CS 365
Introduction to Scientific Modeling
Lecture 5: Metabolic Scaling Theory

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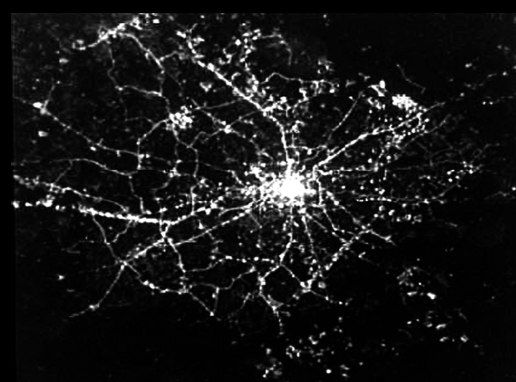
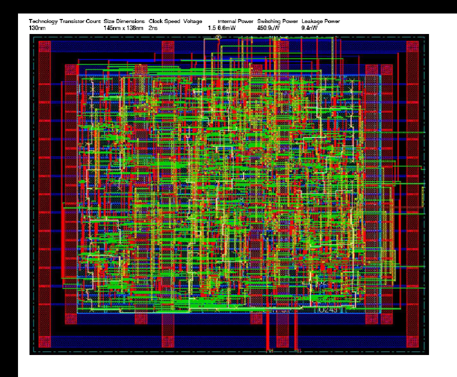
Physical and Geometric constraints determine network architecture and growth

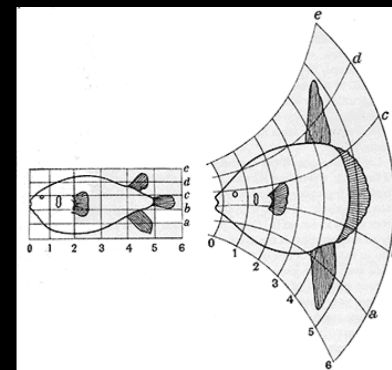
- Network capacity limits performance as systems scale
- Metabolism, response times, power consumption
- Are universal patterns in system behavior predictable from the scaling properties of distribution networks?





On Growth and Form
D'Arcy Thompson (1917)

- Attempted to account for differences in the forms of related animals
  - Through relatively simple mathematical transformations
- Structuralism vs. Survival-of-the-Fittest
- Structuralism: Physical laws govern the form of species, in addition to evolution



So: http://www.gopixpic.com

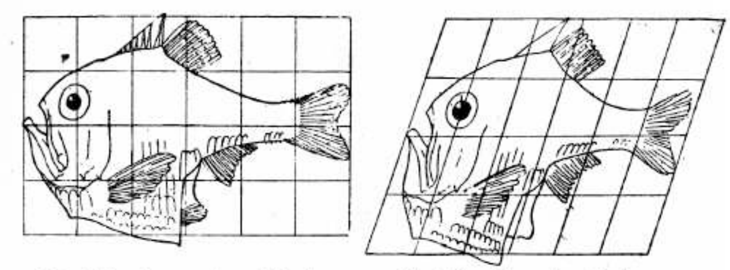


Fig. 517. *Argyropelecus Olfersi.* Fig. 518. *Sternoptyx diaphana.*

So: wikipedia

Allometry: The study of relationships between body size and shape

Metabolic Scaling Theory

A general theory for the origin of allometric scaling laws in biology (1997)

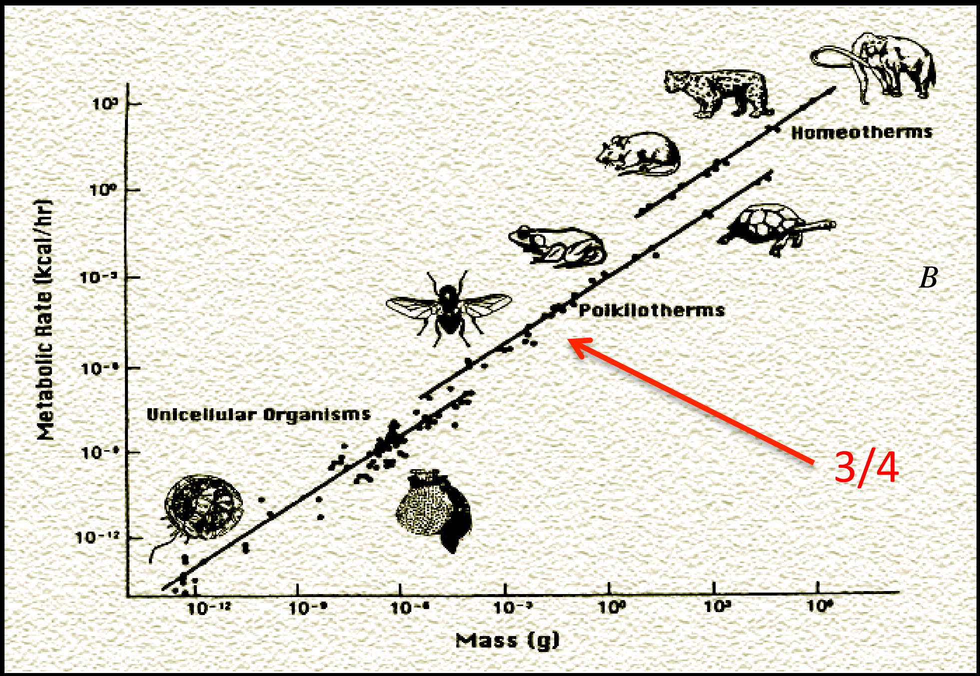
Jim Brown

Geoff West

In biology, larger animals (with centralized distribution networks) are slower

So: bioweb.uwlax.edu

Kleiber's Law



Hemmingson, 1960

- Observed metabolic scaling

$$B \propto M^{3/4}$$

- B is the rate of energy (oxygen) use
  - Mass specific scaling
- B is the master biological rate that governs
  - Ecological interactions
  - Food webs & ecosystem dynamics $\propto M^{-1/4}$
- Other biological rates
  - Biological times $\propto M^{1/4}$

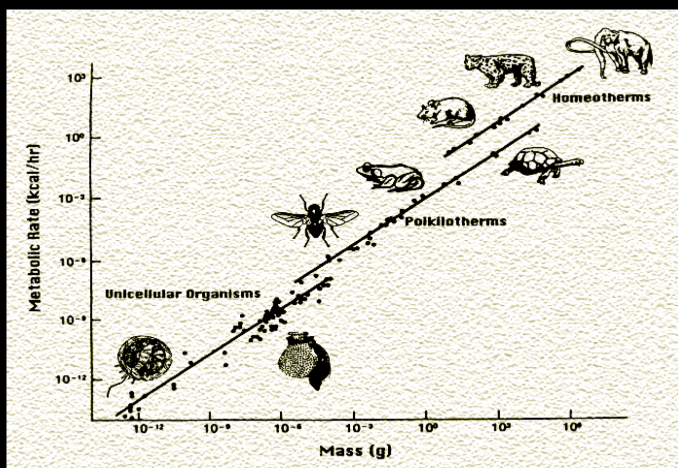
Howard Odum

"there is a unity of the single system of energy, ecology, and economics …

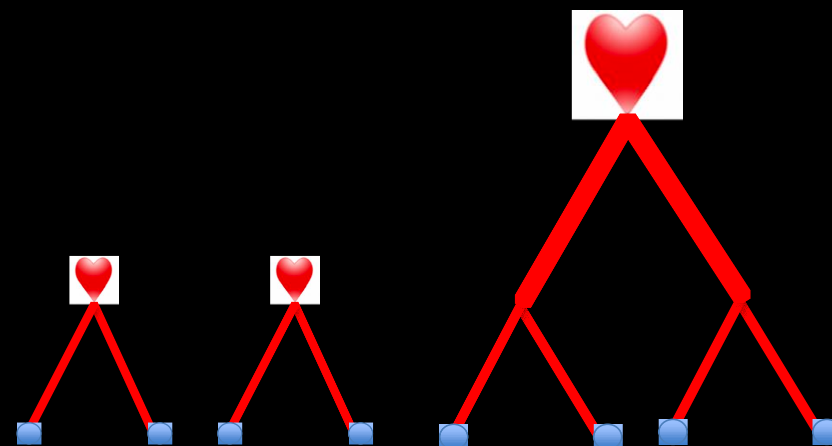
Let us here seek common sense overview which comes from overall energetics"

The Cost of Getting Big

West, Brown, and many others



Kleiber's Law



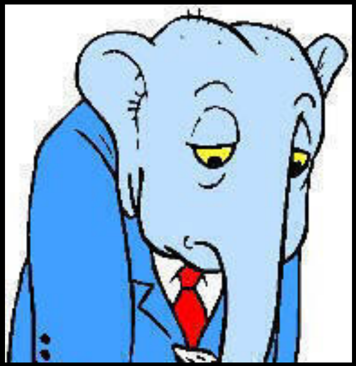
$$B = aM^{3/4}$$

$$V_{net} = kN_c^{(D+1)/D}$$

The Cost of Getting Big
Strategies

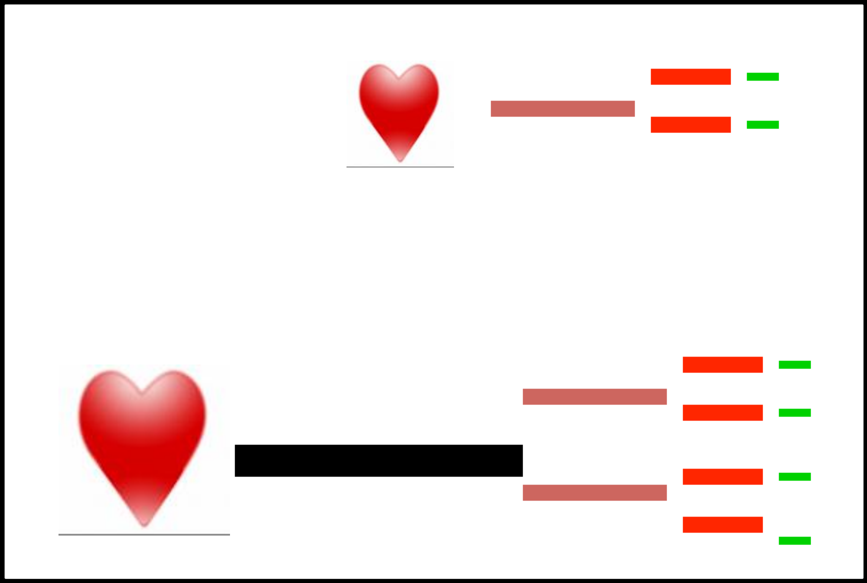
- Slow down the processing speed of terminal components (cells)
  - Mother Nature
  - Scale network as a constant fraction of body size
- Dedicate enough extra network footprint to maintain processing speed
  - Computer architecture (until recently)
  - Use 3rd dimension to accommodate wire scaling demands
- Intermediate solutions
  - Lymph nodes
  - Road networks

HOW DO CITIES SCALE?



Metabolic Scaling Theory

- Larger organisms require larger networks
  - Pipe lengths (L) are longer
  - Cross-section areas (A) are larger
  - # capillaries increases more slowly than pipe volume: $N = cV^{3/4}$
  - Metabolism: $B = cM^{3/4}$



Increasing volume (mass) 100 times increases delivery rate 30 times

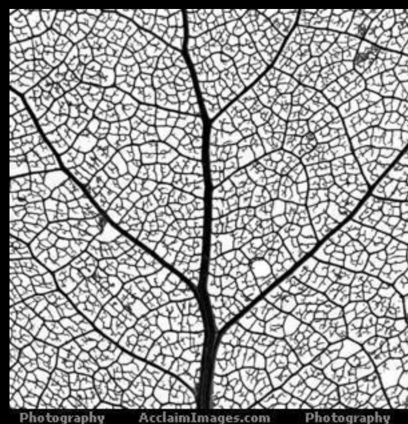
Diminishing returns: Network size grows faster than network delivery rate

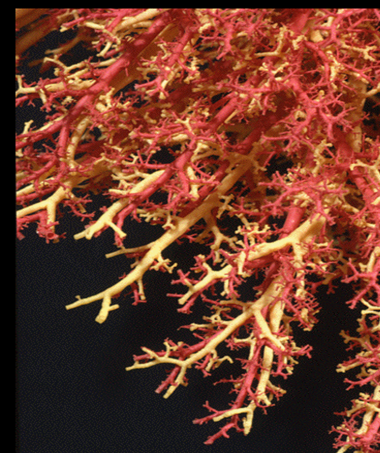
Elements of the Theory

Metabolic Ecology: A Scaling Approach (2012)

- All cells need nutrients and oxygen
  - Delivered by an internal space-filling hierarchical (fractal) distribution network
  - This assumption has been adjusted in later versions of the theory
- The final branch of the network (capillary) is constant size, independent of organism size (invariant terminal units)
- Energy required to distribute resources is minimized (network design is optimized)
- Network is area preserving at every level of hierarchy

Organisms have evolved networks to distribute energy efficiently





Social insects use networks to acquire energy and communicate




Network scaling concepts

- Network volume (V_{net}) increases faster than number of capillaries (N_c)

$$V_{net} \propto N_c^{4/3}$$

  - Diminishing returns
- Each capillary is the same: $B \propto N_c$

$$V_{net} \propto B^{4/3}$$

- Biological constraint, blood volume is a constant percentage of mass: $V_{net} \propto M$

$$B \propto M^{3/4}$$

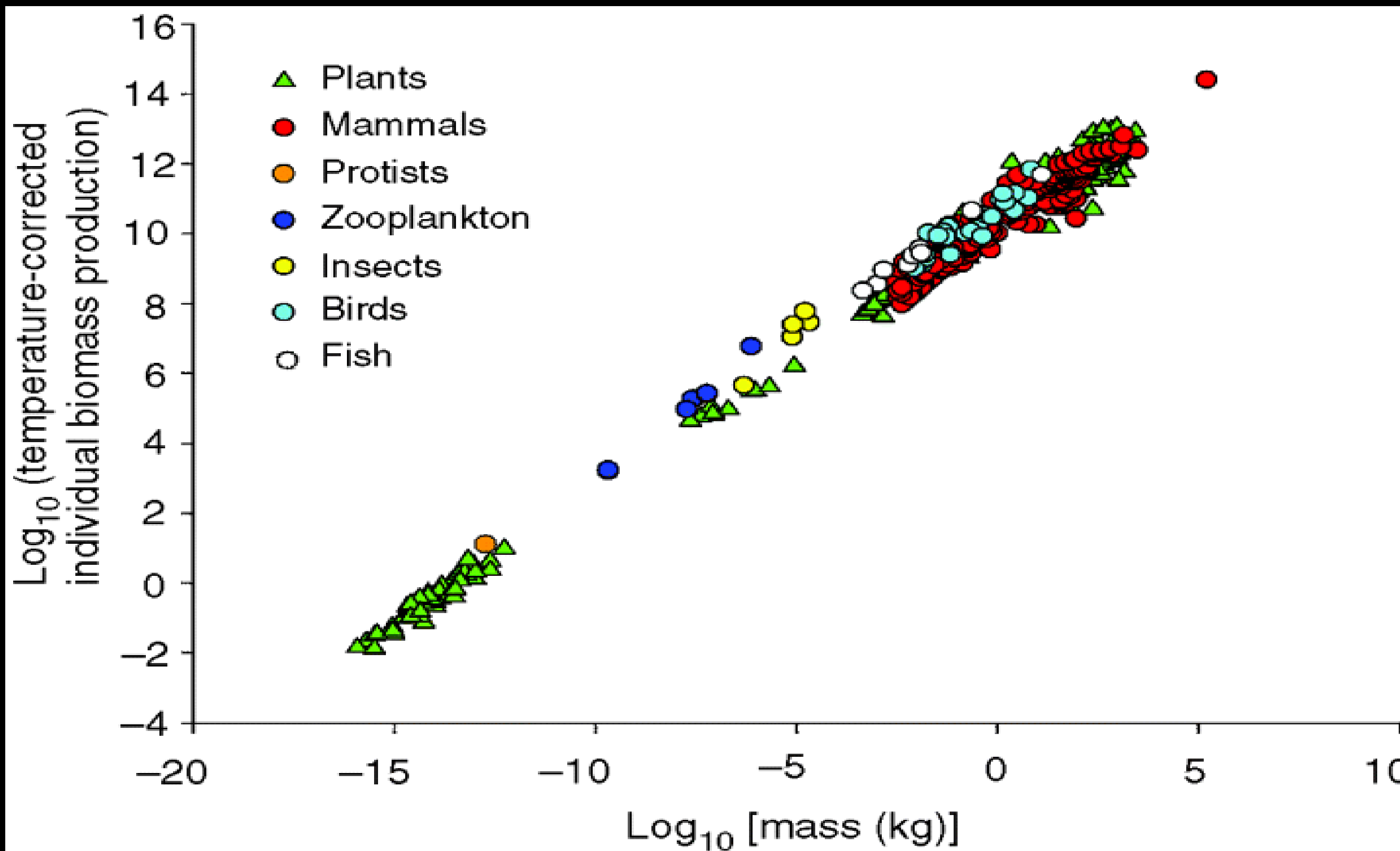
- Controversy, but accepted that centralized distribution networks generate

$$V_{net} \propto N_c^{(D+1)/d}$$

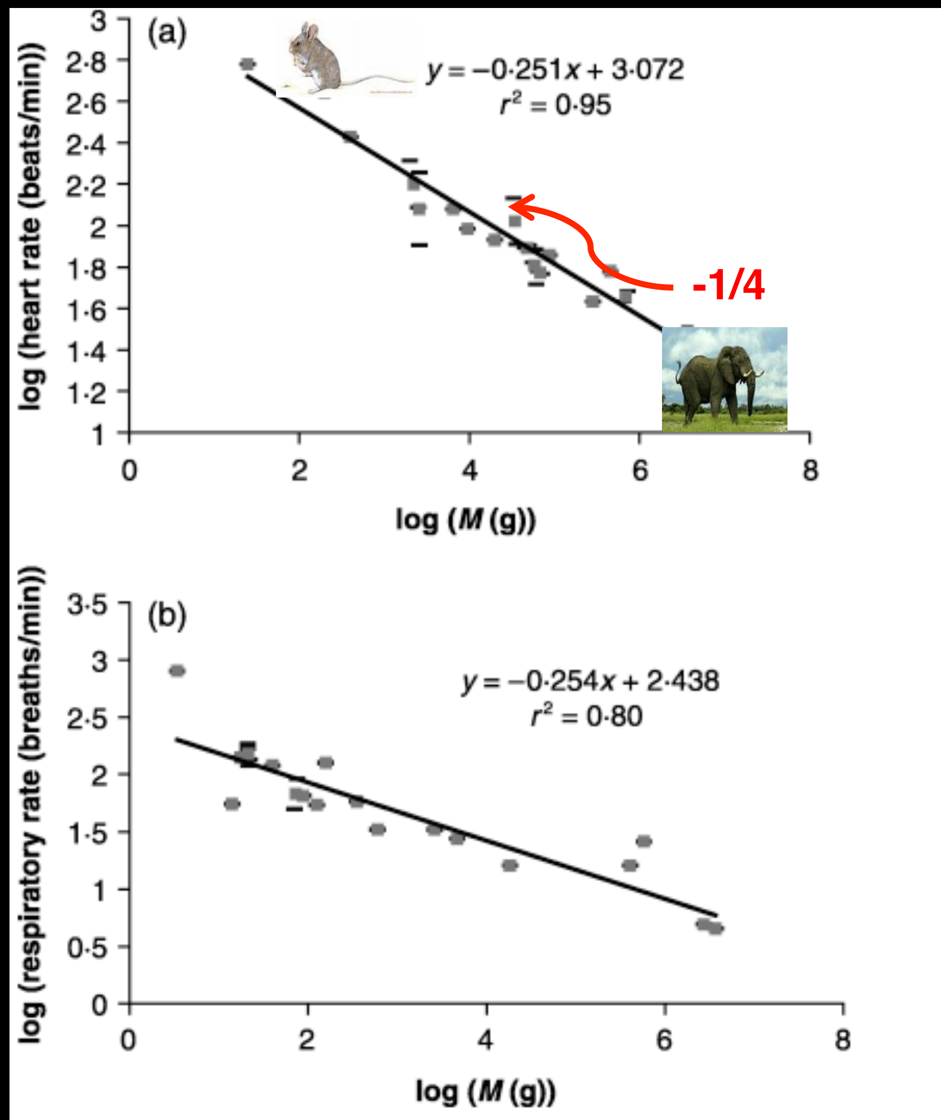
Network scaling accurately predicts rates and times

- Physiology
- Individual growth
- Population growth
- Reproduction
- Disease spread
- Lifespan
- Photosynthesis and carbon flux

Biomass Production: $P \propto M^{3/4}$

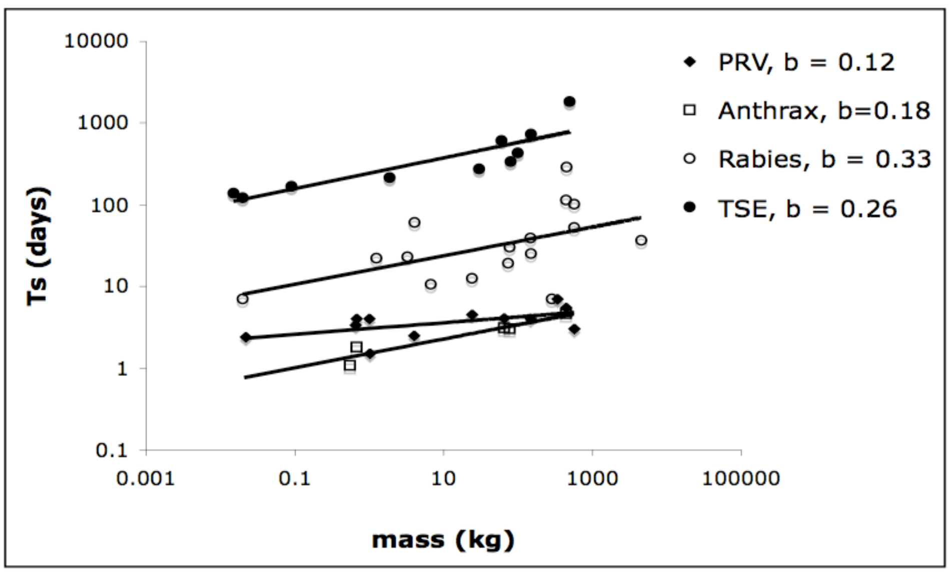


Physiological Rates: $P \propto M^{-1/4}$

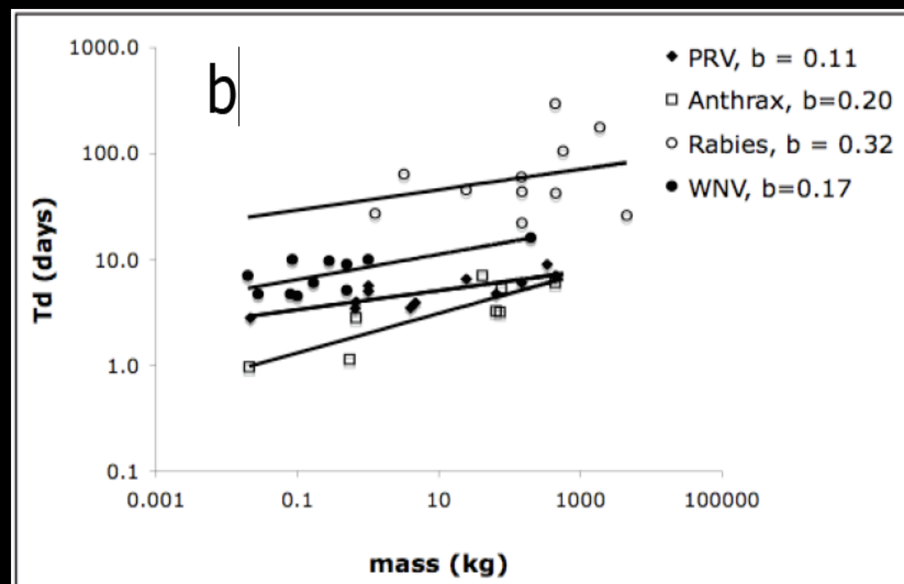


Scaling times associated with disease

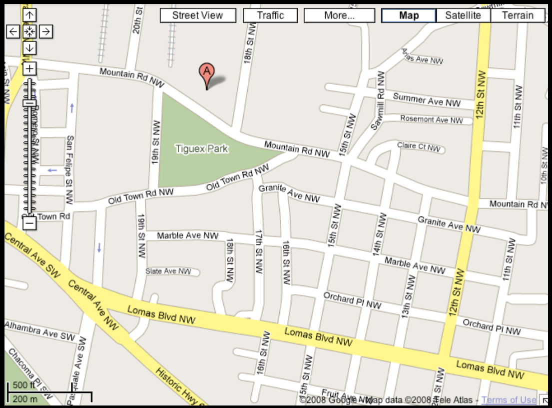
Time to first symptoms

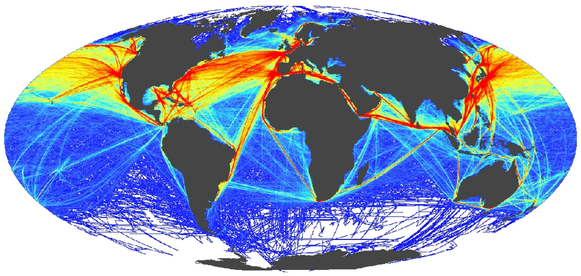


Time to death

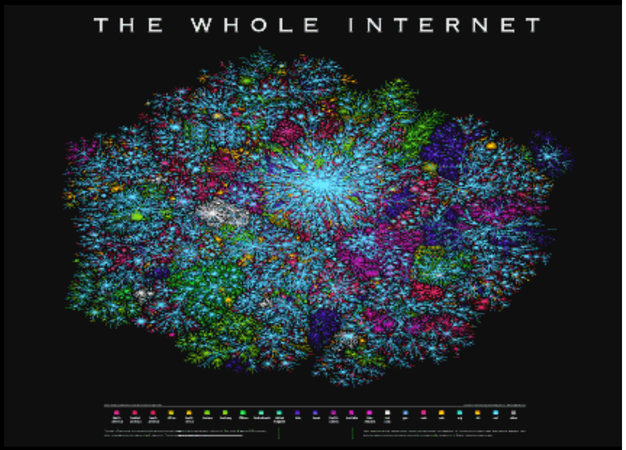


Human engineered networks span the globe



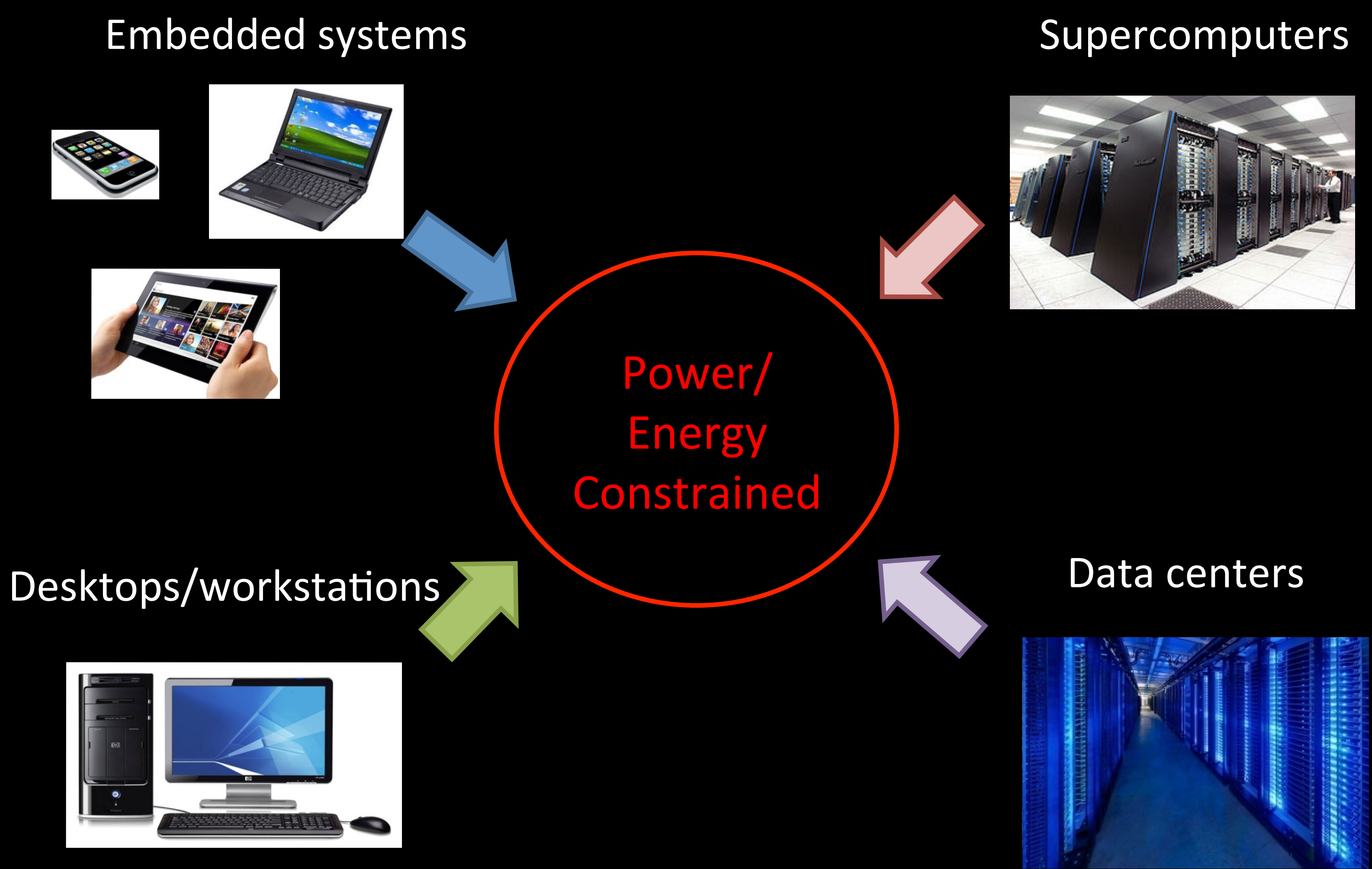


Global Shipping Routes
Halpern et al Science 2008

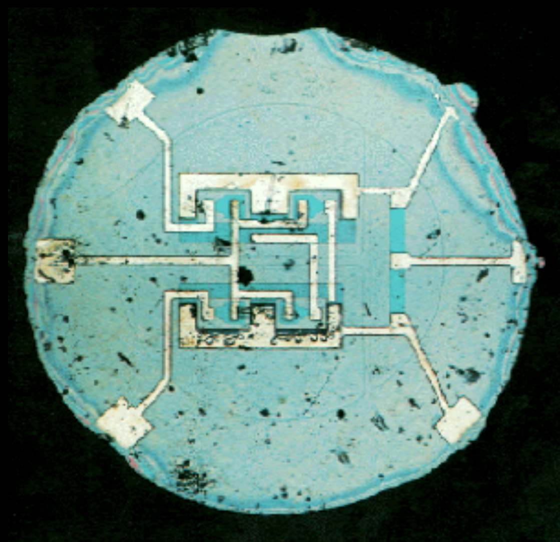


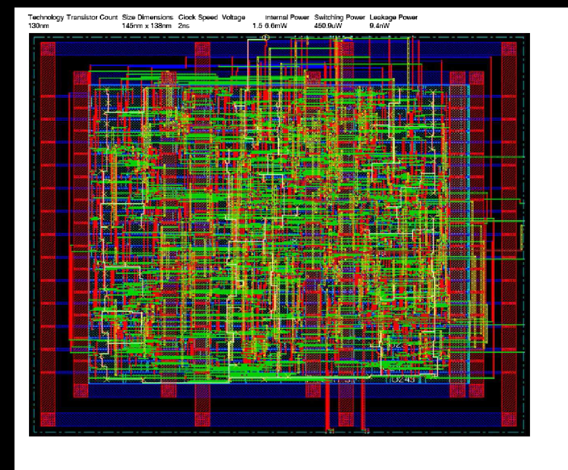
And are subject to similar constraints

Power Consumption



Predicting and Minimizing Power in Microprocessors



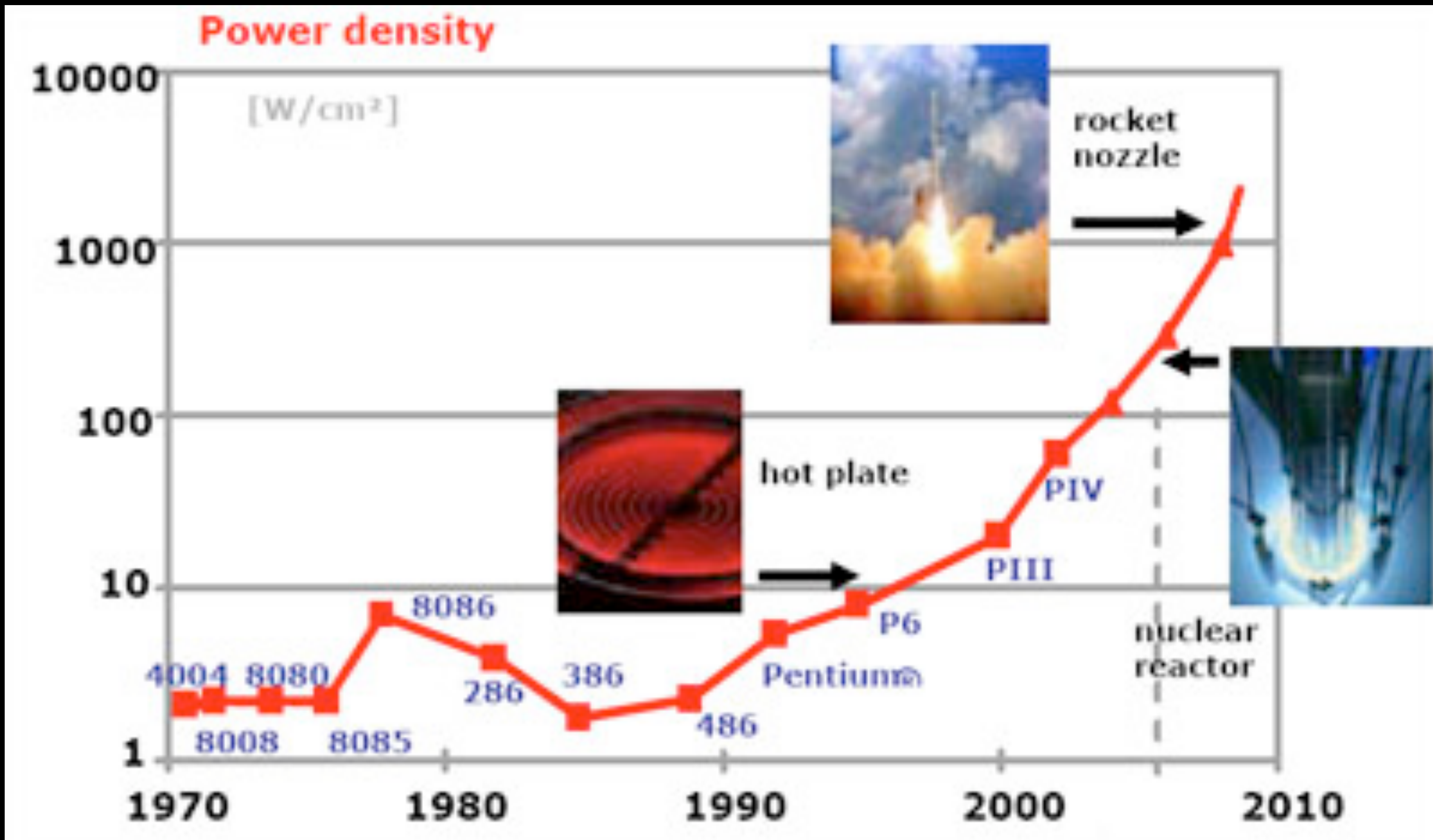


Evolution of computer architecture

- Modern microprocessors contain ~1 billion transistors
- Operate at power ensities (W/m^2) approaching a nuclear blast
- Wire scaling drives power demand on single-core chips

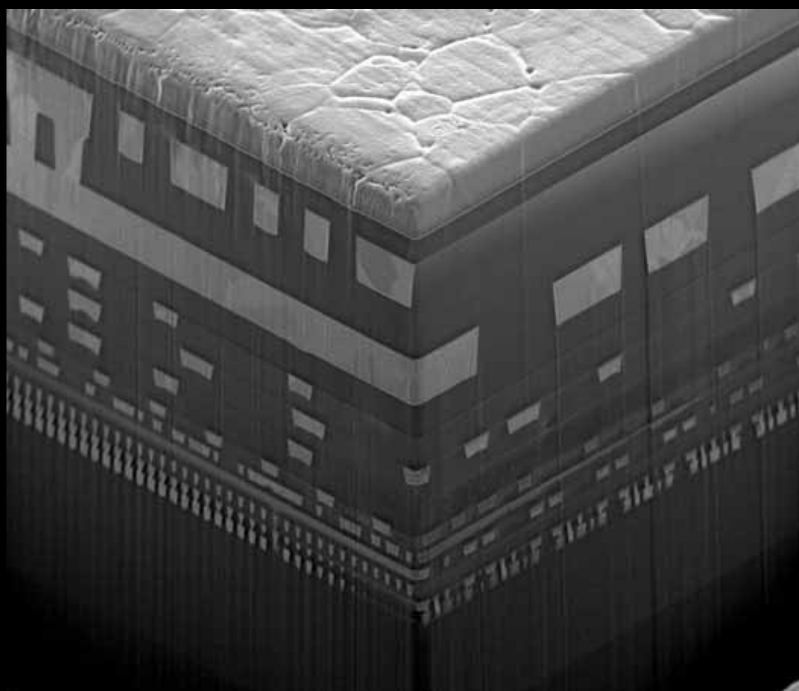
Why Power Scaling Matters

"The Cloud Begins with Coal" (2013)



SO: http://www.nanowerk.com/spotlight/spotid=1762.php

Wire Scaling



Cross section of twelve layers of interconnect

Map MST Model to Chips

(G. Bezerra, J. Brown, S. Forrest, M. Moses)

System	Node	Network	Resource	Organization
Organism	Service volume	Vascular systems	Blood	Centralized
Microprocessor	Transistor	Interconnect	Bits	Decentralized

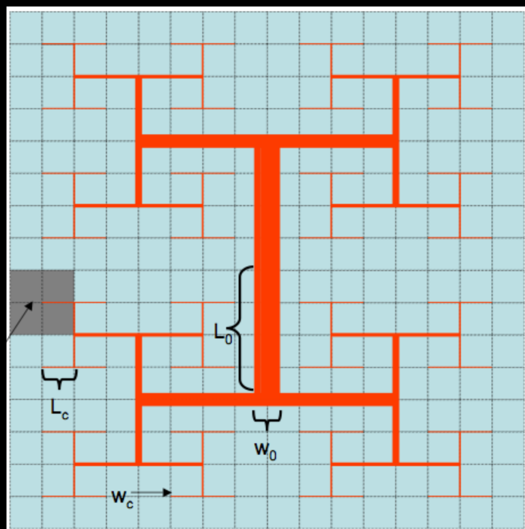
Complications:

- Interconnect is not purely hierarchical
- Transistor size is not constant
- Organisms are 3D, Chips are 2.5 D

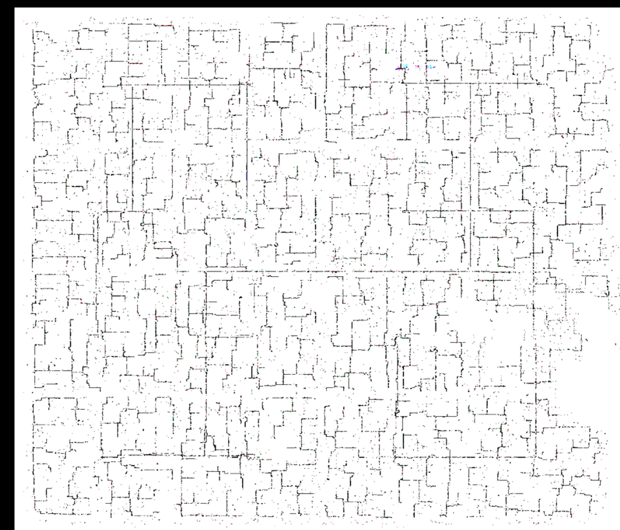
Scaling in Computing

- Computational complexity (Cook, 1970): Algorithm scaling
- Empirical observations but little theoretical framework elsewhere
  - Moore's Law, Rent's Rule, the 80/20 rule for software
  - Simulations

Clock Trees

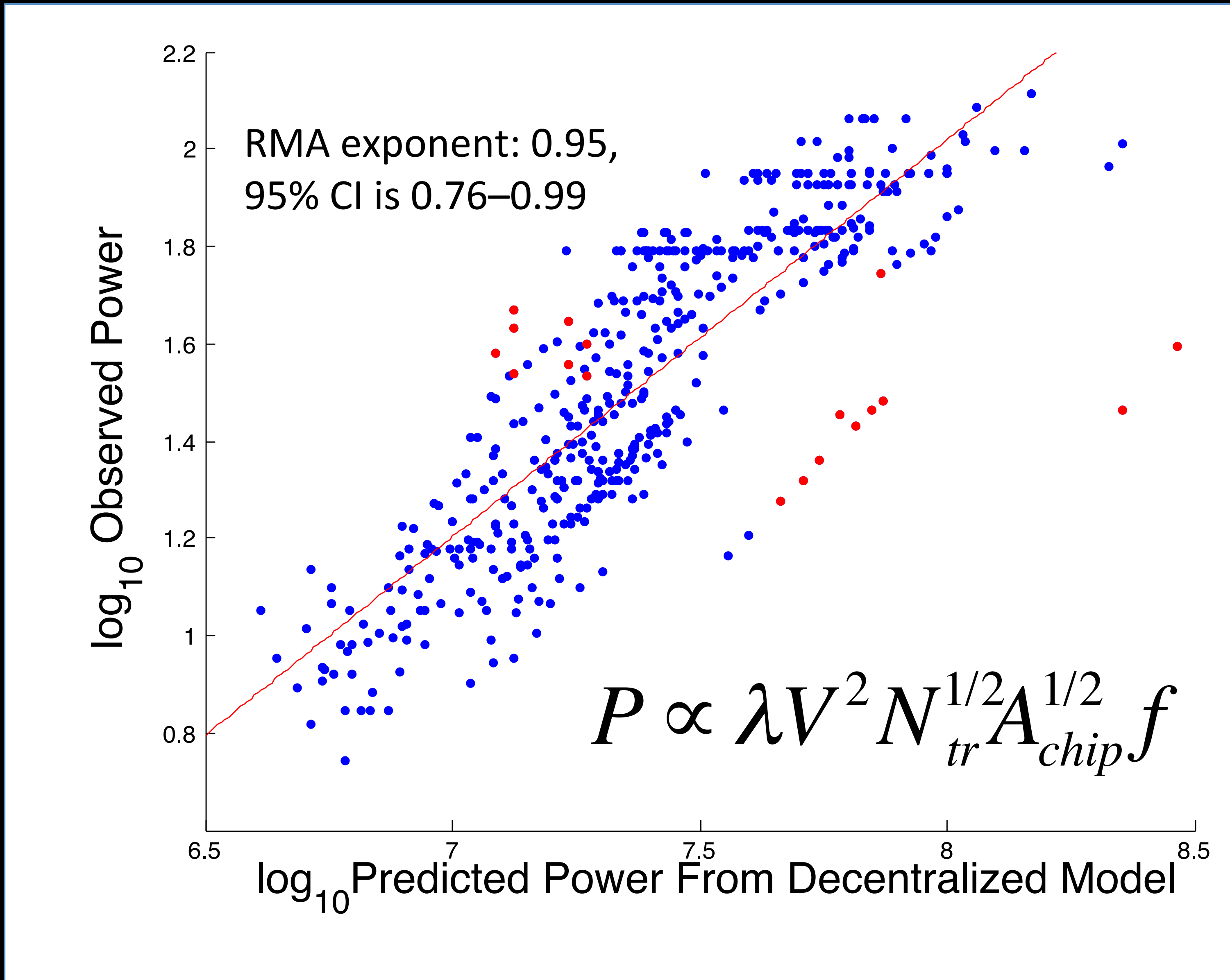


H-Tree Design



An Actual Clock Tree

- Clock trees consumed ~40% of chip power budget
- H-trees
  - Cross-sectional area preserving
  - Space filling
  - Equal path length to each point (helps skew)

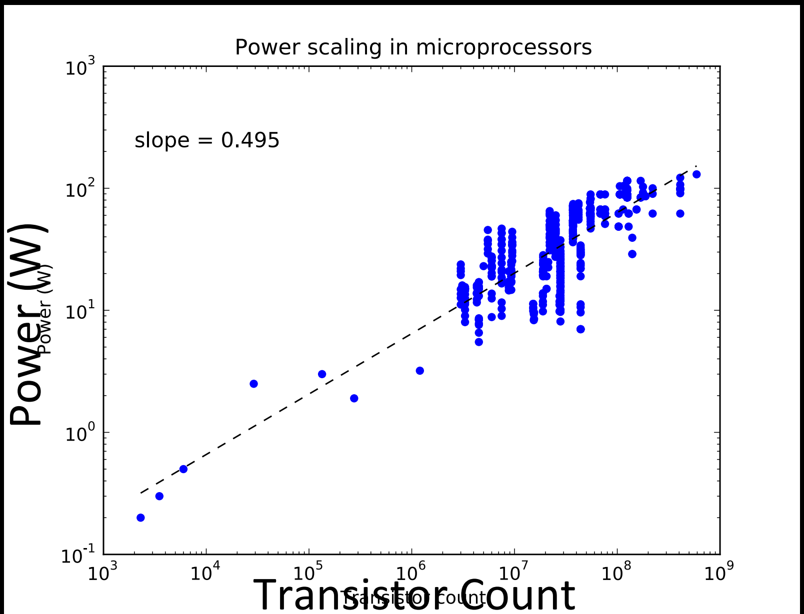


Engineering and economic imperatives are similar to natural selection

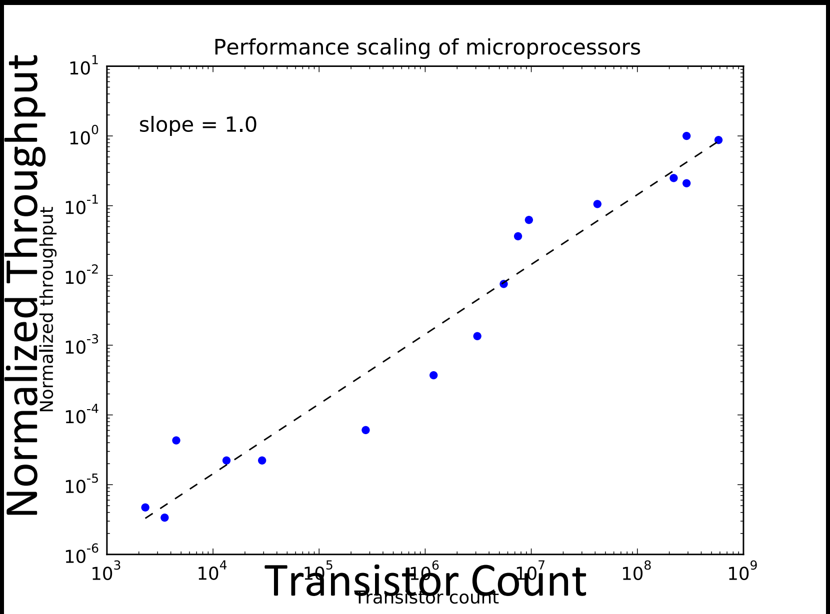
Next Steps

- Order-of-magnitude and beyond
- Multi-core architectures
- The energy-delay product

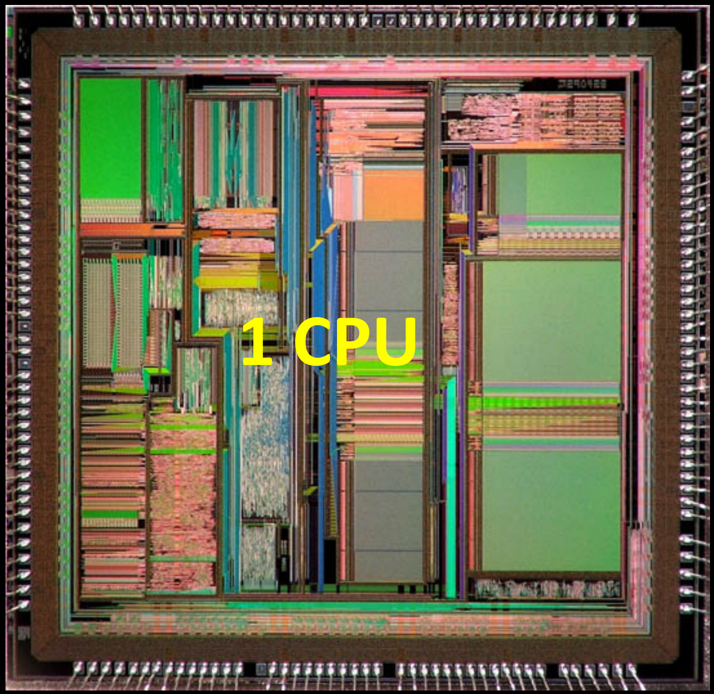
Power scaling in microprocessors



Performance scaling of microprocessors

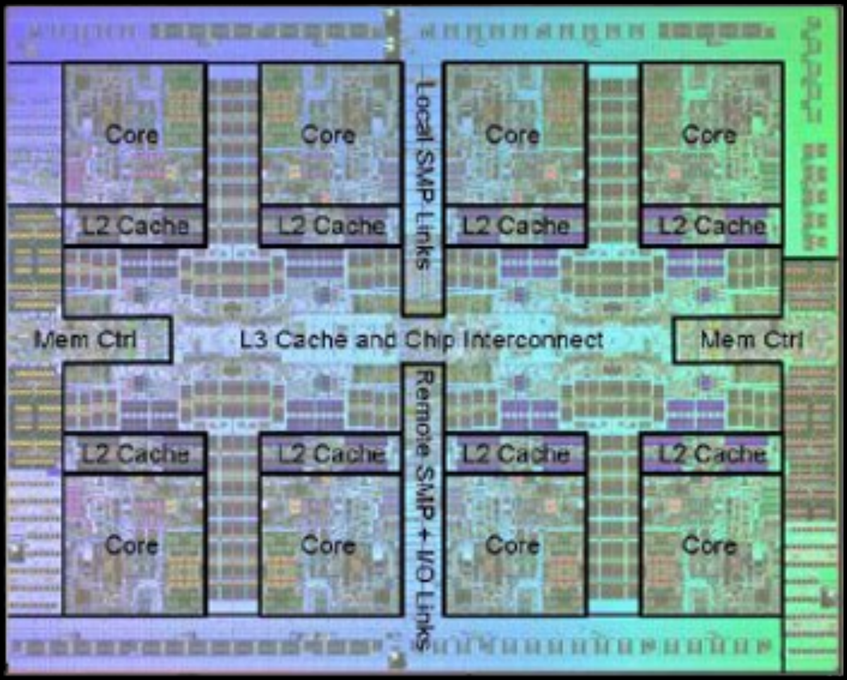


MONOLITHIC



- Performance only
- Serial processing
- High clock frequency
- High power consumption

MULTI-CORE



- Power aware
- Parallel processing
- Low clock frequency
- Energy efficient
- Shared resources (L3 cache)

Physical and Geometric constraints determine network architecture and growth

- Network capacity limits performance as systems scale
- Metabolism, response times, power consumption
- Are universal patterns in system behavior predictable from the scaling properties of distribution networks?