |  |  |
| 5 | 1 | \$ | 10,000 | 10.00\% | 5.00\% | 0.00\% | \$ | 1,500 |
| 6 | 2 | \$ | 10,500 | 10.00\% | 5.00\% | 0.00\% | \$ | 1,575 |
| 7 | 3 | \$ | 11,025 | 10.00\% | 5.00\% | 0.00\% | \$ | 1,654 |
| 8 | 4 | \$ | 11,576 | 10.00\% | 5.00\% | 0.00\% | \$ | 1,736 |

The contribution information from columns B, C, D, and E is totaled in column $F$. Thus, when the value column is
18 calculated, columns B, C, D and E and not needed.

## Quiz: Value at End of Each Year

|  | A | B | C | D | E | F |  | G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | Annuity Account with Increasing Annual Contributions |  |  |  |  |  |  |
| 3 | Year | Annual Salary | Pre-tax Employee Contribution | Employer Contribution | Total Dollar Amount of Annual Contribution | Annual Investment Return |  | alue |
| 4 | 0 |  |  |  |  |  | \$ | - |
| 5 | 1 | \$ 10,000 | 10.00\% | 5.00\% | \$ 1,500 | 20.00\% | \$ | 1,500 |
| 6 | 2 | \$ 10,500 | 10.00\% | 5.00\% | 1,575 | 20.00\% | \$ | 3,375 |

What equation should be entered in G5 and filled down?
a) $=$ E5 + F5*B5
b) $=$ G4*F5 + E5*F5
c) $=\mathrm{G} 4+\mathrm{F} 5^{*} \mathrm{~B} 4$
d) $=G 4+G 4 * F 5$

## ROW(), ROW(reference) Function

The ROW () function returns the row number of its location.

|  | A | B |
| :--- | :--- | :--- |
| 1 | John | $=\operatorname{ROW}()$ |
| 2 | Paul |  |
| 3 | Ringo |  |
| 4 | George |  |$\downarrow$


|  | A | B |
| :--- | :--- | ---: |
| 1 | John | 1 |
| 2 | Paul | 2 |
| 3 | Ringo | 3 |
| 4 | George | 4 |


|  | A | B | C |
| :---: | :---: | :---: | :---: |
| 7 | Oct 12 | $=\operatorname{ROW}()-\operatorname{ROW}(\$ A \$ 7)+1$ |  |
| 8 | Oct 15 |  |  |
| 9 | Oct 26 |  |  |


|  | A | B |
| :--- | :---: | ---: |
| 7 | Oct 12 | 1 |
| 8 | Oct 15 | 2 |
| 9 | Oct 26 | 3 |

## MOD(number, divisor) Function

In computing, the modulo operation finds the remainder of division of one number by another.
The second number is called the divisor.

|  | $A$ | B | C |
| :---: | ---: | :---: | :--- |
| 1 | 1 | $=\mathrm{MOD}(\mathrm{A} 1,4)$ |  |
| 2 | 2 |  |  |
| 3 | 3 |  |  |
| 4 | 4 |  |  |
| 5 | 5 |  |  |
| 6 | 6 |  |  |
| 7 | 7 |  |  |
| 8 | 8 |  |  |
| 9 | 9 |  |  |
| 10 | 10 |  |  |


|  |  | A |
| :---: | ---: | ---: |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 0 |
| 5 | 5 | 1 |
| 6 | 6 | 2 |
| 7 | 7 | 3 |
| 8 | 8 | 0 |
| 9 | 9 | 1 |
| 10 | 10 | 2 |

## Using MOD() and ROW() Together

What is the result of filling down the equation in B2?

|  | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Divisor | $3]$ |  |  |
| 2 |  | $=\mathrm{MOD}(\mathrm{ROW}()-1, \$ \mathrm{~B} \$ 1)$ |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |
| 7 |  |  |  |  |
| 8 |  |  |  |  |
| 9 |  |  |  |  |


|  | A | B |
| :---: | :---: | :---: |
| 1 | Divisor | 3 |
| 2 |  | 1 |
| 3 |  | 2 |
| 4 |  | 0 |
| 5 |  | 1 |
| 6 |  | 2 |
| 7 |  | 0 |
| 8 |  | 1 |
| 9 |  | 2 |

## Quiz: ROW() and MOD()

If the equation shown in cell B2 is filled down through B9, then what value will be displayed in cell B6?

|  | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Divisor | $5]$ |  |  |
| 2 |  | $=\mathrm{MOD}$ | ROW()-1, | \$B\$1) |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |
| 7 |  |  |  |  |
| 8 |  |  |  |  |
| 9 |  |  |  |  |

a) 0
b) 1
c) 2
d) 3
e) 4

## "Logical" Equation

A logical equation is an equation that returns either the value TRUE or FALSE.

|  | A | B |
| :---: | :---: | :---: |
| 1 | TRUE |  |
| 2 | FALSE |  |
| 3 | TRUE |  |
| 4 | FALSE |  |
| 5 |  |  |
| 6 | FALSE |  |



## Quiz: Logical Equation

If the equation in C 2 is filled down through C 7 , then what will be displayed in C5, C6 and C7?

|  | A | B | C |
| ---: | ---: | ---: | :---: |
| 1 | Roll 1 | Roll 2 | Win |
| 2 | 1 | 6 | $=\mathrm{A} 2+\mathrm{B} 2=7$ |
| 3 | 2 | 5 |  |
| 4 | 3 | 4 |  |
| 5 | 4 | 4 | $?$ |
| 6 | 6 | 1 | $?$ |
| 7 | 6 | 2 | $?$ |

a) FALSE, FALSE, FALSE
b) FALSE, TRUE, FALSE
c) FALSE, TRUE, TRUE
d) TRUE, FALSE, FALSE
e) TRUE, TRUE, TRUE

## Fill Background on Every Odd Row

|  | A | B | C | D | G |  | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | Year | Annual Salary | Pre-tax Employee Contribution | Employer Contribution | Annual Investment Return |  | Value |
| 4 | 0 |  |  |  |  | \$ | - |
| 5 | 1 | \$ 10,000 | 10.00\% | 5.00\% | 20.00\% | \$ | 1,500 |
| 6 | 2 | \$ 10,250 | 10.00\% | 5.00\% | 20.00\% | \$ | 3,338 |
| 7 | 3 | \$ 10,506 | 10.00\% | 5.00\% | 20.00\% | \$ | 5,581 |
| 8 | 4 | \$ 10,769 | 10.00\% | 5.00\% | 20.00\% | \$ | 8,312 |
| 9 | 5 | \$ 11,038 | 10.00\% | 5.00\% | 20.00\% | \$ | 11,631 |
| 10 | 6 | \$ 11,314 | 8.00\% | 5.00\% | 20.00\% | \$ | 15,428 |
| 11 | 7 | \$ 11,597 | 8.00\% | 5.00\% | 20.00\% | \$ | 20,021 |
| 12 | 8 | \$ 11,887 | 8.00\% | 5.00\% | 20.00\% | \$ | 25,570 |
| 13 | 9 | \$ 12,184 | 8.00\% | 5.00\% | 20.00\% | \$ | 32,268 |
| 14 | 10 | \$ 12,489 | 8.00\% | 5.00\% | 20.00\% | \$ | 40,345 |

## Fill Background on Every Odd Row

## Method 1: Format as Table



## Fill Background on Every Odd Row

## Method 2: Conditional Formatting

| New Formating Rule | ? |
| :--- | :--- |
| Select a Rule Type: |  |
| Format all cells based on their values |  |
| Format only cells that contain |  |
| Format only top or bottom ranked values |  |
| Format only values that are above or below average |  |
| F Use a formula to determine which cells to format |  |

## Edit the Rule Description:

Format values where this formula is true:
$=\operatorname{MOD}(\operatorname{ROW}(), 2)=1$

$\square$ which does NOT give the correct balance? last period's balance + interest + new deposit

- cumulitive Interest + cumulitive deposits
- last period's balance + last period's balance*periodic rate of return + new deposit

