CS 361
Data Structures & Algs
Lecture 14

Prof. Tom Hayes
University of New Mexico
10-7-2010

Last Time

- Inside BFS
  - Running Time
  - Implementation
  - Degrees & Degree sums
  - Properties of BFS and DFS trees
- Identifying trees in the wild

Today

More tree forensics

READING: Finish reading Chapter 3

identifying BFS & DFS trees

Where is the root?

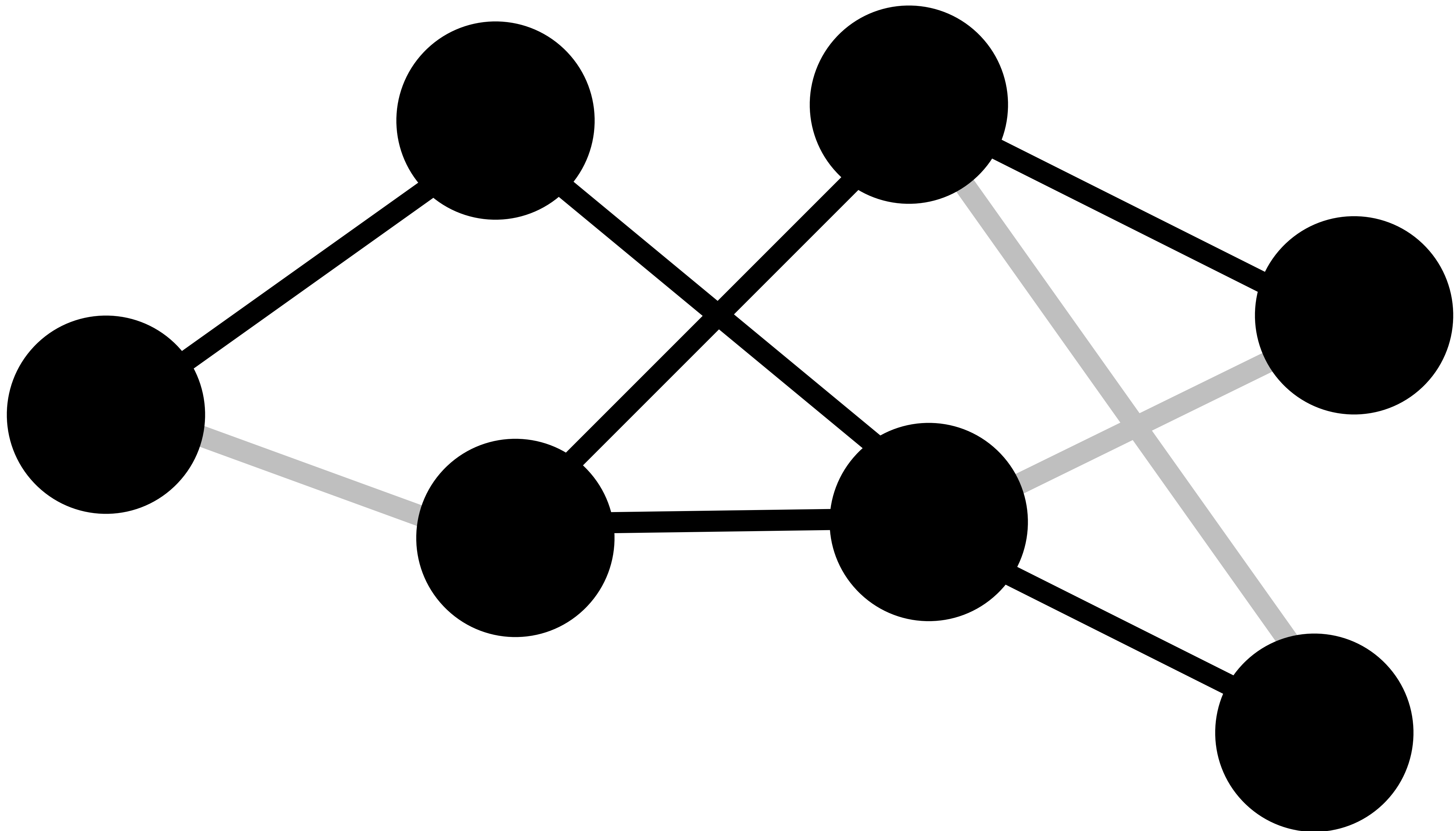
Key: Edges left out of the tree:

- in BFS, level difference is {-1, 0, 1}
  - not incident with the root.
- in DFS, join a node to its ancestor

Welcome to the morgue!

Somebody constructed this tree.
Please help me identify it.

BFS or DFS tree?



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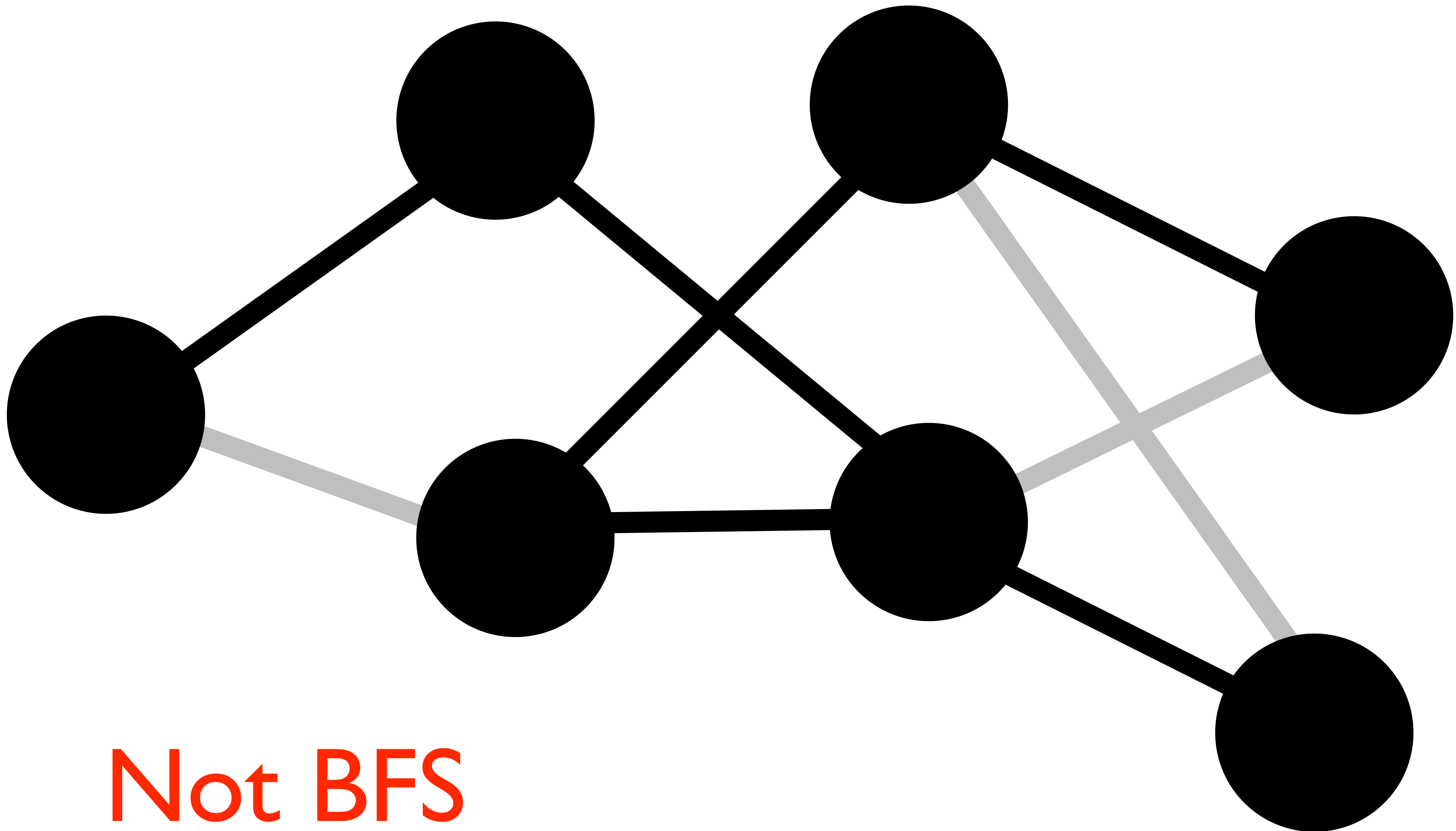
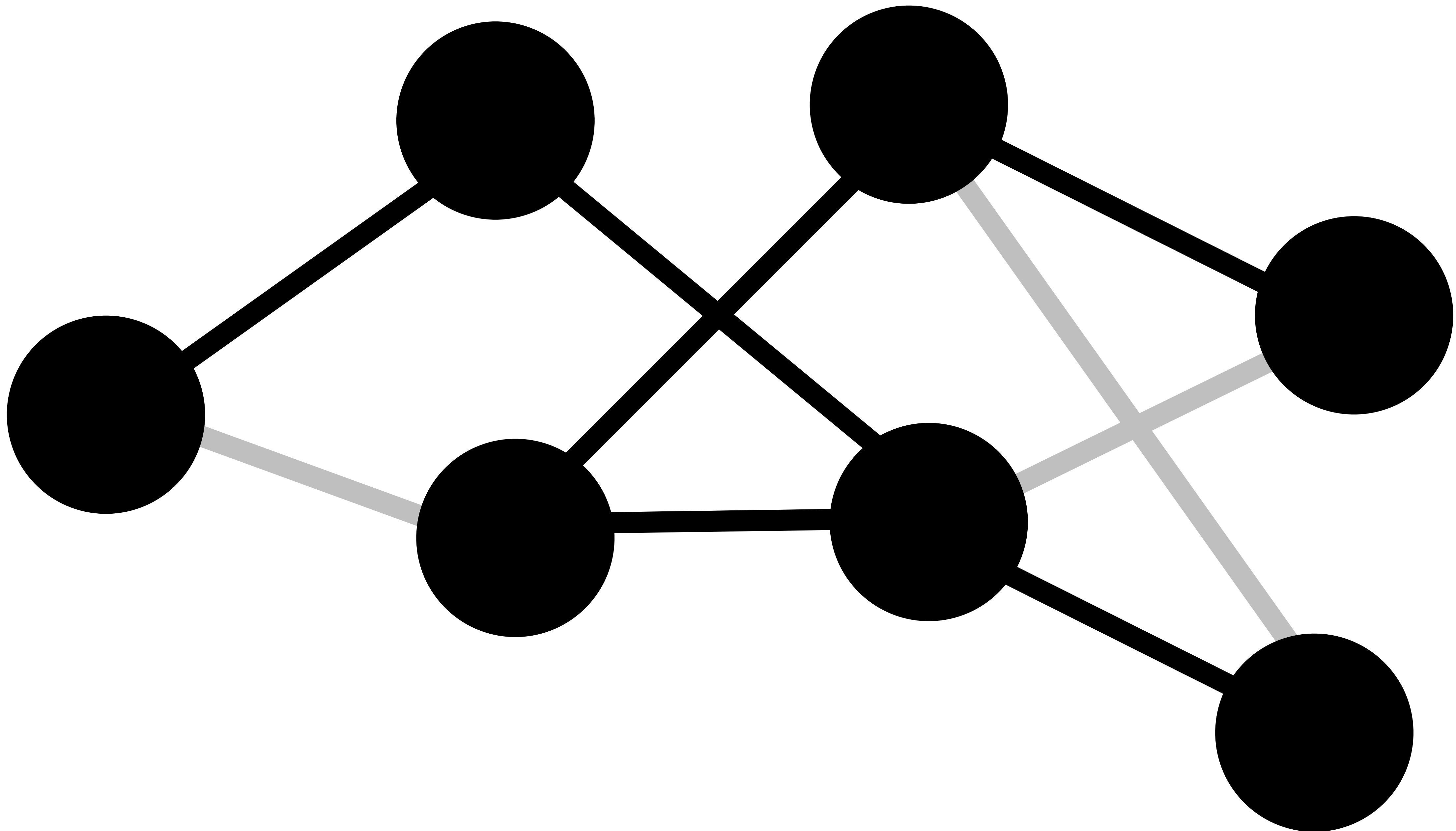


Not BFS

Where is the root (DFS)?



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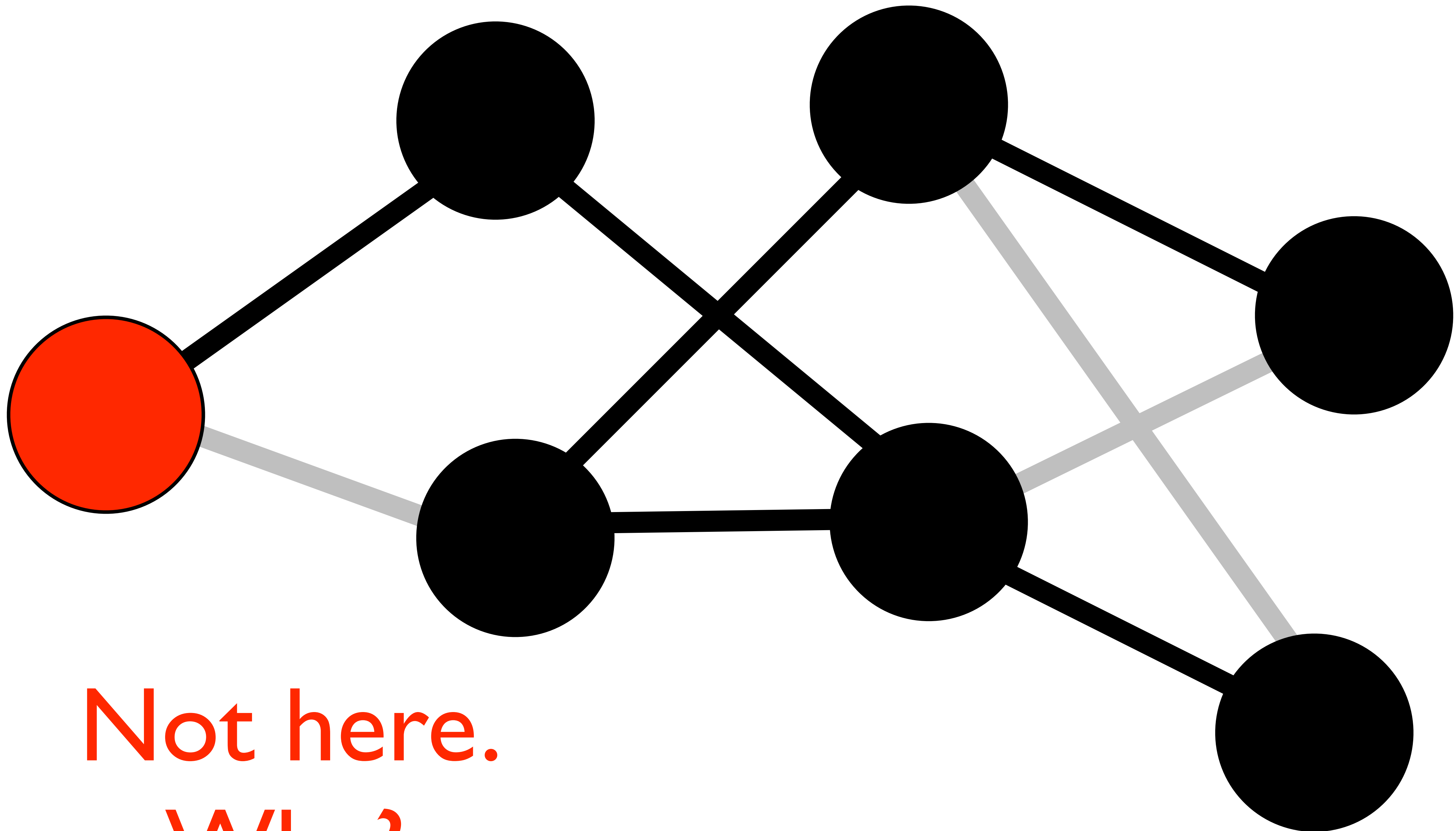

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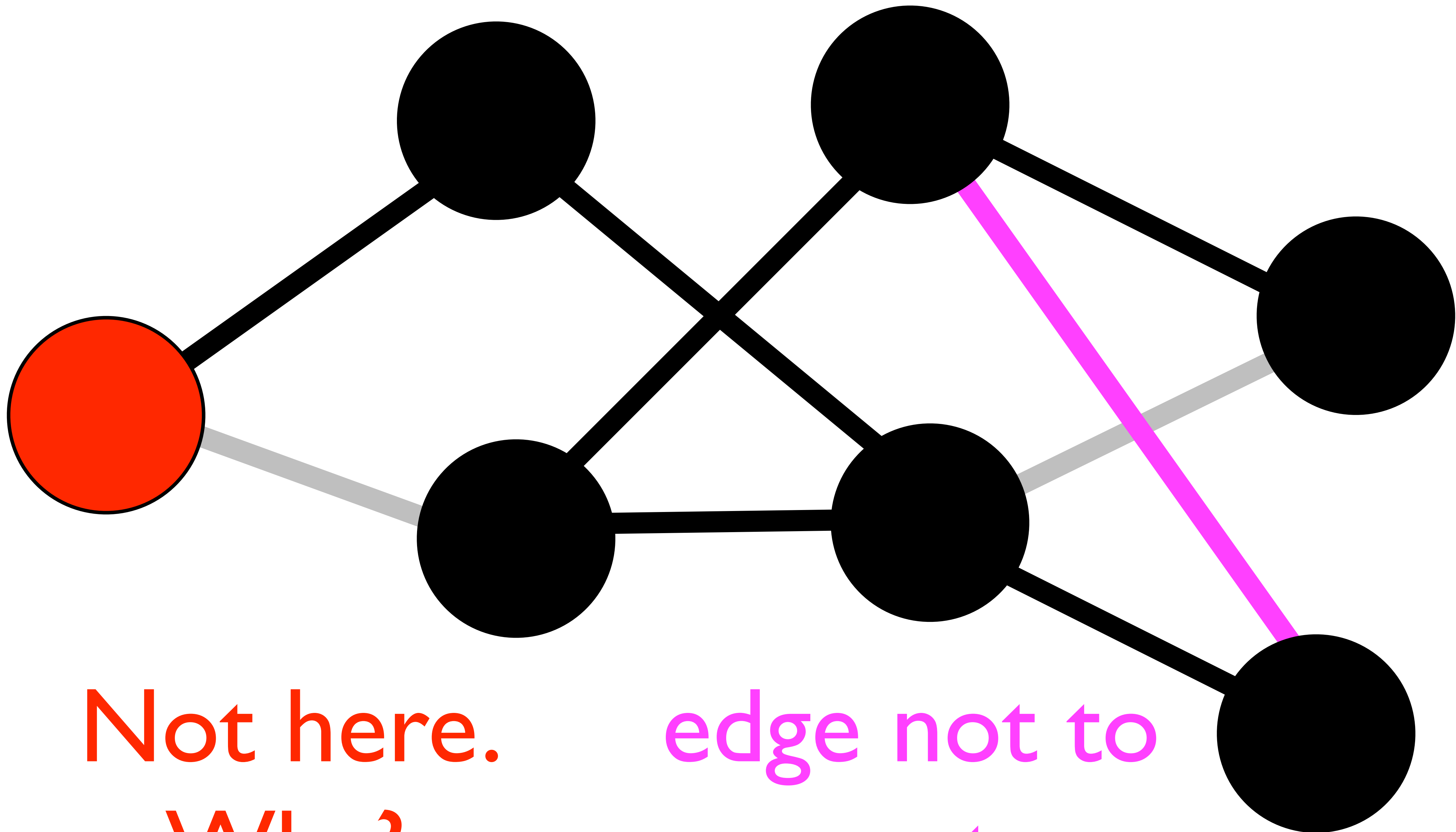

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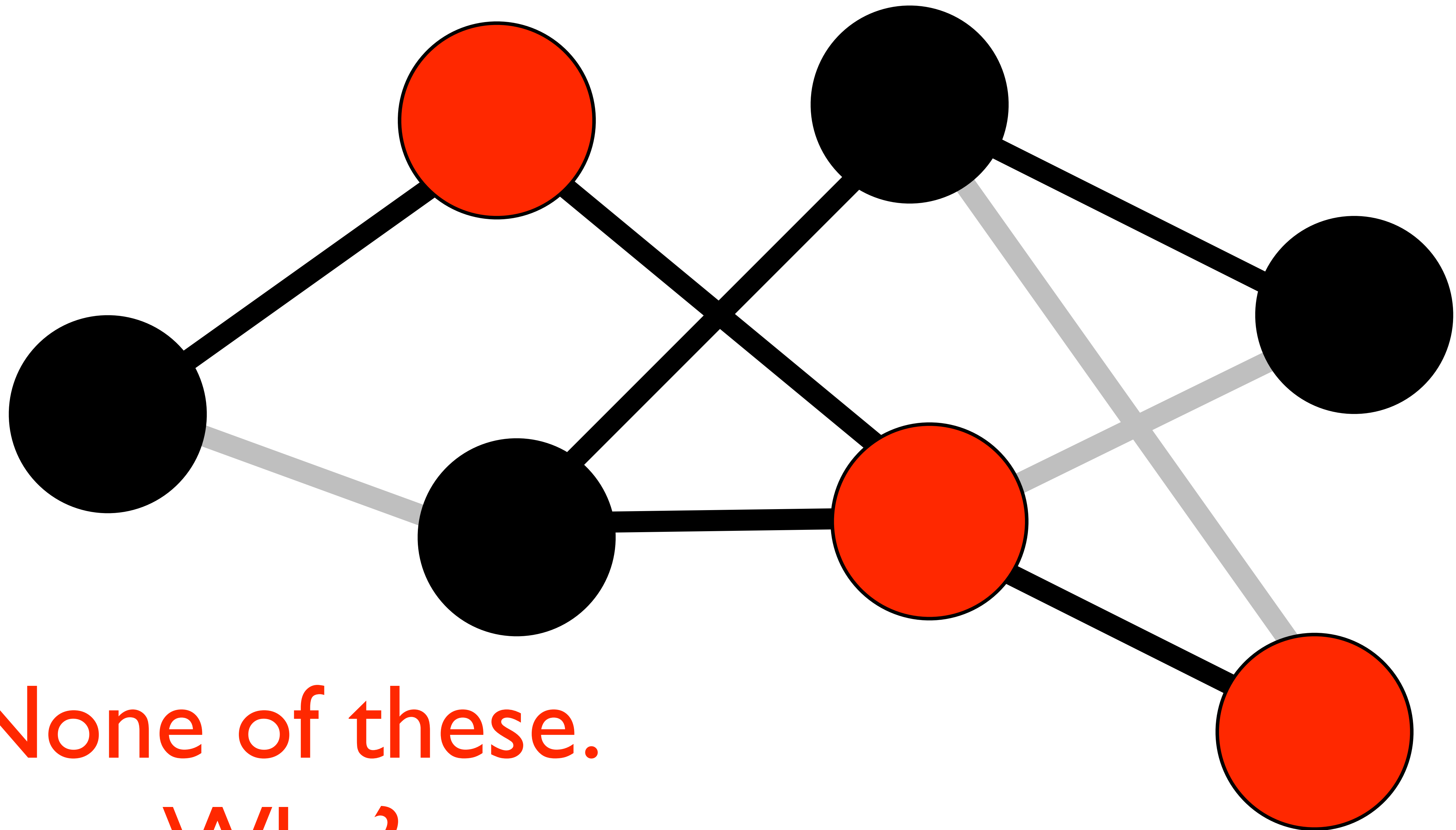

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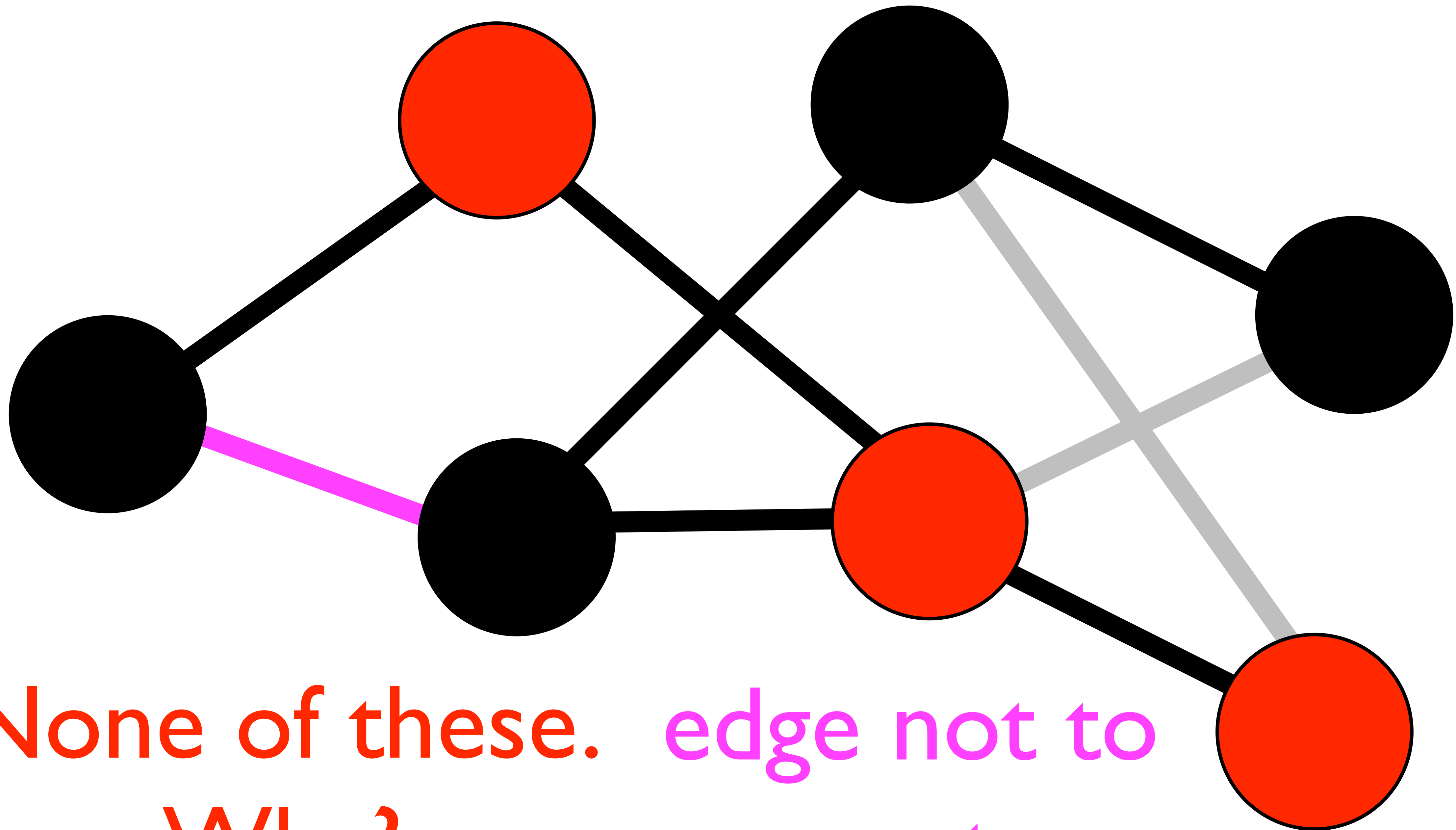

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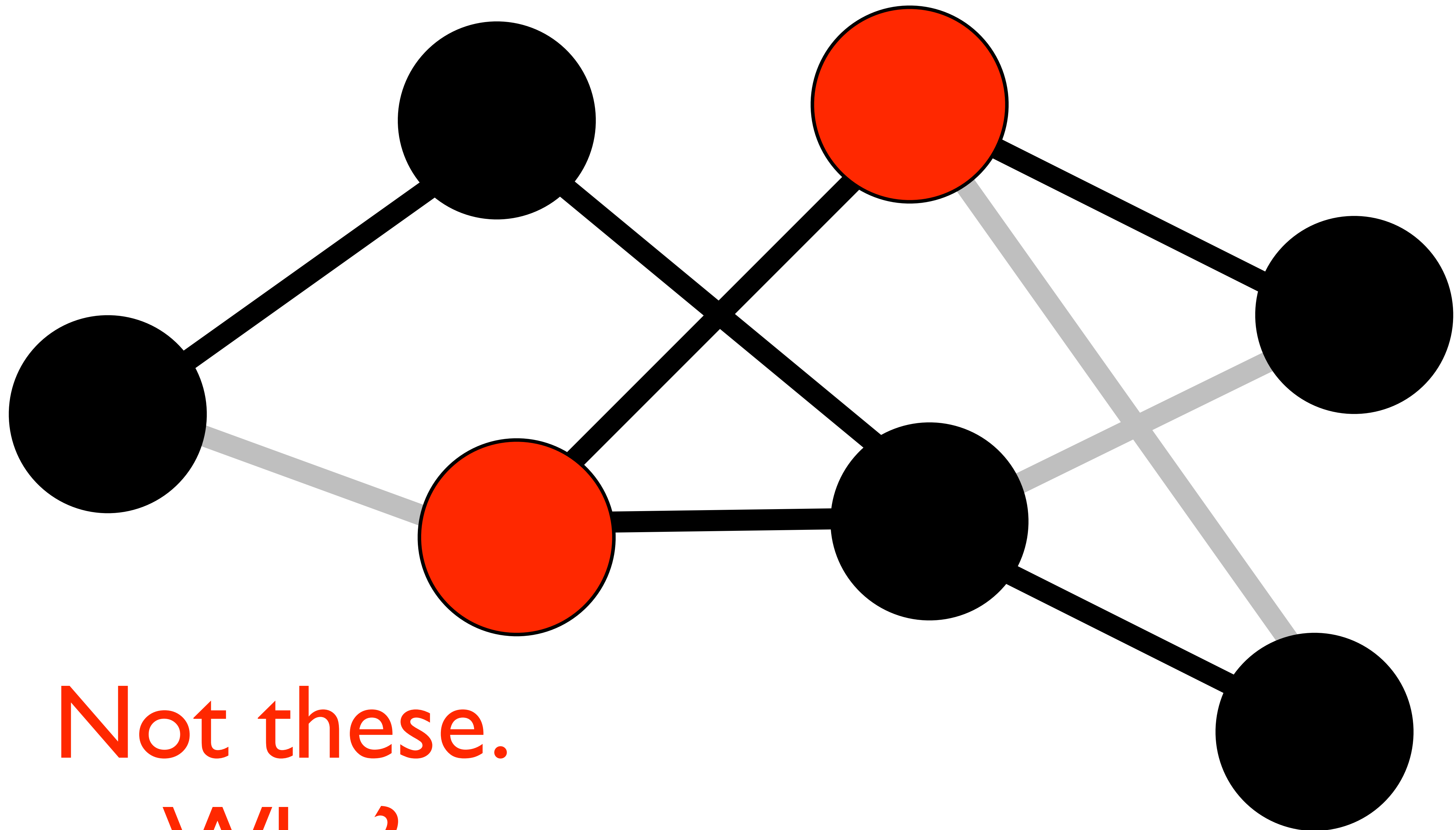
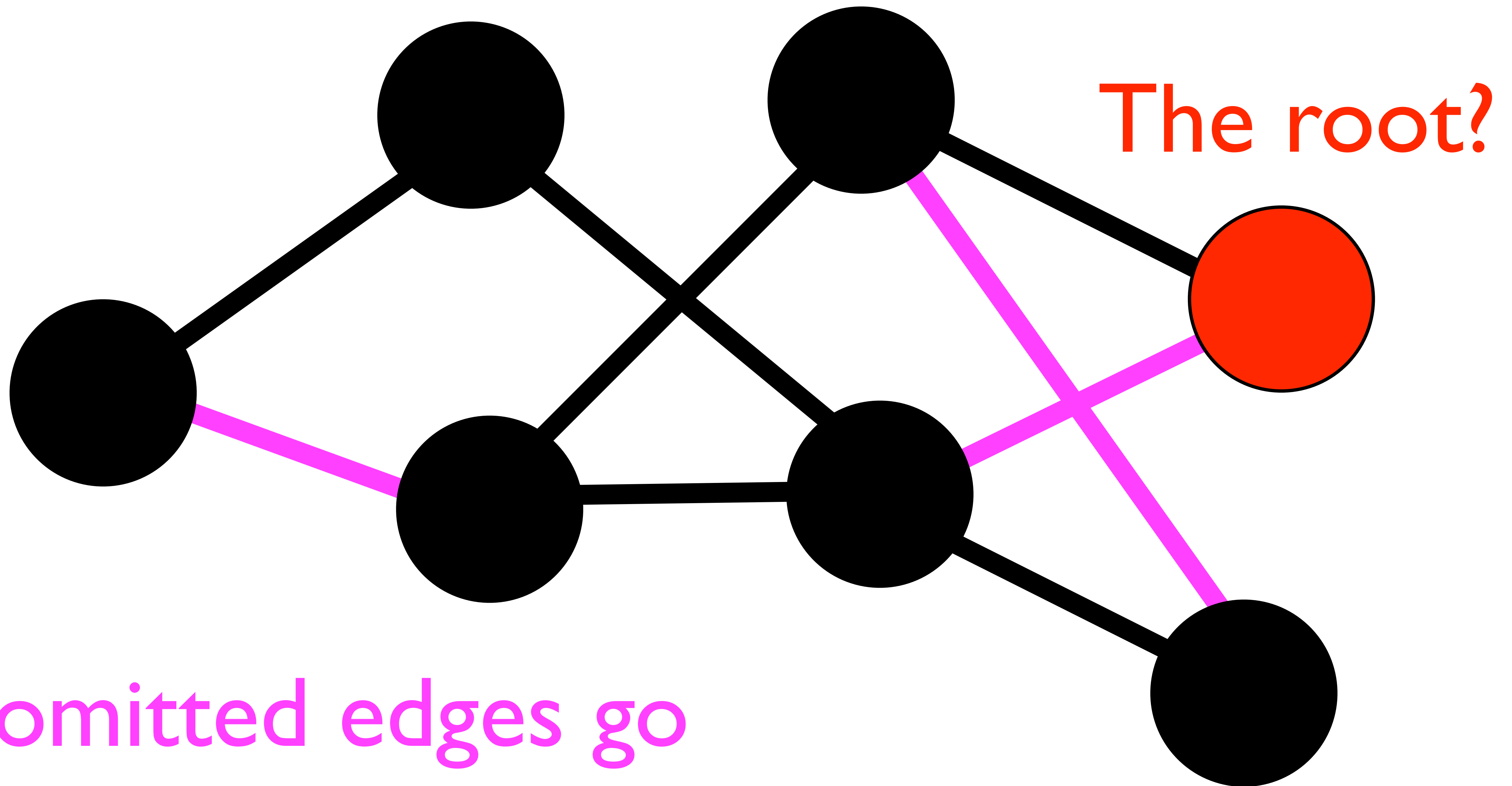


Where is the root (DFS)?

Not these.
Why?

edge not to
ancestor

Where is the root (DFS)?



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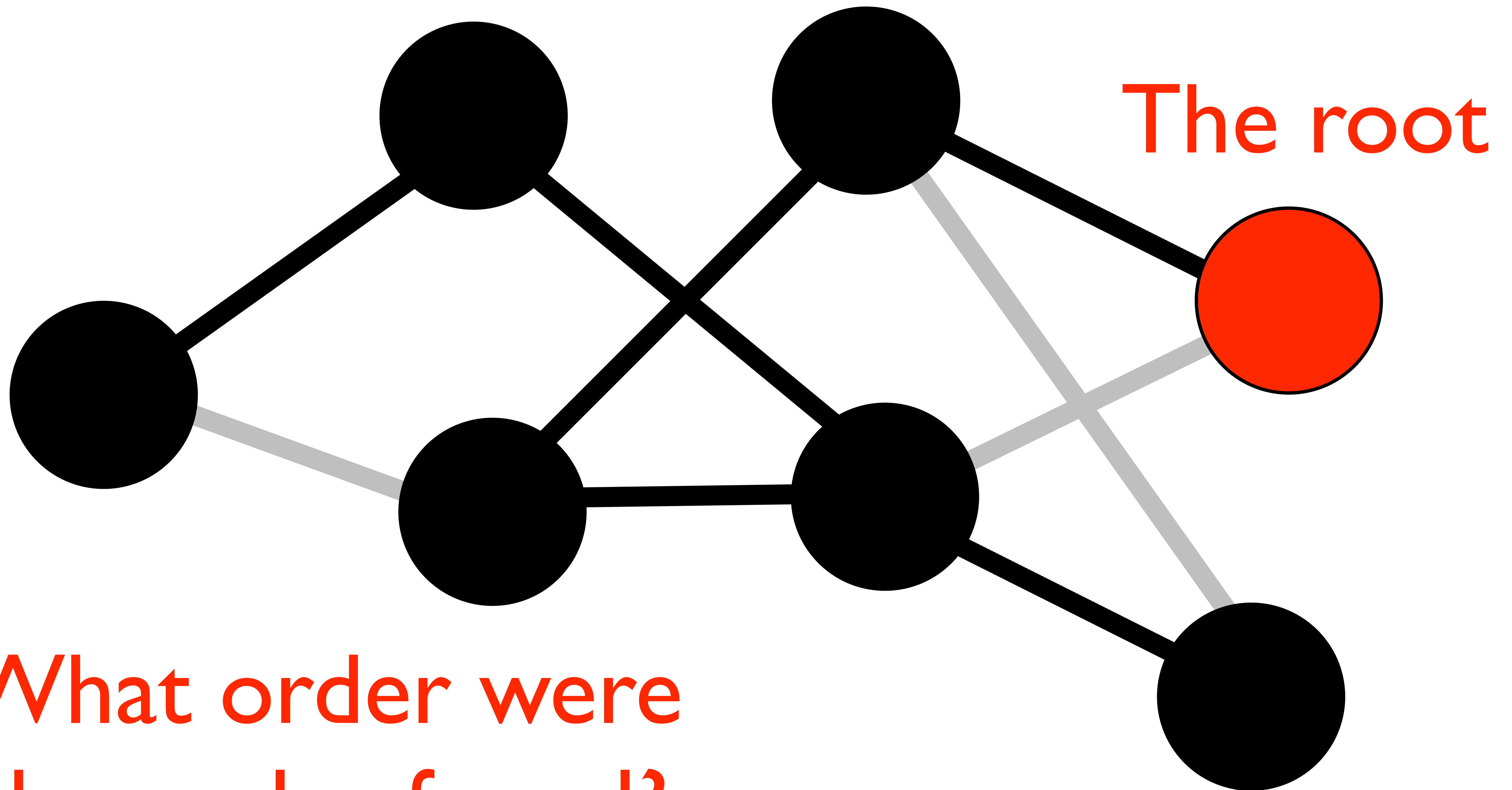

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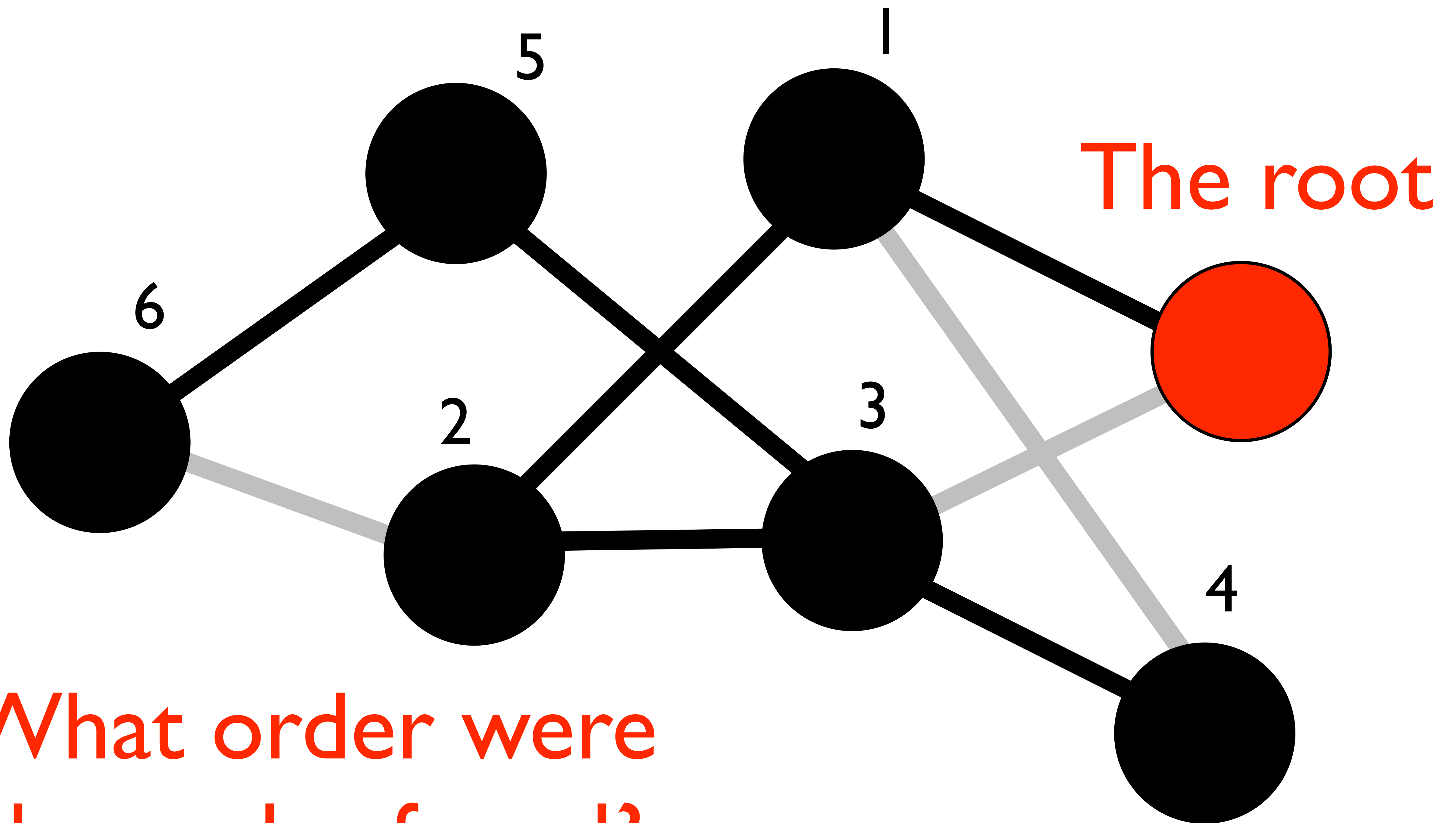

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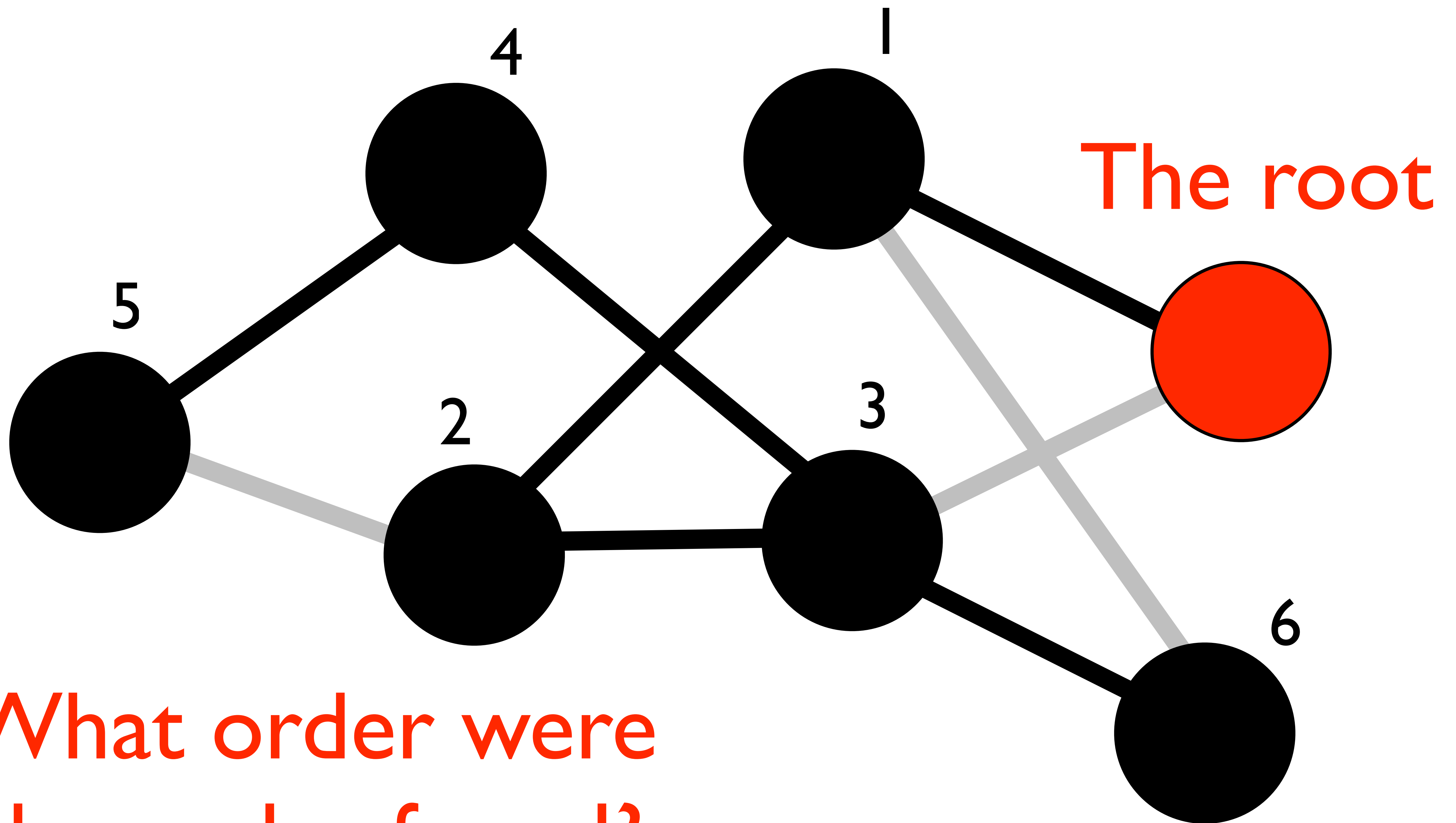

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Given: Connected graph G and vertex u. DFS and BFS, both started from u, give us the same tree, T.

Prove: G = T.

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Since T is a DFST, we know: ....

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Homework: Finish this proof.

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Level 0 = {s}. Level($n/2 + 1$) contains t.

No two levels overlap. Some level i has size 1! ($1 \le i \le n/2$) HW: Finish this proof.

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Input: An undirected graph G.

Output: A cycle in G, if one exists, else throw new UnexpectedTreeException( )

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BFS or DFS?

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Let T be a DFS tree for G, rooted at u.
Now what?

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Let T be a DFS tree for G, rooted at u. Now what? Suppose {v,w} is a back-edge for T.

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Let T be a DFS tree for G, rooted at u. Now what? Suppose {v,w} is a back-edge for T. w or v is ancestor of the other

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Let T be a DFS tree for G, rooted at u. Now what? Suppose {v,w} is a back-edge for T. Say w is ancestor of v.

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Let T be a DFS tree for G, rooted at u. Now what? Suppose {v,w} is a back-edge for T. Say w is ancestor of v. Found cycle: v, p(v), p(p(v)), ..., w, v.

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Let T be a DFS tree for G, rooted at u. Now what? Suppose {v,w} is a back-edge for T. Say w is ancestor of v. Found cycle: v, p(v), p(p(v)), ..., w, v.

What's left?

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What's left? 1) Why is this cycle?

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What's left? 1) Why is this cycle? 2) What if there is no back-edge?

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What's left? 1) Why is this cycle?
2) What if there is no back-edge?
3) What if G is not connected?

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Let T be a DFS tree for G, rooted at u. Now what? Suppose {v,w} is a back-edge for T. Say w is ancestor of v. Found cycle: v, p(v), p(p(v)), ..., w, v.

HW: Finish proof

DFS: recursive version

```
Initialize: Mark all nodes Unfound
DFS(v):
  Mark v as Found
  For (w in Adj[v]):
    If (w is Unfound)
      {parent(w) = v;  DFS(w);}
```

DFS: iterative version

```
Initialize:
Active = new stack with just s.
Mark all nodes Unfound
while (Active is nonempty) {
  v = Active.pop( );
  if (v is Unfound) {
    Mark v Found;
    For (w in Adj[v]):
        Active.push(w);
    }
```

Compare: BFS

```
Initialize:
Active = new queue with just s.
Mark all nodes Unfound
while (Active is nonempty) {
  v = Active.remove( );
  if (v is Unfound) {
    Mark v Found;
    For (w in Adj[v]):
        Active.add(w);
    }
```

Both versions of DFS are implemented using a Stack!

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(Recursive version uses the function-call stack)

Next Topic: Directed Graphs (aka "digraphs")

Directed Graphs

A directed graph is like a graph, but the edges are ORDERED pairs of vertices, rather than unordered pairs of vertices.

For normal graphs, we denote edges in curly braces: {u,v}   (unordered)

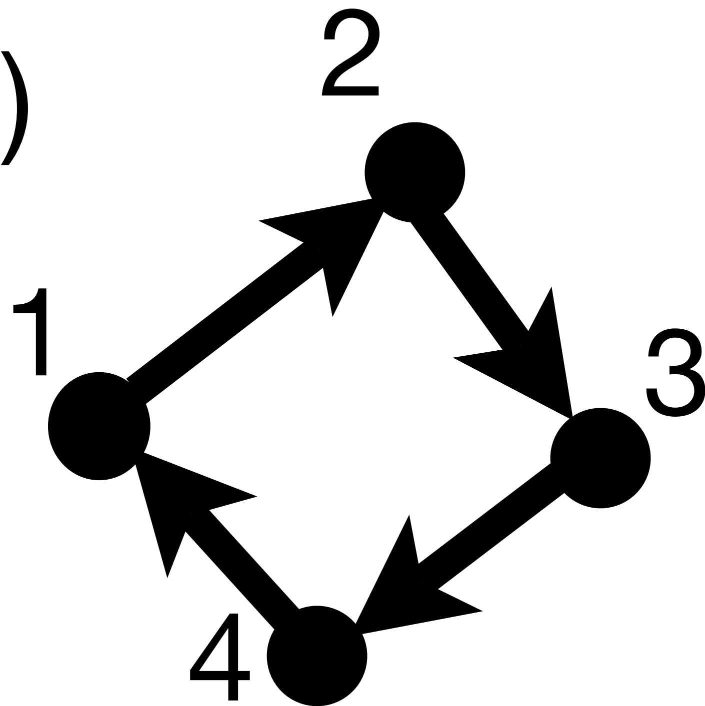
For directed graphs, parentheses: (u,v) (ordered: “from u to v”).
Directed edges are also called “arcs”

Directed Graphs

Example: $V = \{1,2,3,4\}$

$E = \{ (1,2), (2,3), (3,4), (4,1)\}$.

This example is known as a "directed cycle" or "oriented cycle" (of length 4)



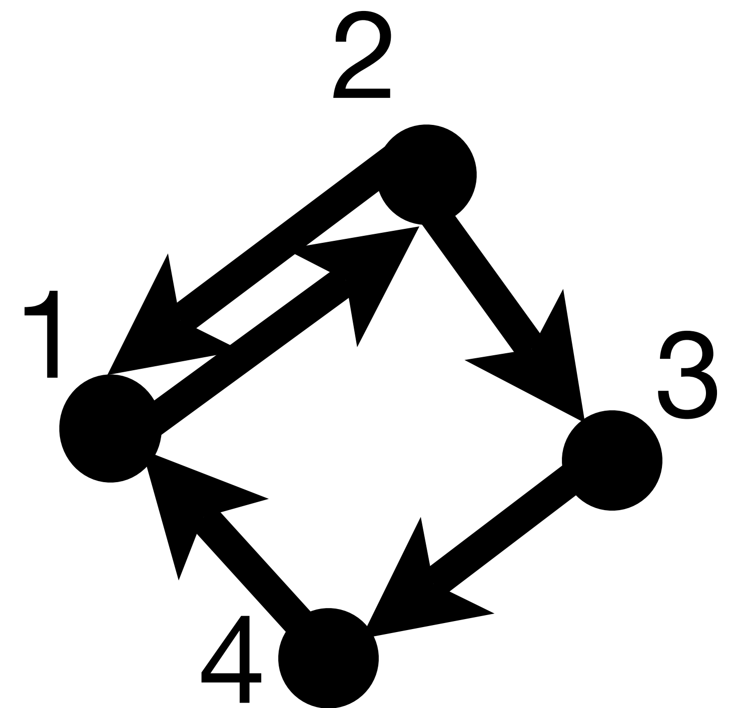
Representing DiGraphs

For directed graphs, we need 2 adjacency lists for each node. edges INTO v, and OUT FROM v.

2 arrays of Lists:

in-neighbors[2] = {1}

out-neighbors[2] = {1,3}

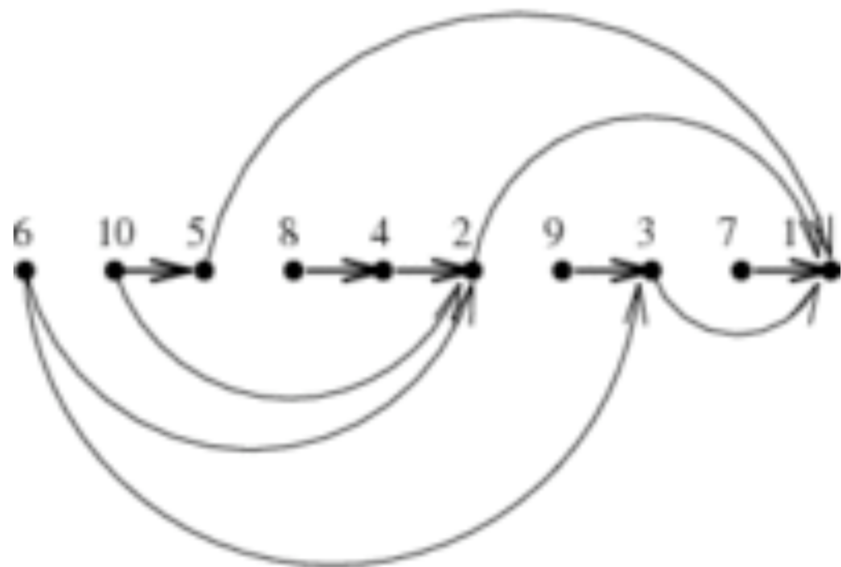


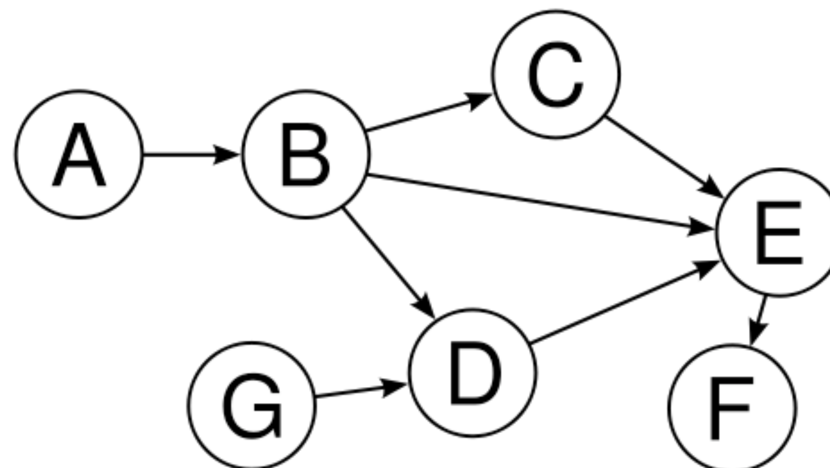
DAGs

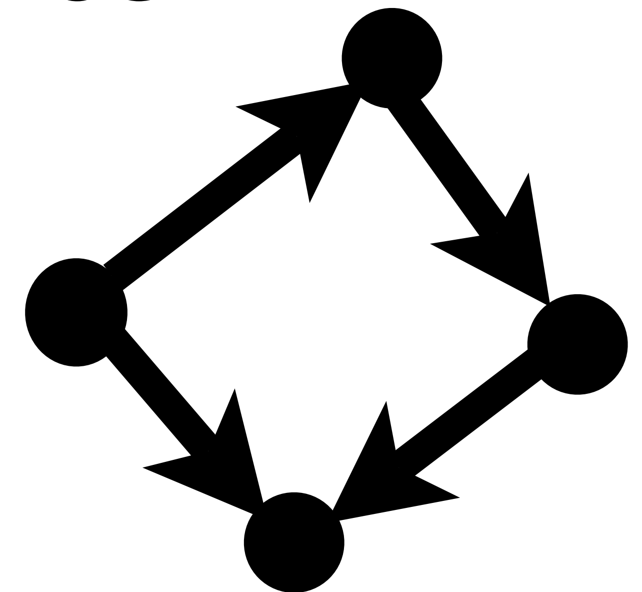
A directed graph is called acyclic if it has no oriented cycles.

Meaning: you can’t get back where you start if you always follow arrows.

The “underlying graph” (just lose all the orientation info) may have cycles.



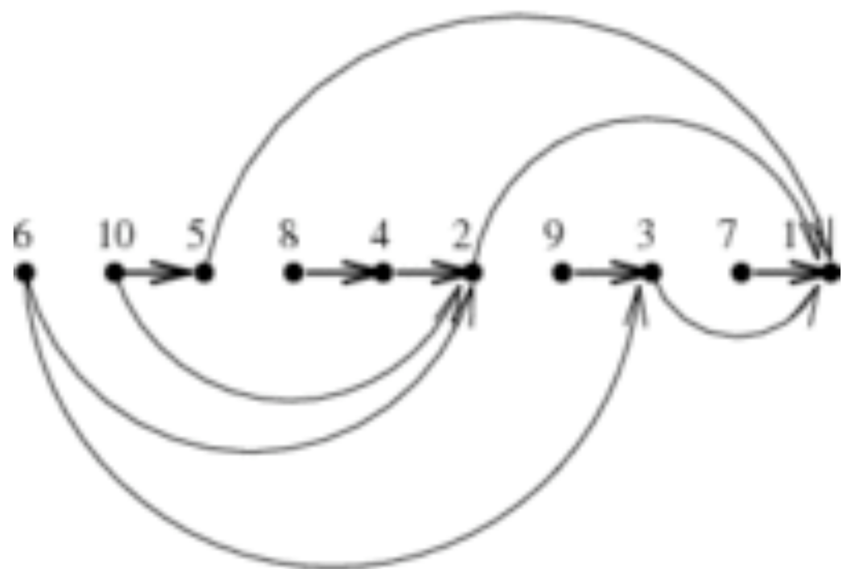


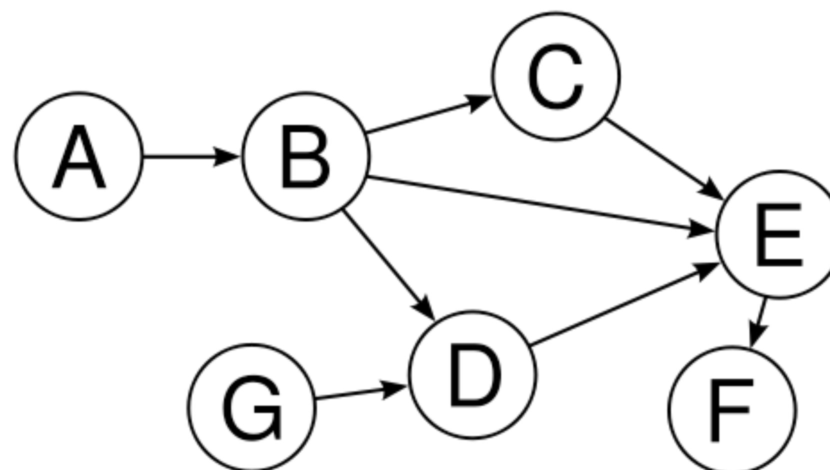


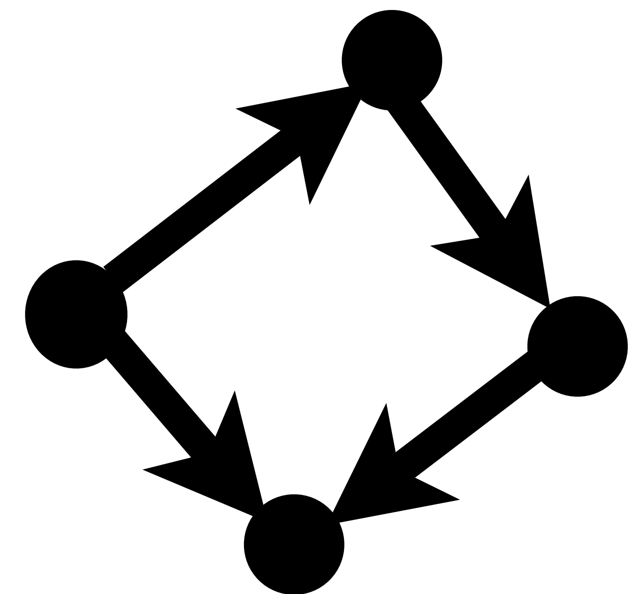
DAGs

Q: How do we tell if a graph is a DAG? Alternatively, how do we find an oriented cycle if there is one?

Look at the Left example. The nodes are in a line, all the edges go left-to-right. This is called a topological sort.







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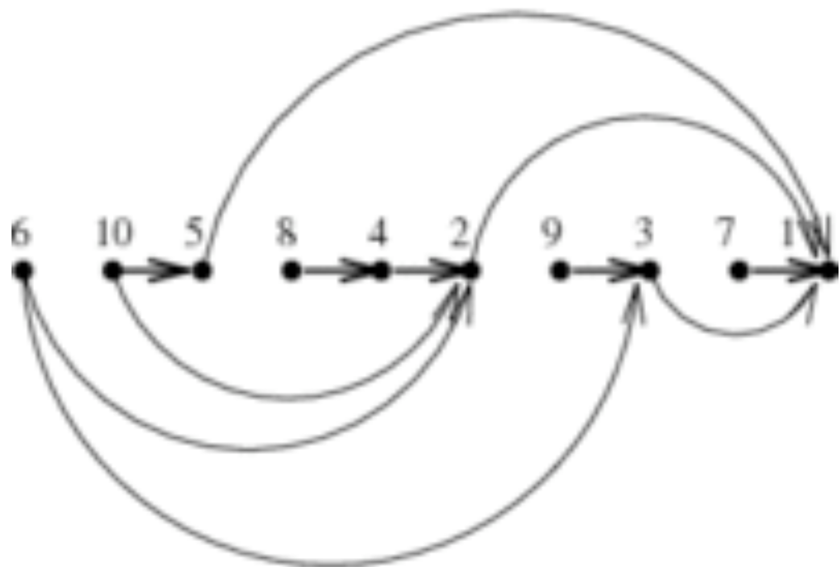
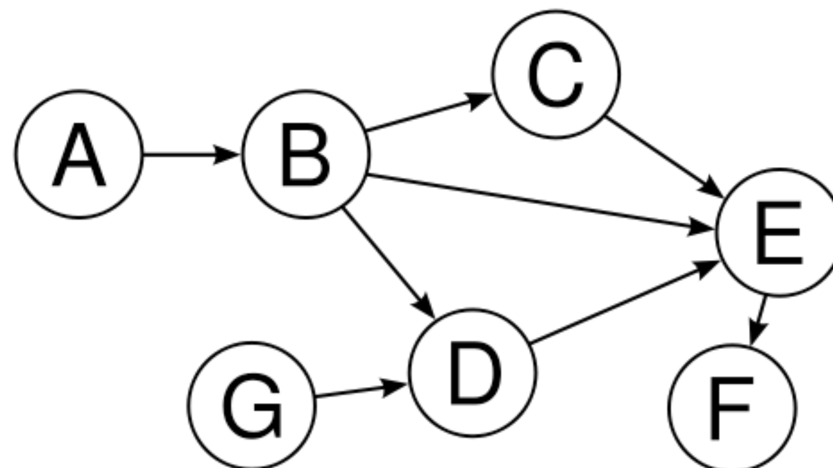
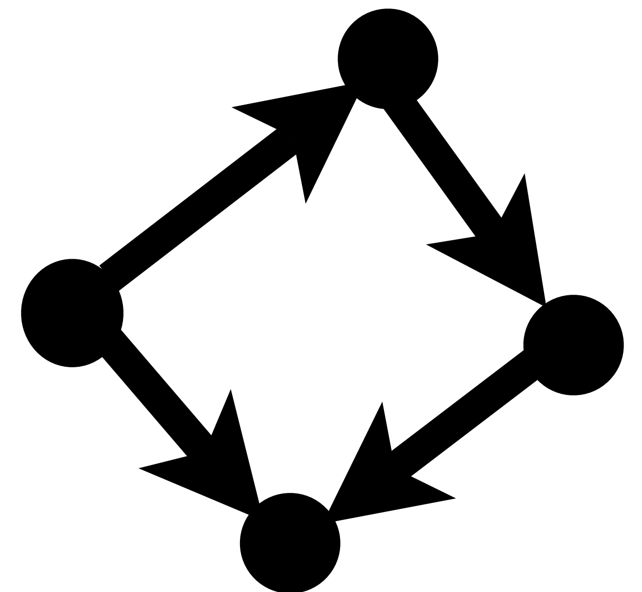
Thm: G has a topological sort iff G is a DAG



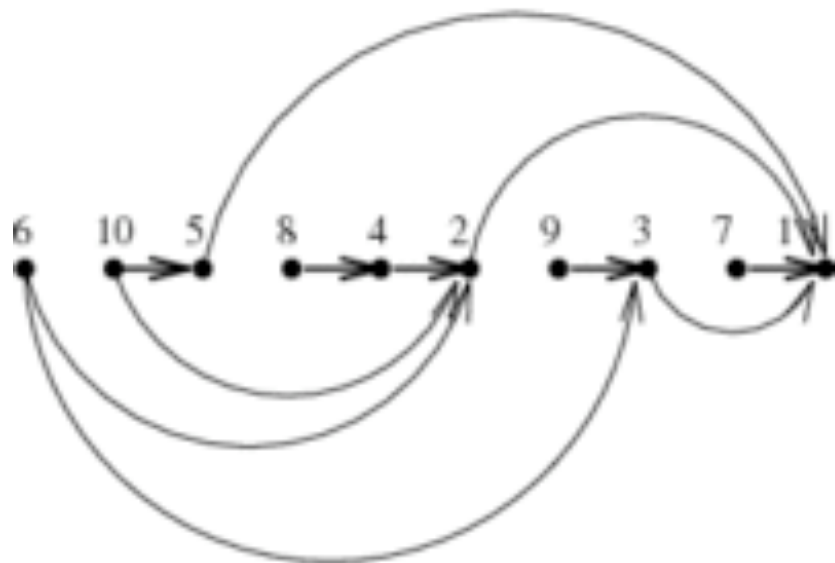


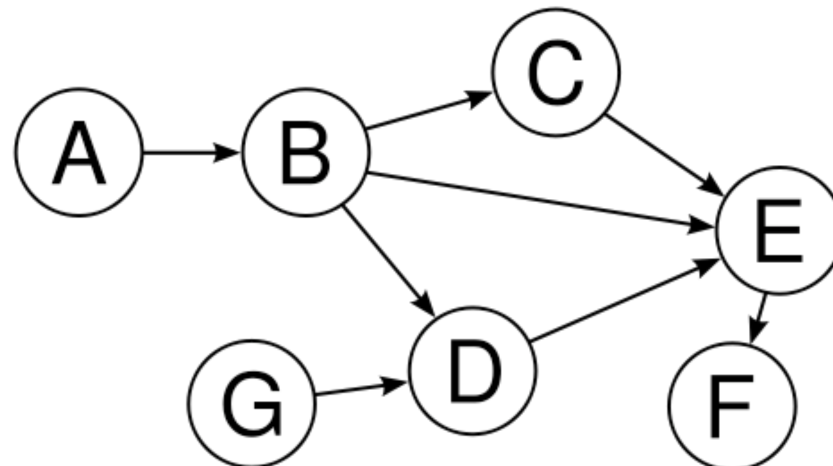


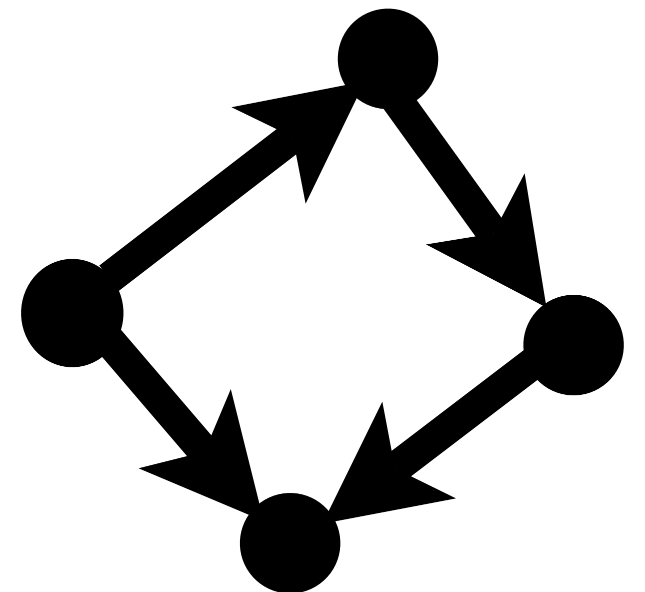
Topological Sorting

Q: How do we tell if a graph has a topo. sort?

Look at the left example. The leftmost node has no in-edges. Is this always the case?







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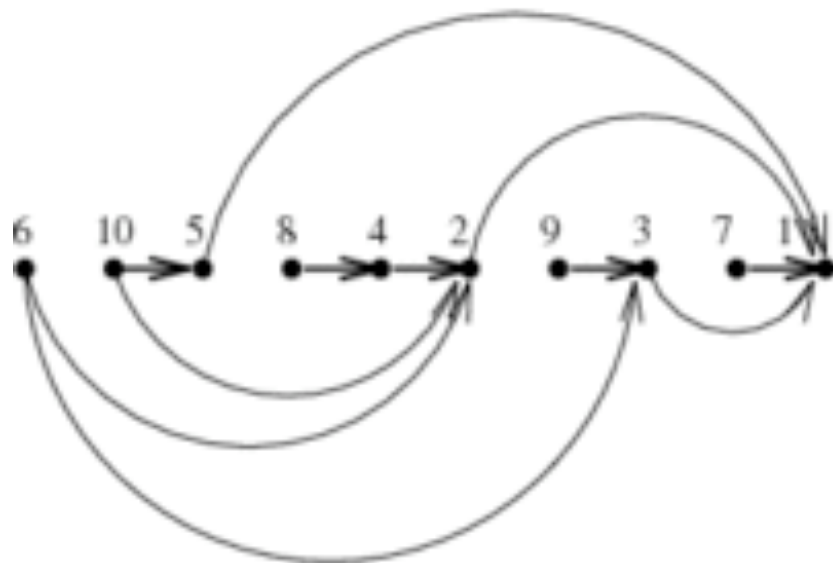
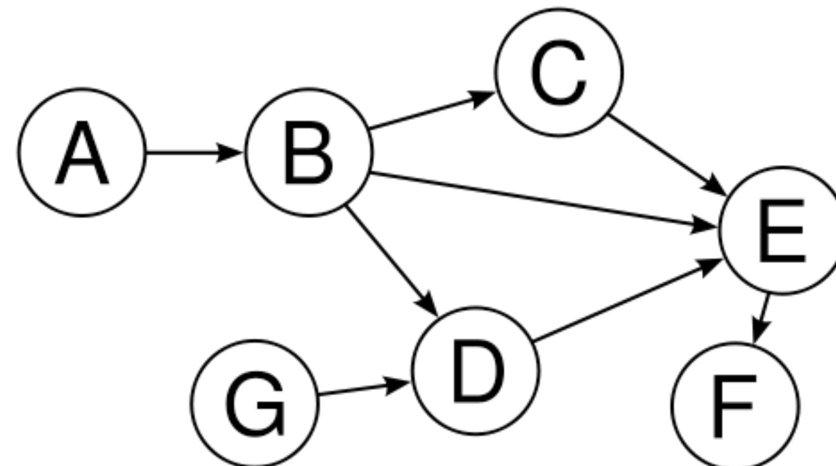
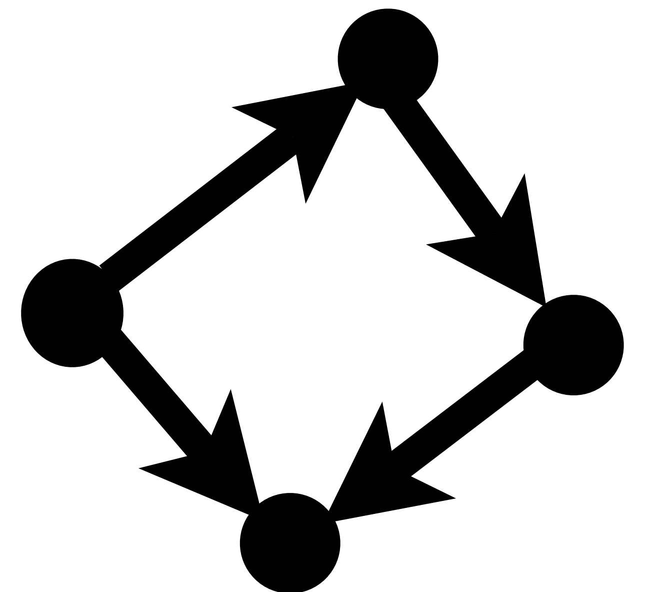
Idea: Find the leftmost node. Recurse!







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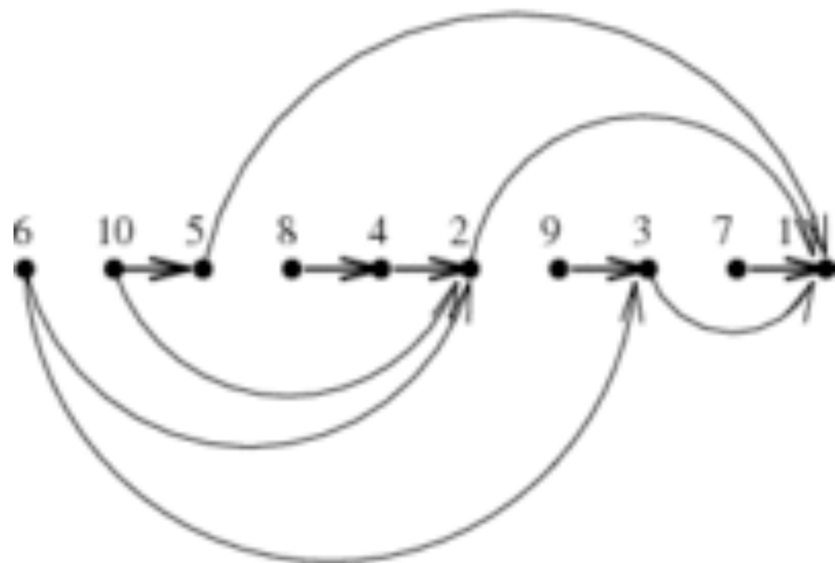
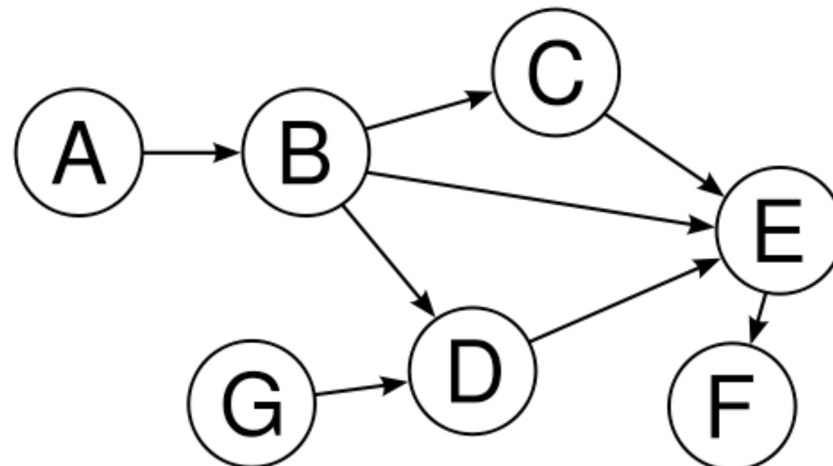
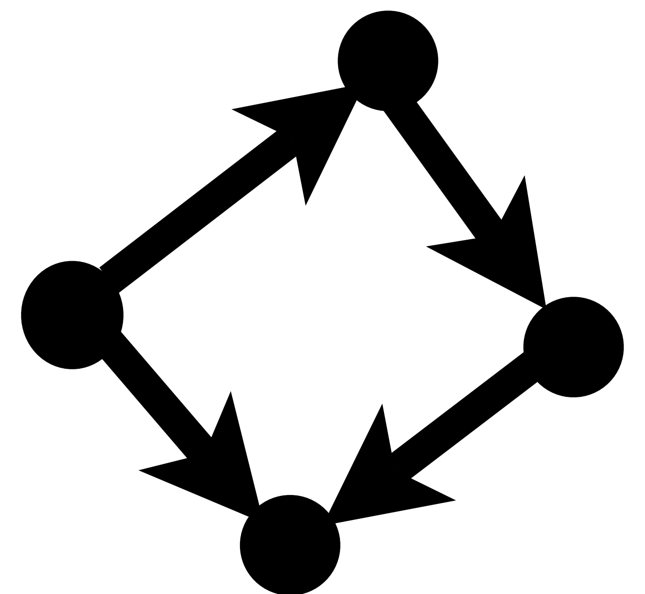
Implementation issues?

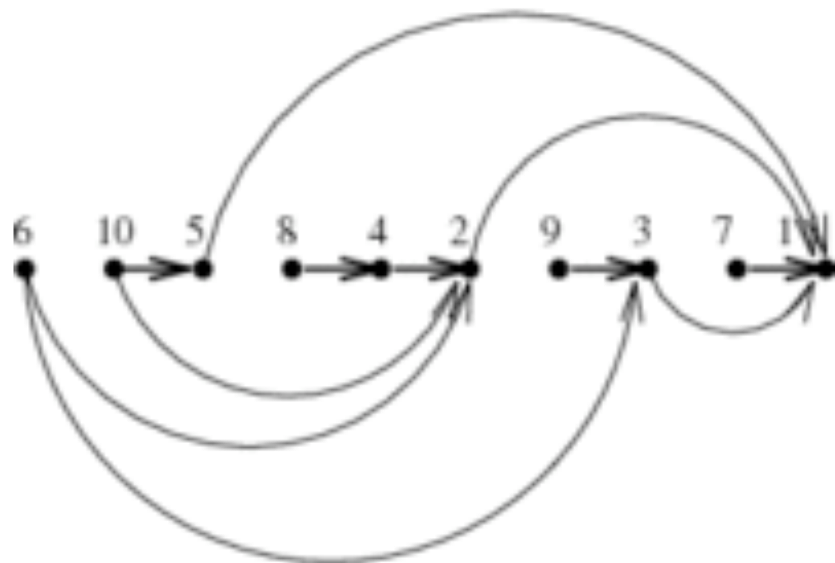


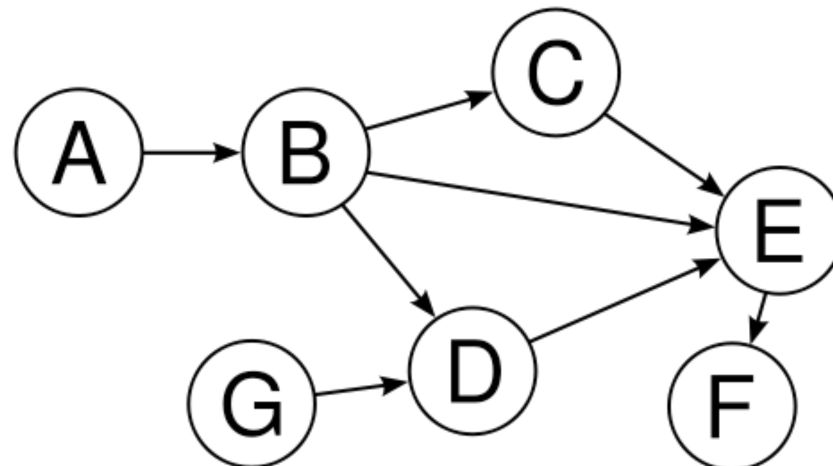


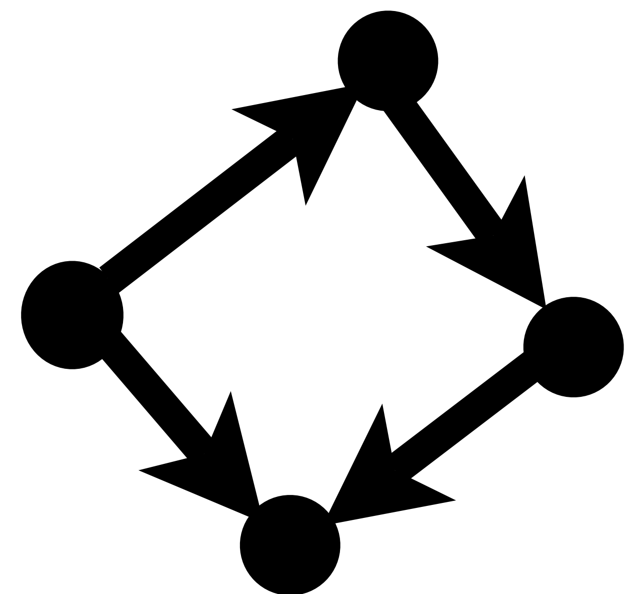


Searching DiGraphs

Question: Find all nodes reachable from s in a directed graph G.



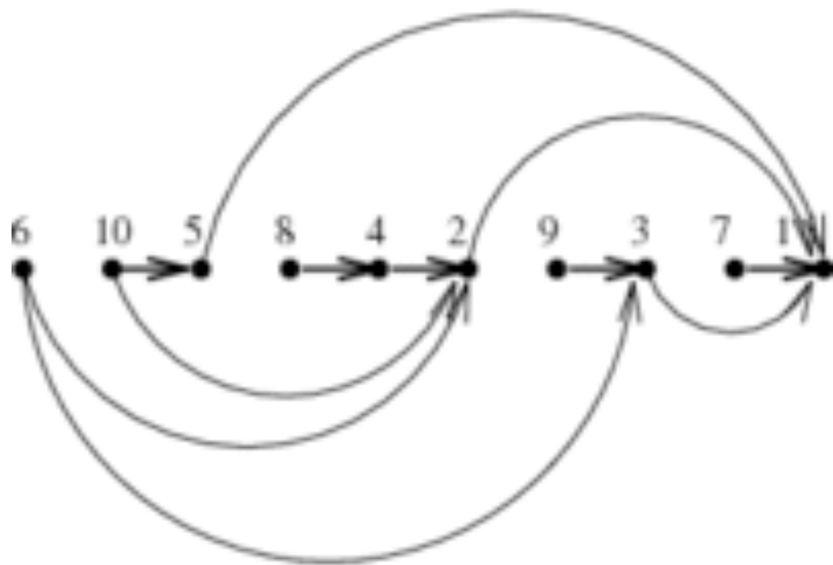


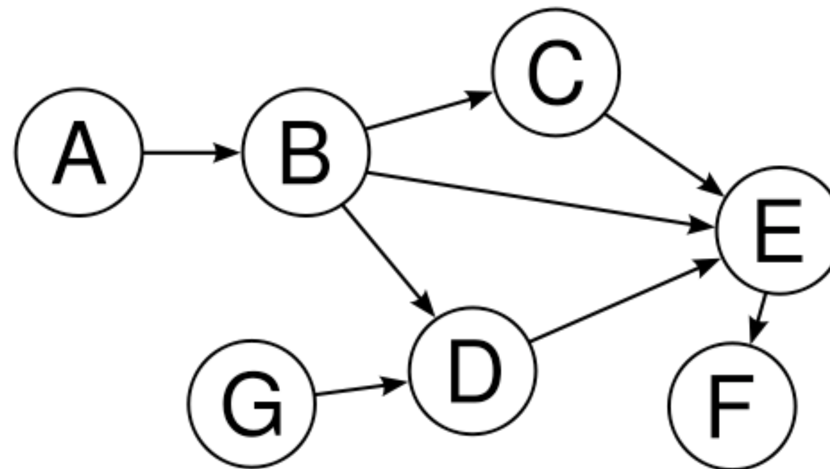


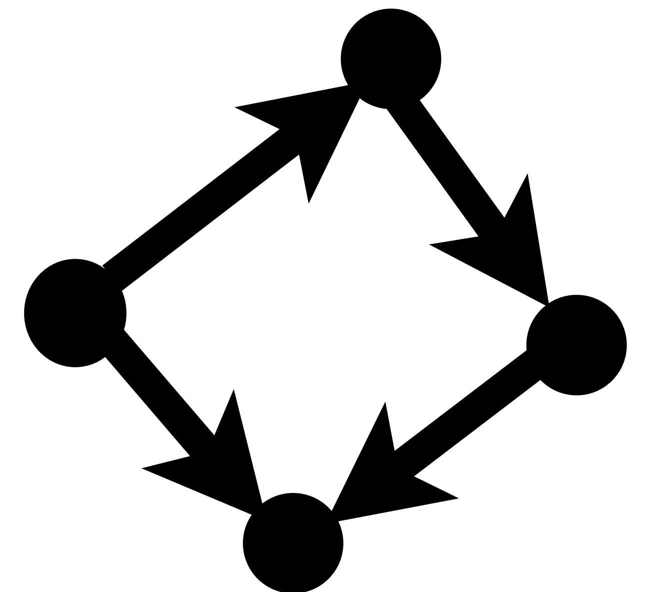
Searching DiGraphs

Question: Find all nodes reachable from s in a directed graph G.

Idea: Slightly modify BFS and DFS algorithms: they should only "find" nodes along out-edges.

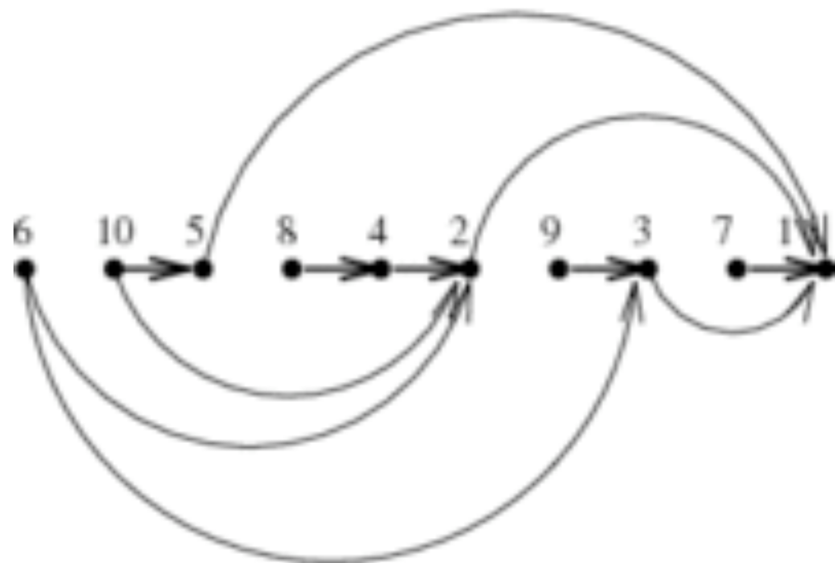


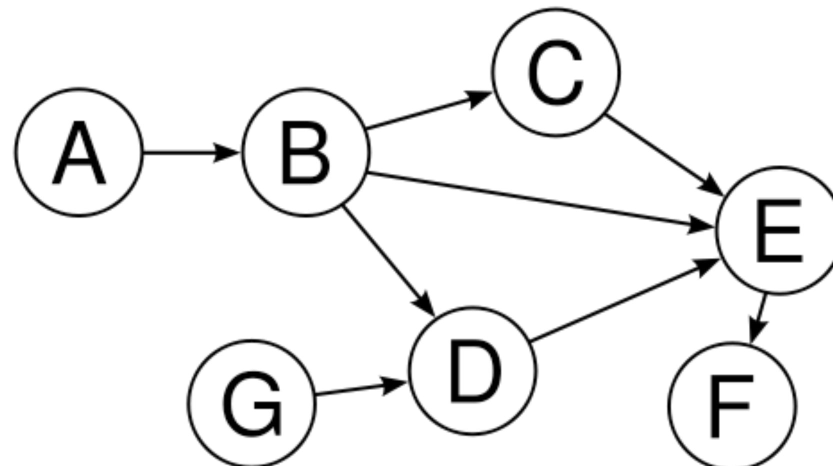


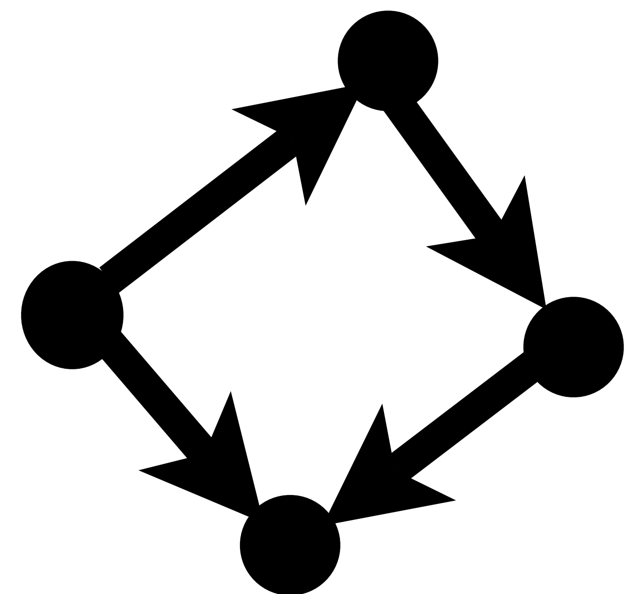


DFS and topo sort

Suppose we run DFS on a directed graph. What can we say about a node v when we mark it as INACTIVE?



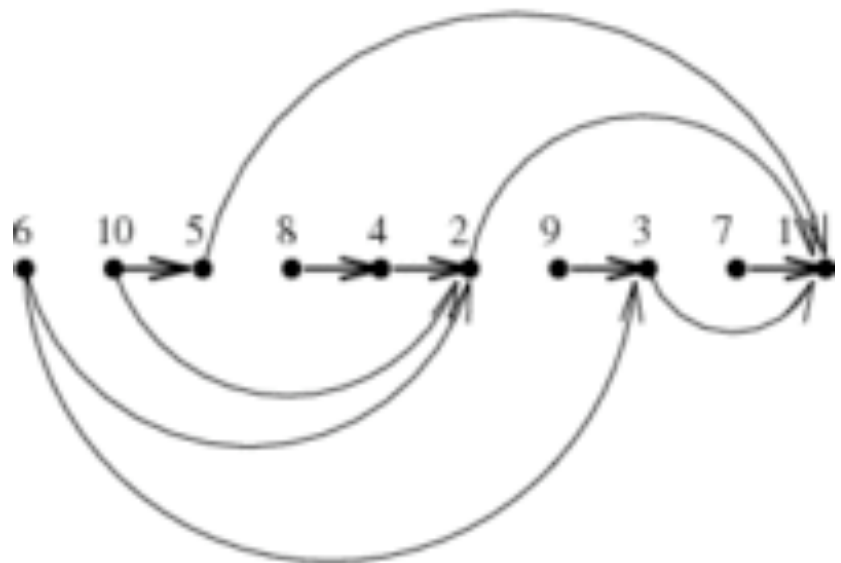


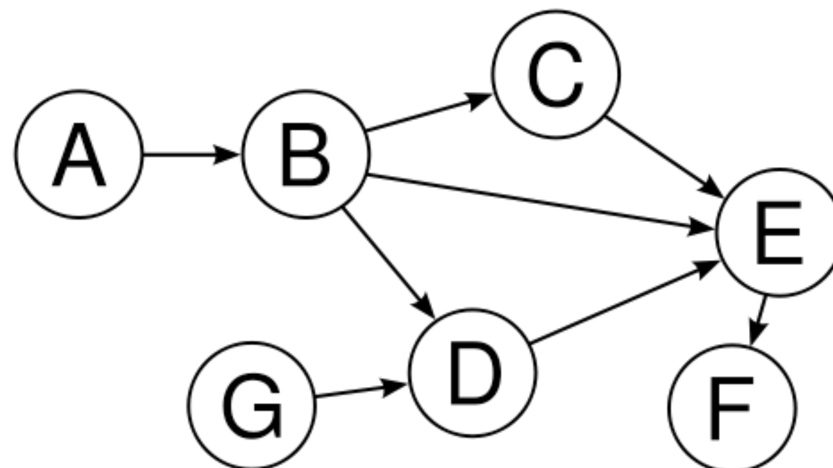


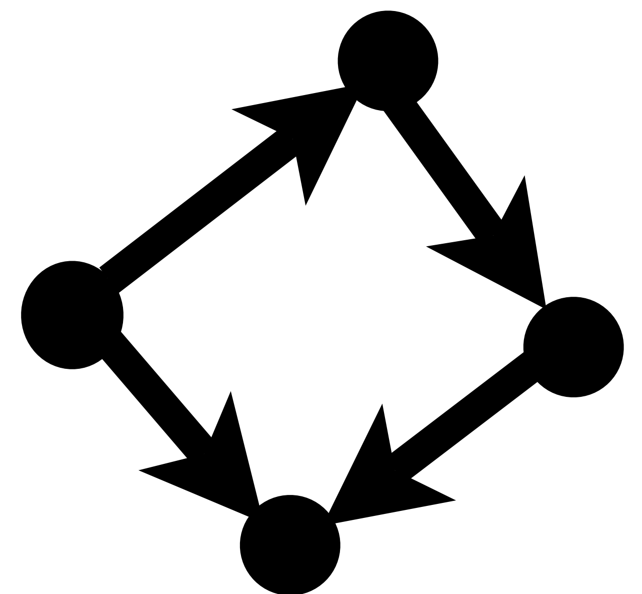
DFS and topo sort

Suppose we run DFS on a directed graph. What can we say about a node v when we mark it as INACTIVE?

Did we fully explore all nodes reachable from v?



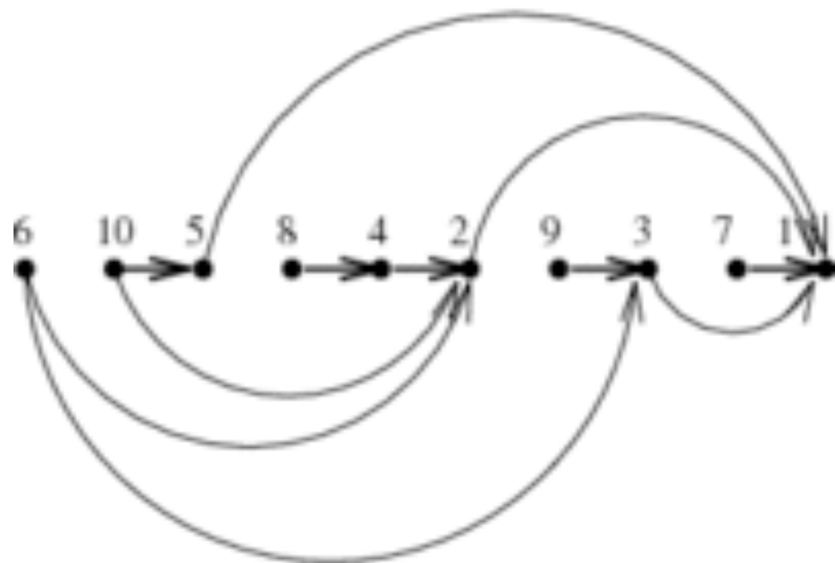


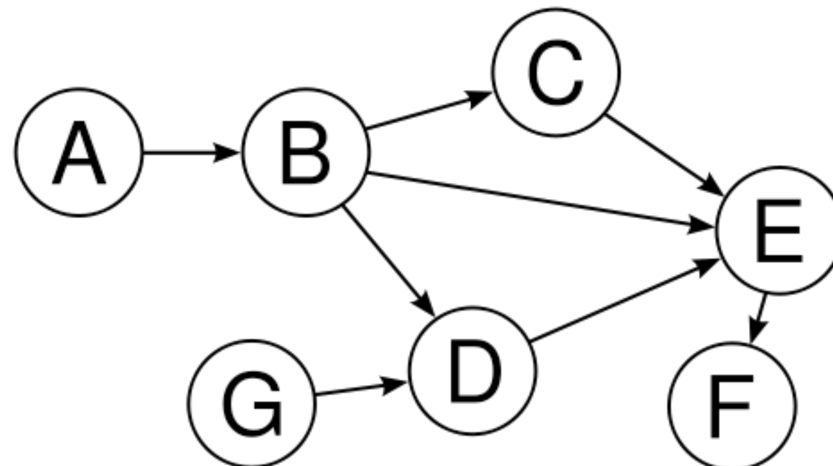


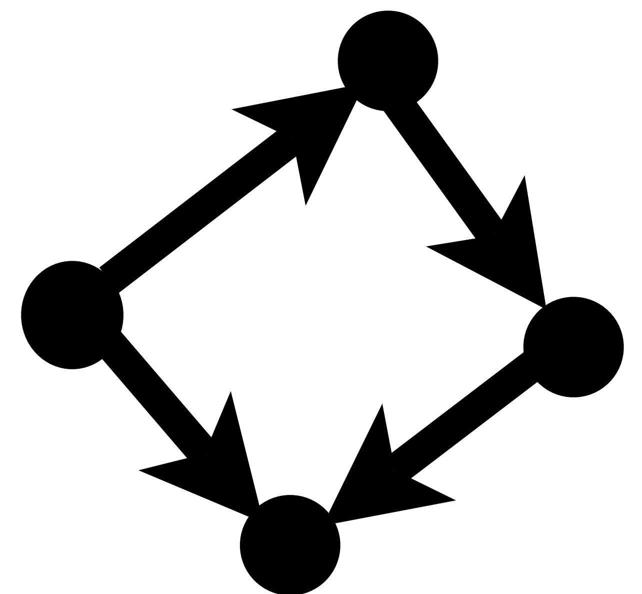
DFS and topo sort

Suppose we run DFS on a directed graph. What can we say about a node v when we mark it as INACTIVE?

Did we fully explore all nodes reachable from v? (to whiteboard)







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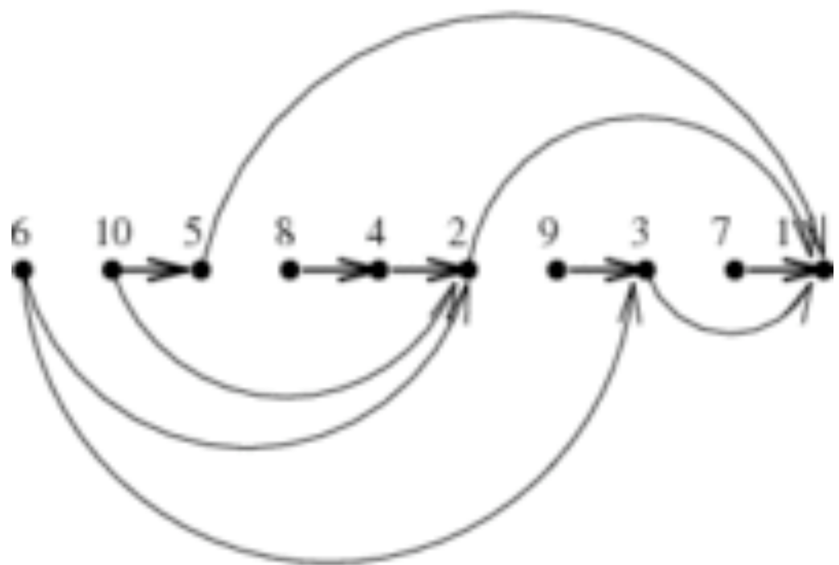

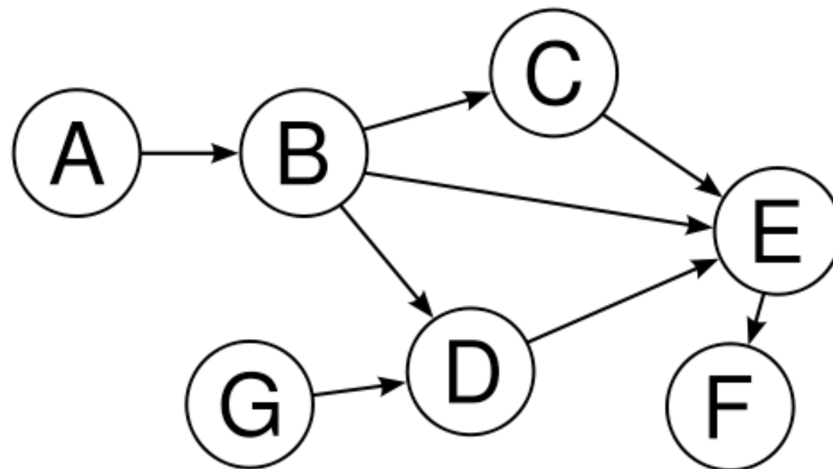

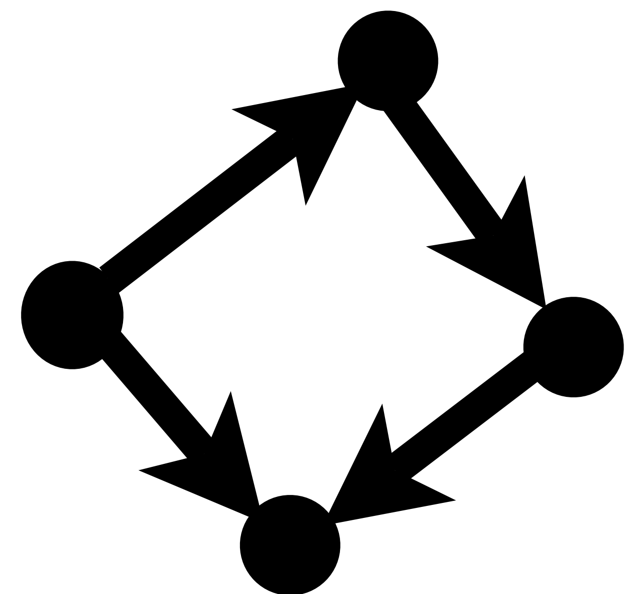

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So: IF we ever find a back-edge to an active node, output the cycle. Otherwise, we know all nodes reachable from v were marked INACTIVE before v was.

Algorithm

Start at a node s. Begin the DFS algorithm, following only out-edges.

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If not all nodes reached, go to next component.