CS 361
Data Structures & Algs
Lecture 15

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University of New Mexico
10-12-2010

Last Time

Identifying BFS vs. DFS trees

- Can they be the same?

Problems 3.6, 3.9, 3.2

- details left as homework. email.

DFS: recursive vs iterative. 2 kinds of stack

Digraphs, directed paths, oriented cycles, DAGs, topological ordering, DFS in digraph

Today

strongly and weakly connected digraphs

new equivalence relation: “strongly connected to”. Strongly connected components. Structure Theorem.

DFS for digraphs. Applications.

Connected Digraphs

An undirected graph is connected when?

Connected Digraphs

Undirected graph G=(V,E). “Connected”?

For every u, v in V, there exists a path from u to v.

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Directed graph G=(V,E). What should “connected” mean?

2 versions: “Strongly connected”

“Weakly connected”

Connected Digraphs

Undirected graph G=(V,E). "Connected"?

For every u, v in V, exists path from u to v.

Directed graph G=(V,E). "Strongly connected" means for every u, v in V, there exists an oriented path from u to v, and an oriented path from v to u.

"Weakly connected" means, ignoring edge directions, the undirected graph is connected.

Components

Undirected G=(V,E). "Component of v"?

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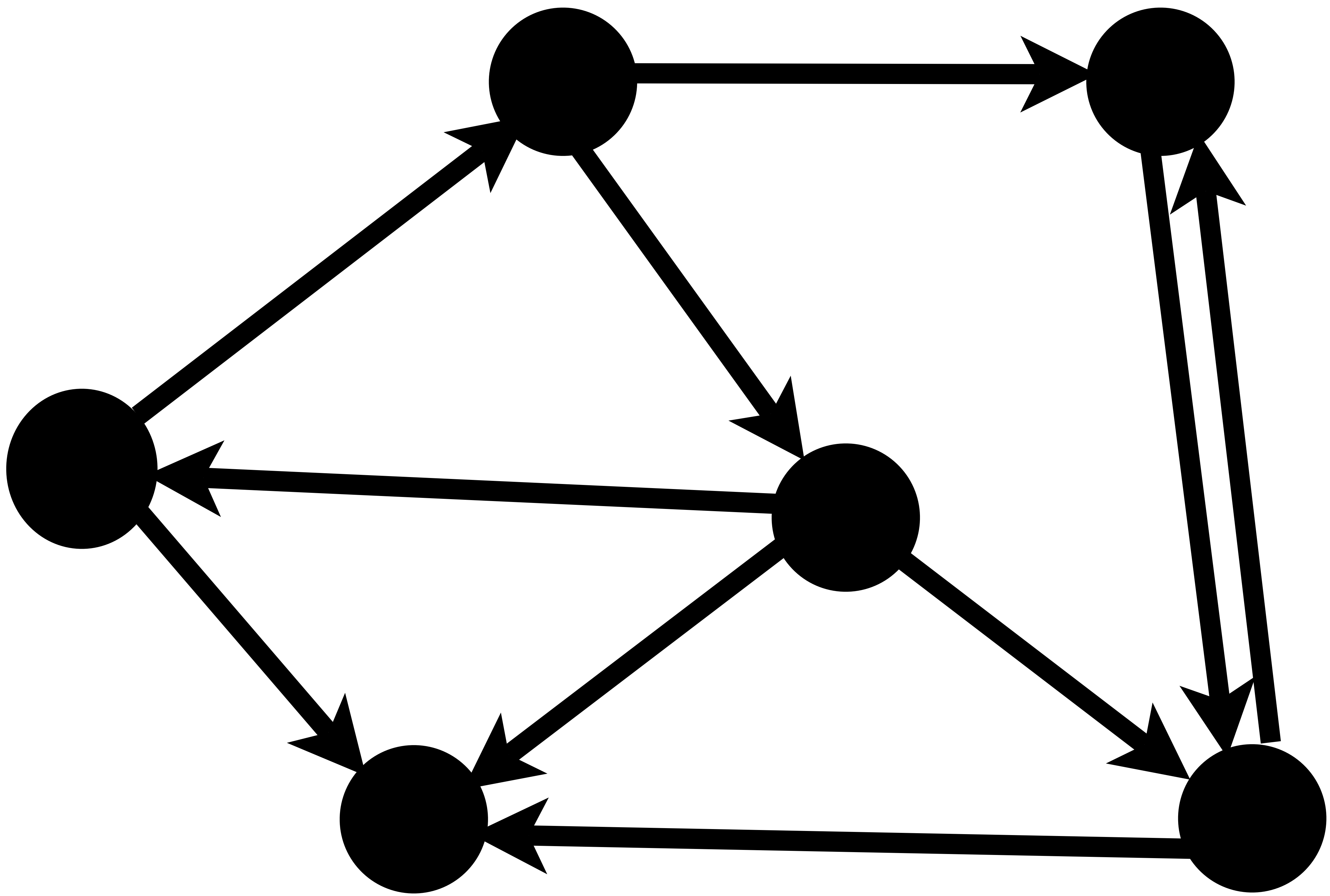
Undirected G=(V,E). "Component of v"? All vertices that have a path to/from v. Recall: "a has a path to b" is an **equivalence relation** on V.

Directed G = (V,E). "Strong component of v"? All vertices w such that w has both an oriented path to v, and from v. "a is in the same strong component as b" is an equivalence relation too.

Example



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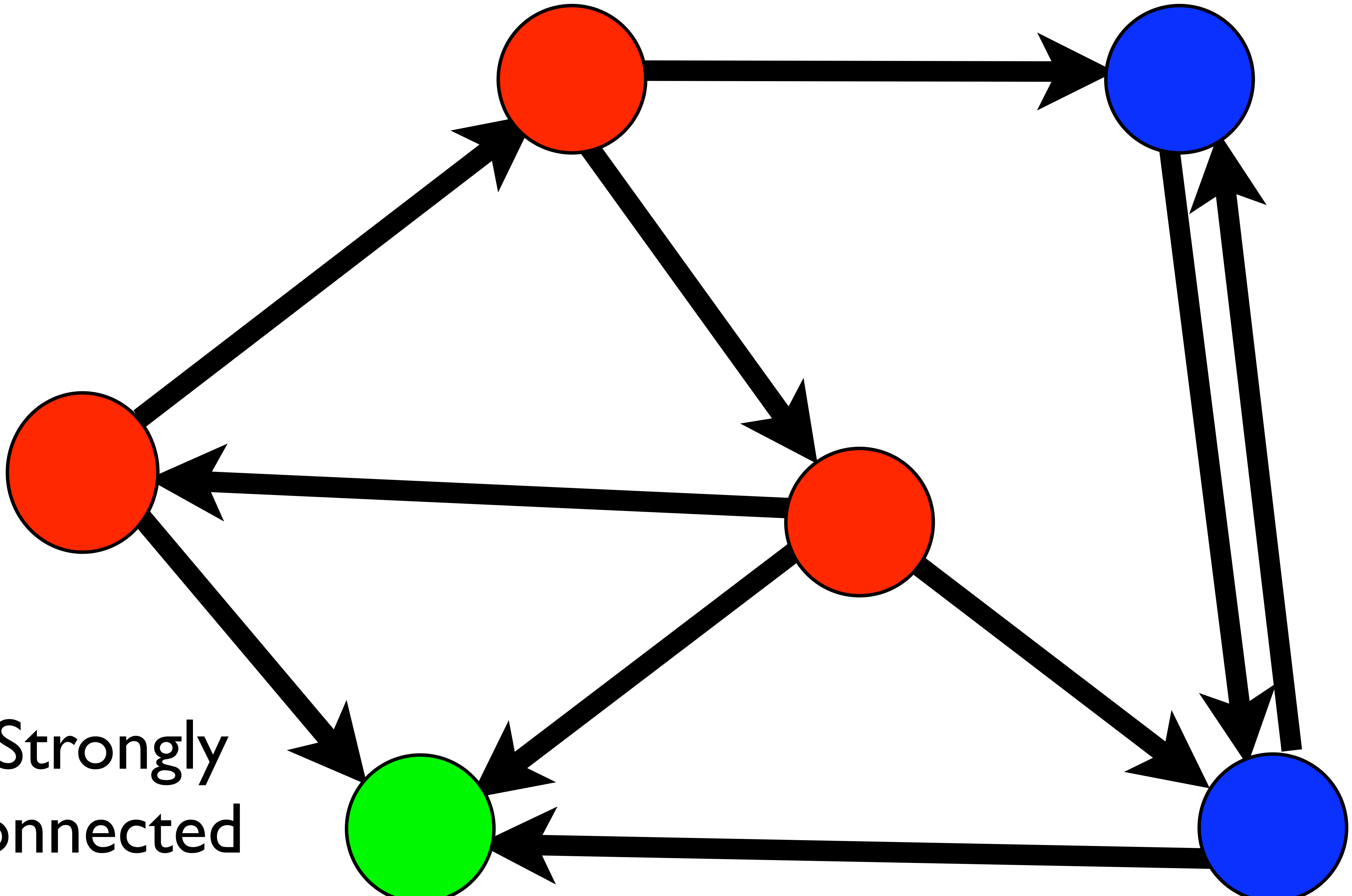


3 Strongly
Connected
Components

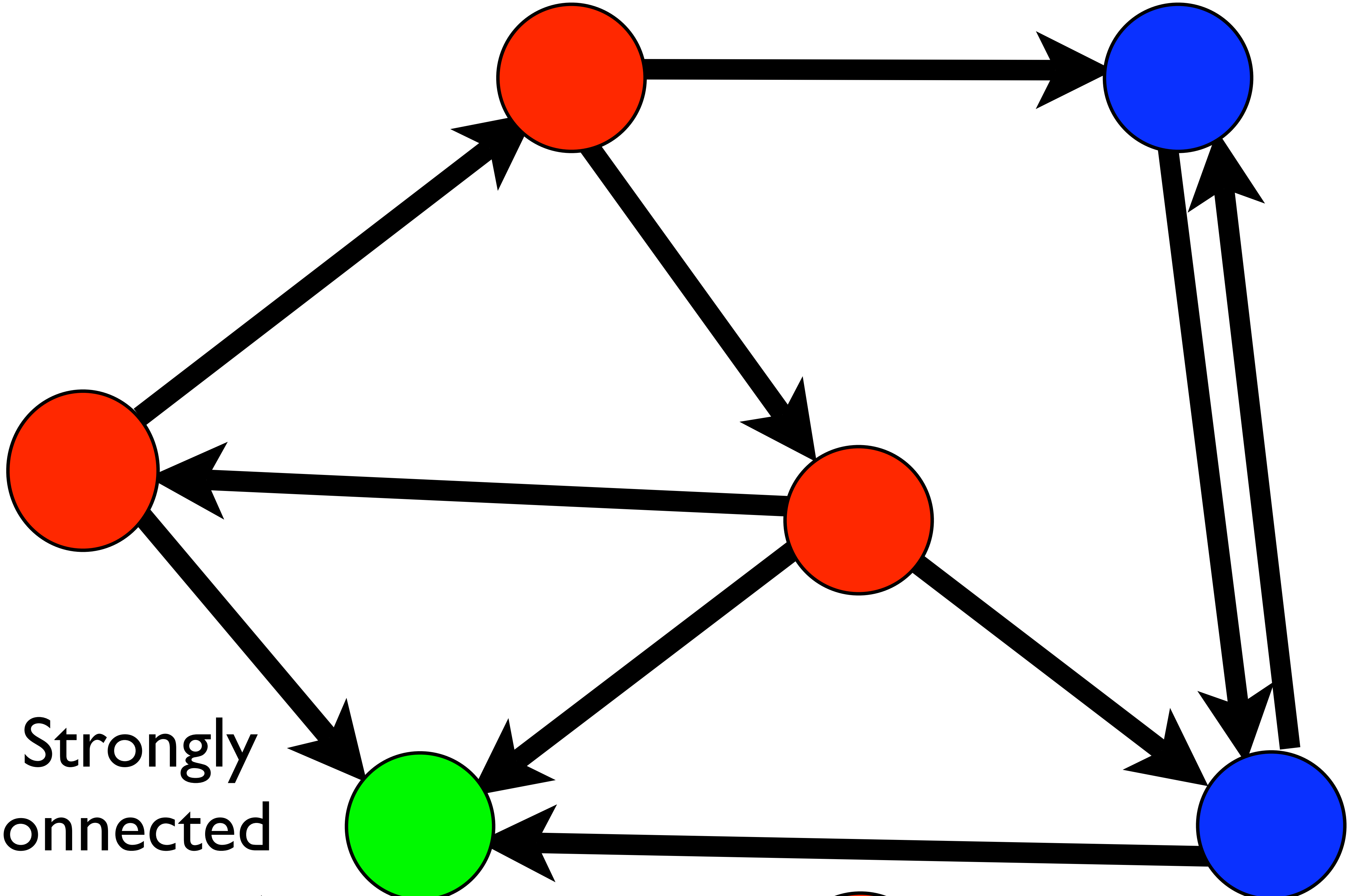
Structure Theorem

The Strong Components graph of G is obtained by “contracting” each strong component of G to a single vertex. Self-loops and multiple edges may result, and are discarded.

1: Strong Components graph is acyclic.

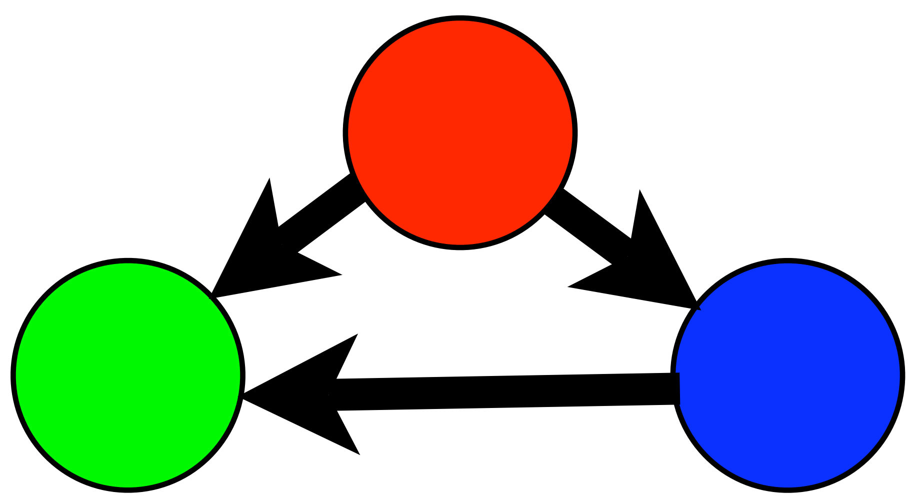
2: G has an oriented path from u to v iff scg(G) has an oriented path from [u] to [v]. (paths of length 0 count).

3: G is acyclic iff G = scg(G).



3 Strongly Connected Components

contracted version:



One Technicality

In an undirected graph, the shortest a cycle can be is length 3. Why? No edge or node may be repeated.

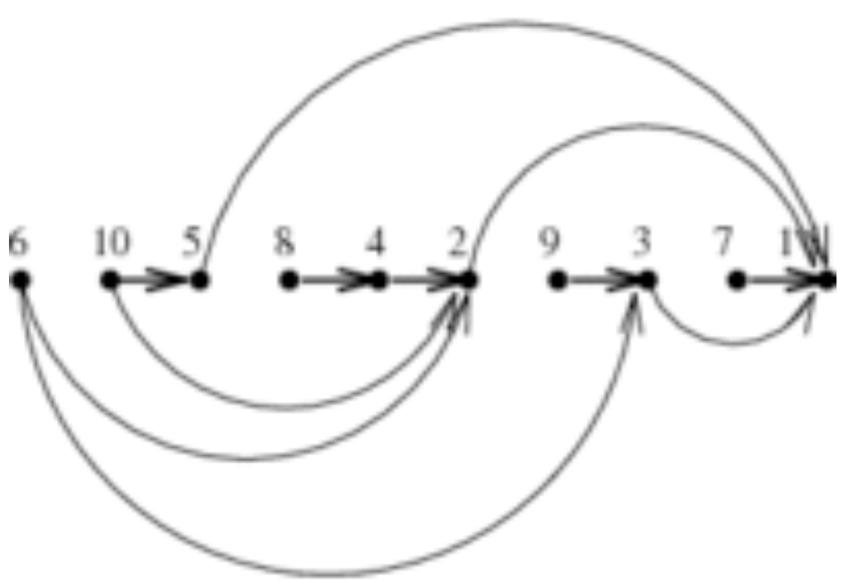
In a directed graph, there can be cycles of length 2, or even 1. Why? Edges in opposite directions don't count as repeats

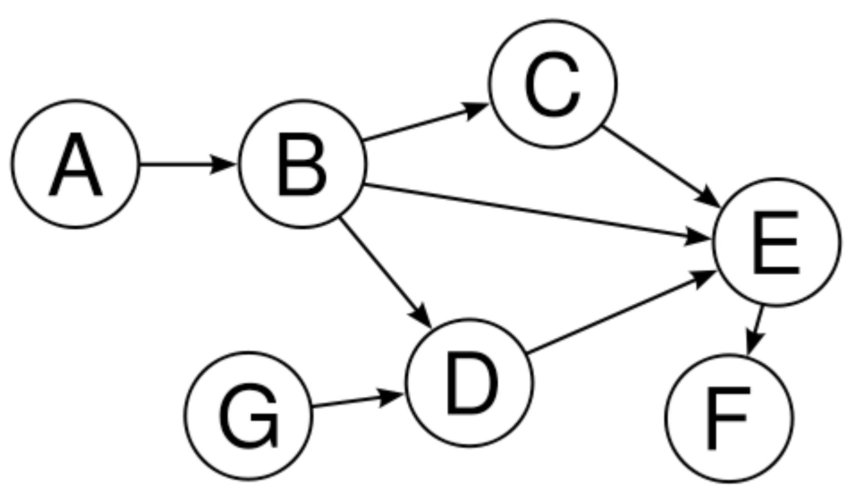
DAGs

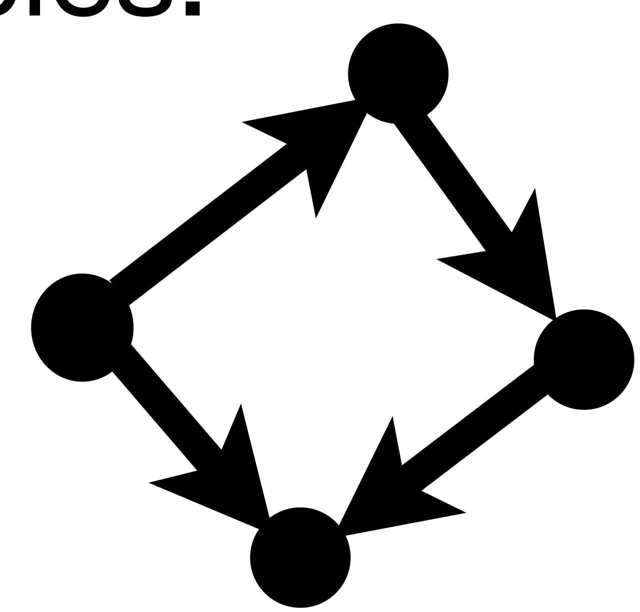
A directed graph is called acyclic if it has no oriented cycles.

Meaning: you can't get back where you start if you always follow arrows.

The "underlying graph" (just lose all the orientation info) may have cycles.



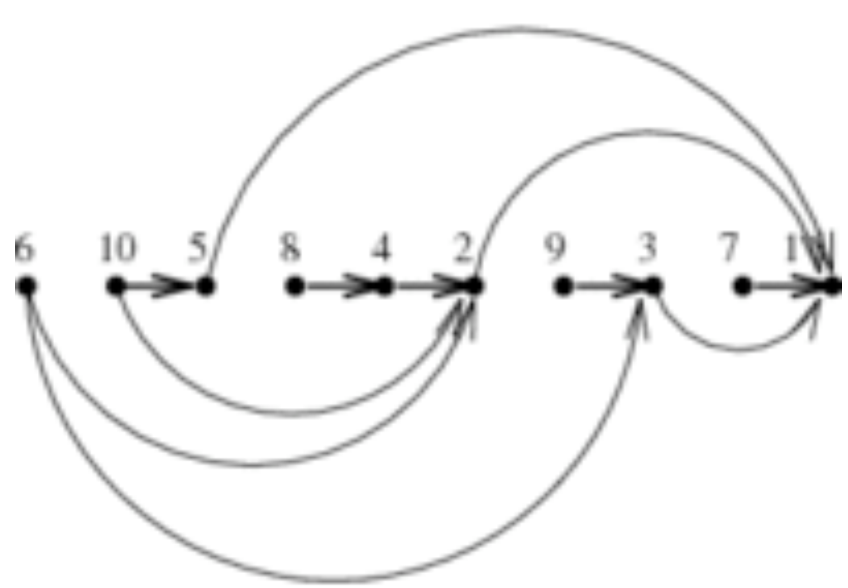


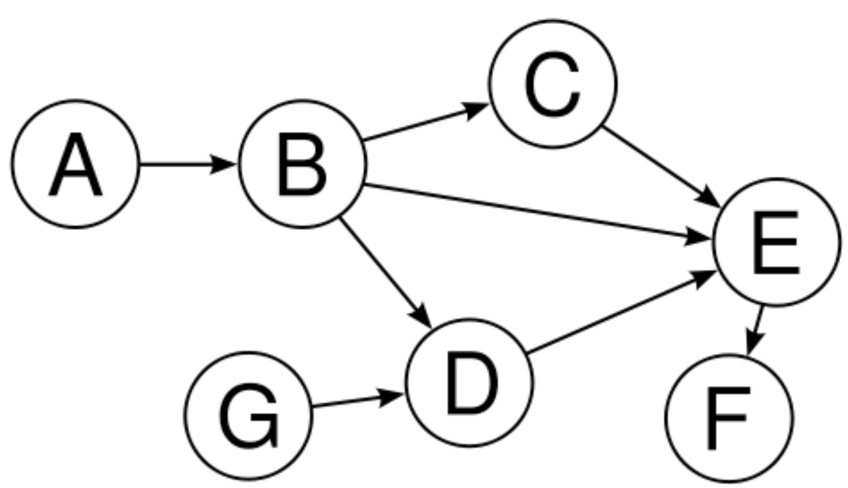


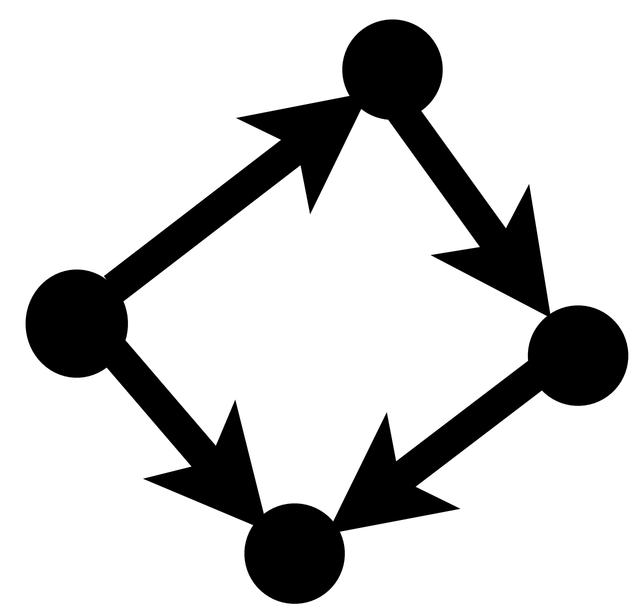
DAGs

Q: How do we tell if a graph is a DAG? Alternatively, how do we find an oriented cycle if there is one?

Look at the Left example. The nodes are in a line, all the edges go left-to-right. This is called a topological sort.







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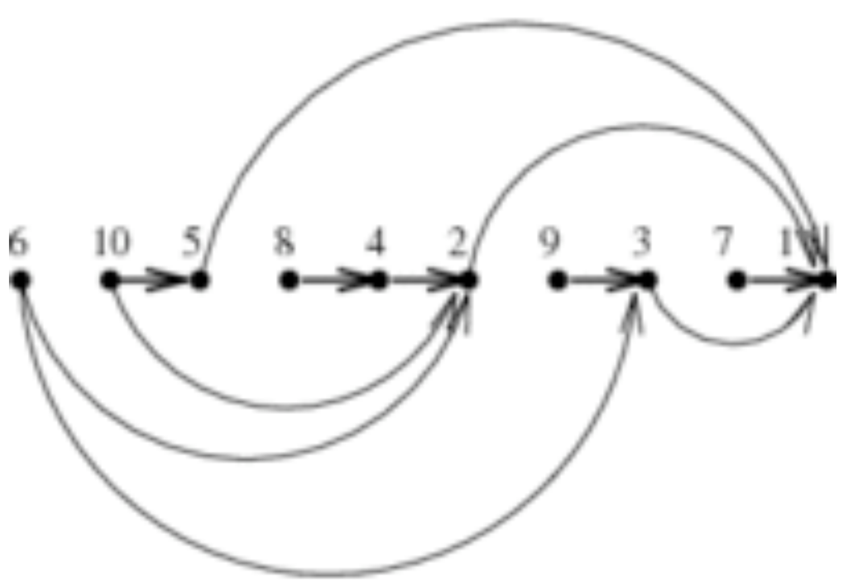
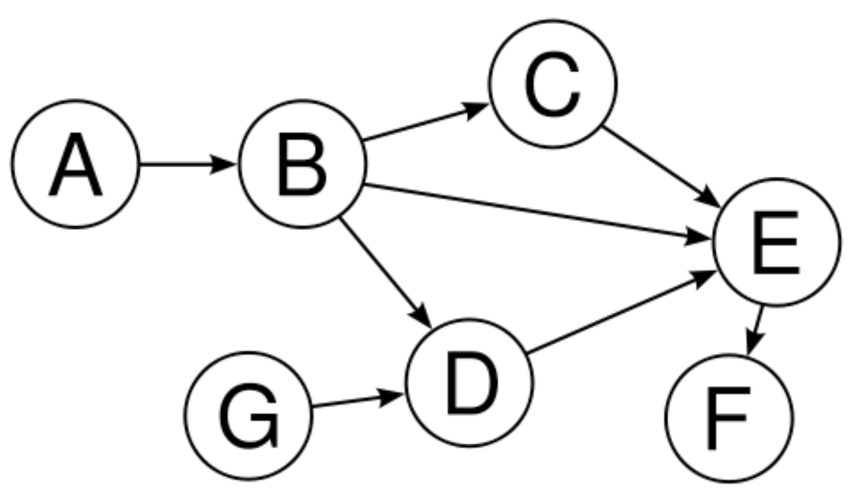
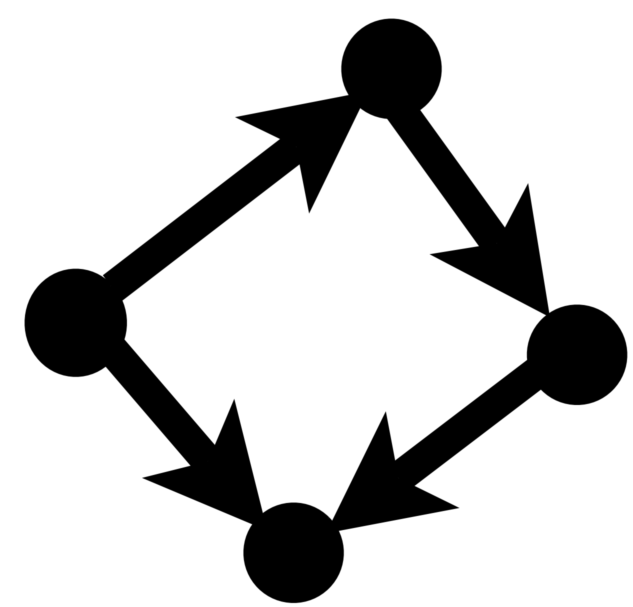
Thm: G has a topological sort iff G is a DAG







Topological Sorting

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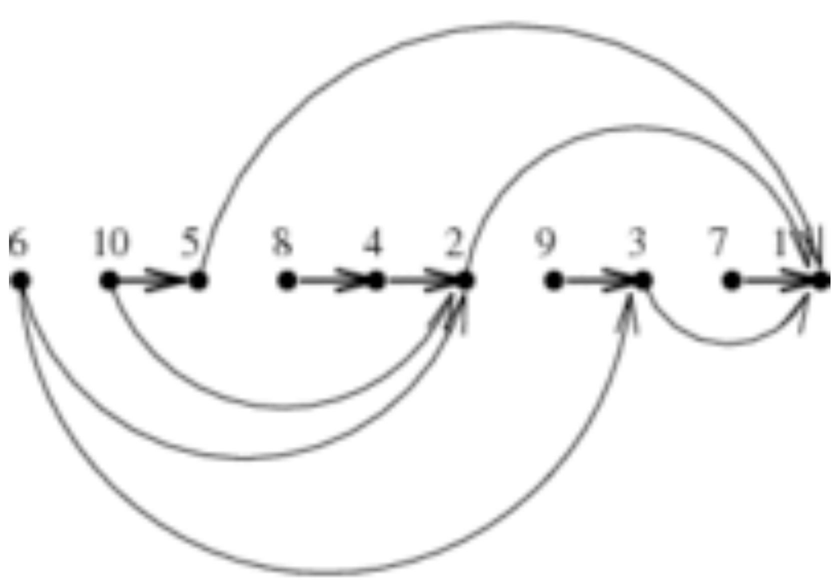

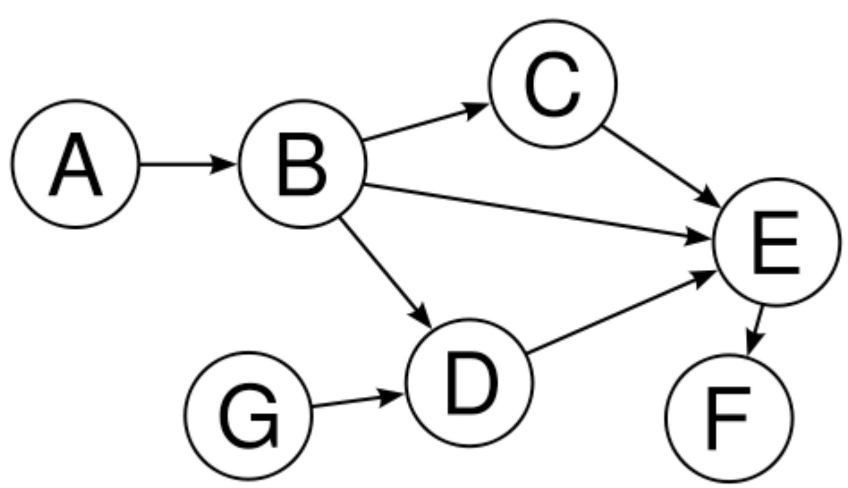

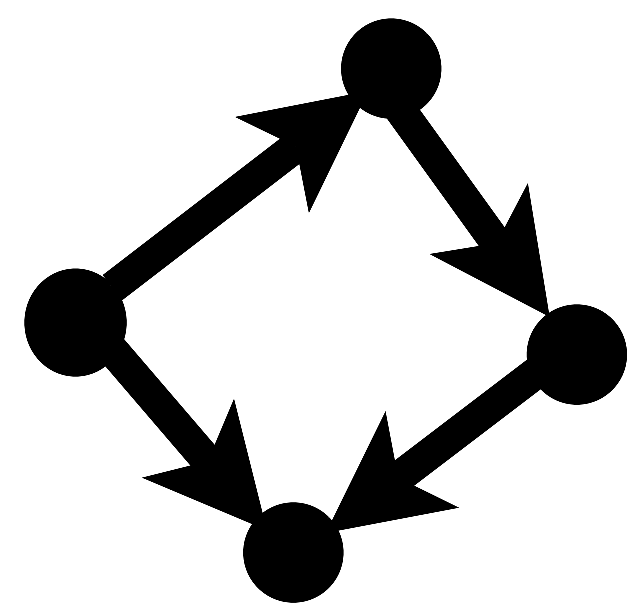

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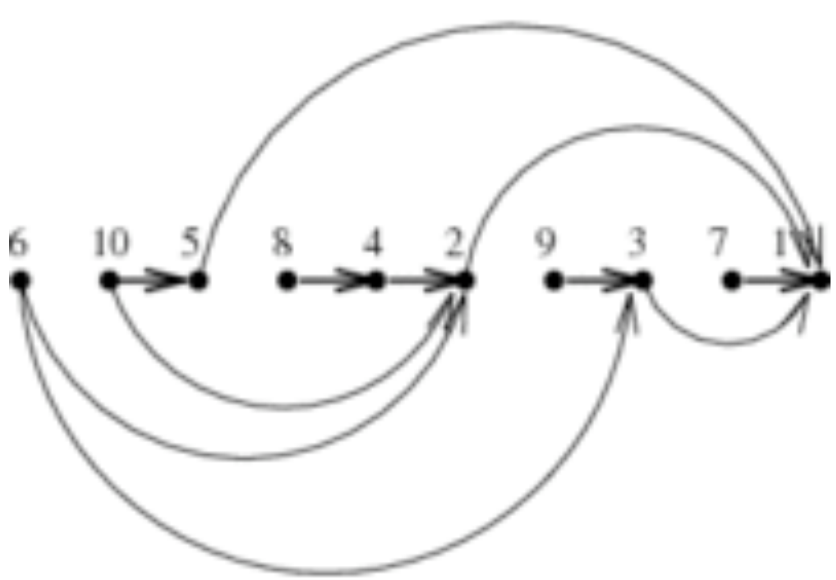

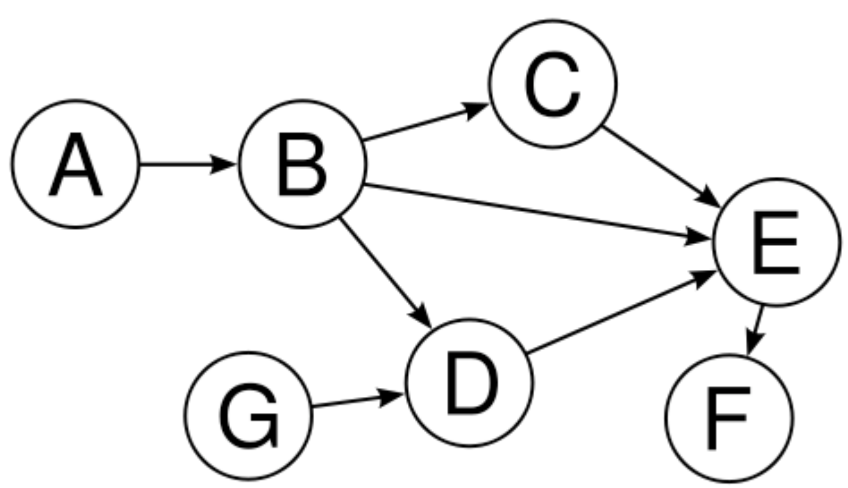

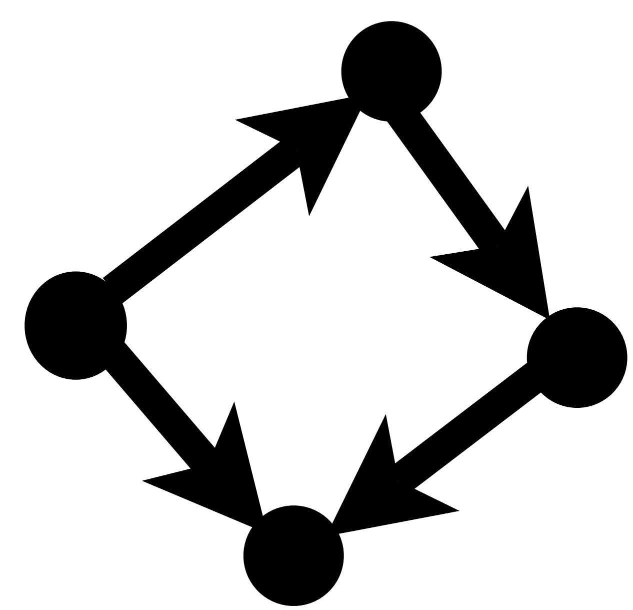

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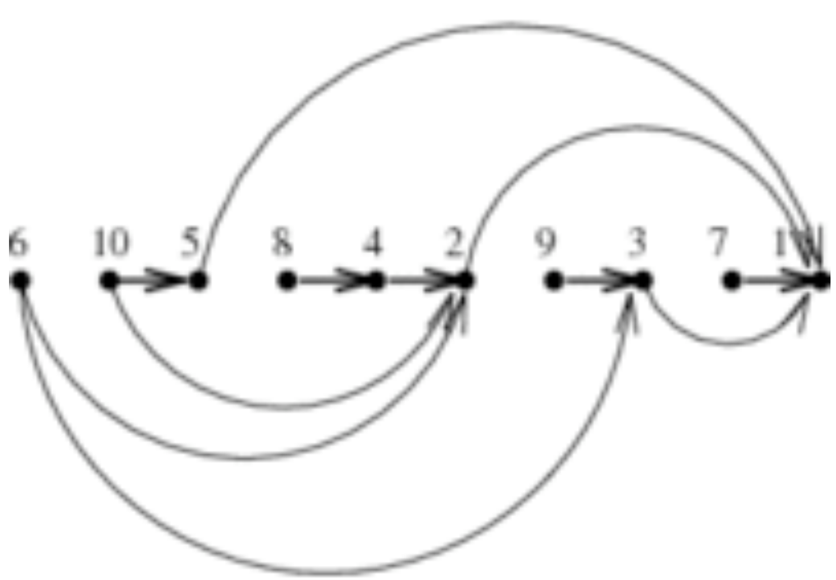
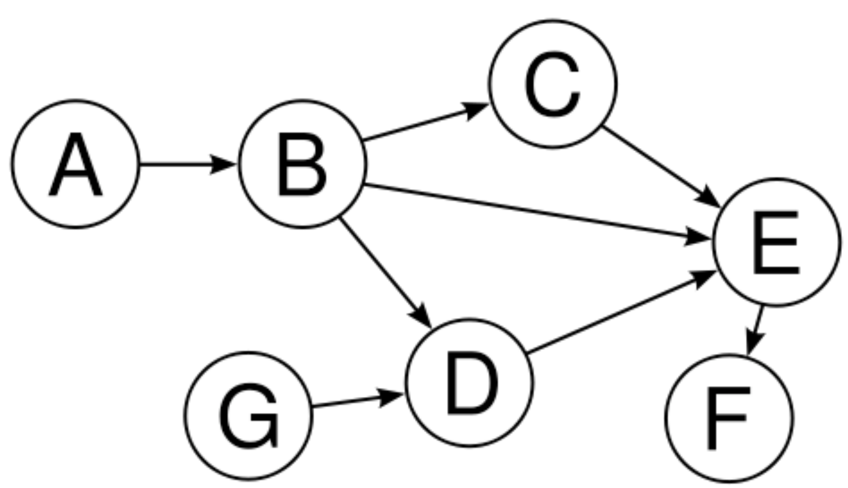
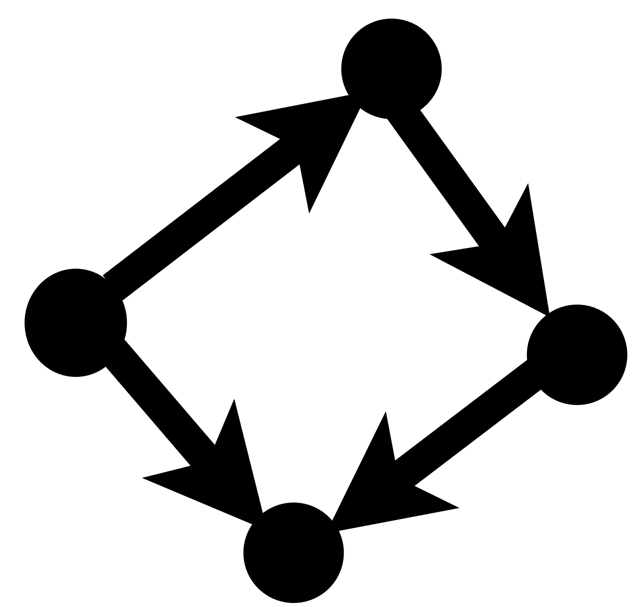
Implementation issues?

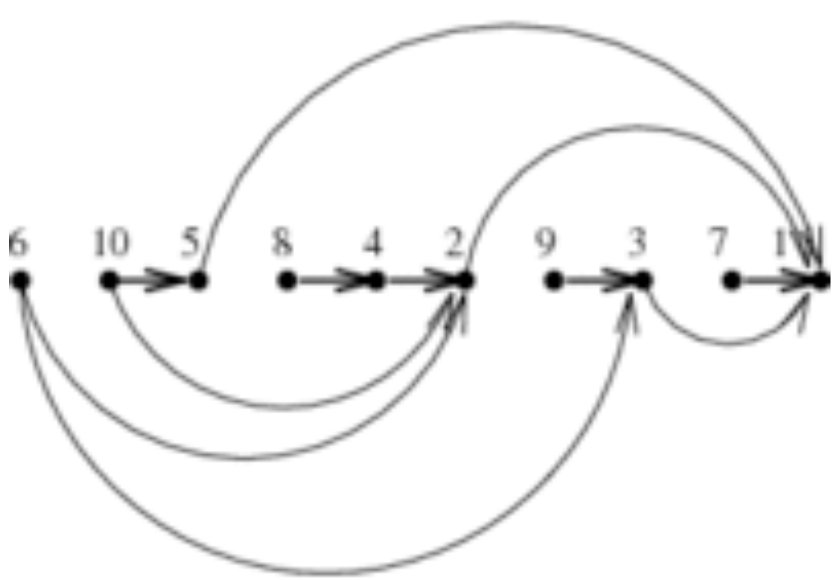


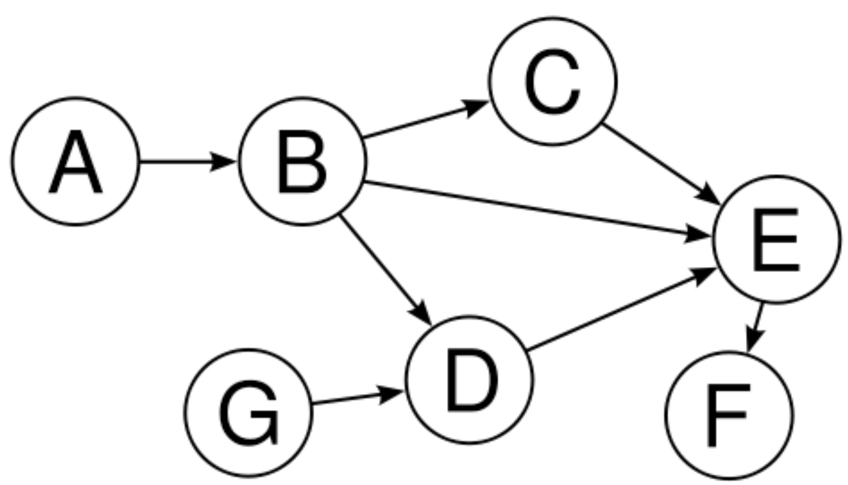


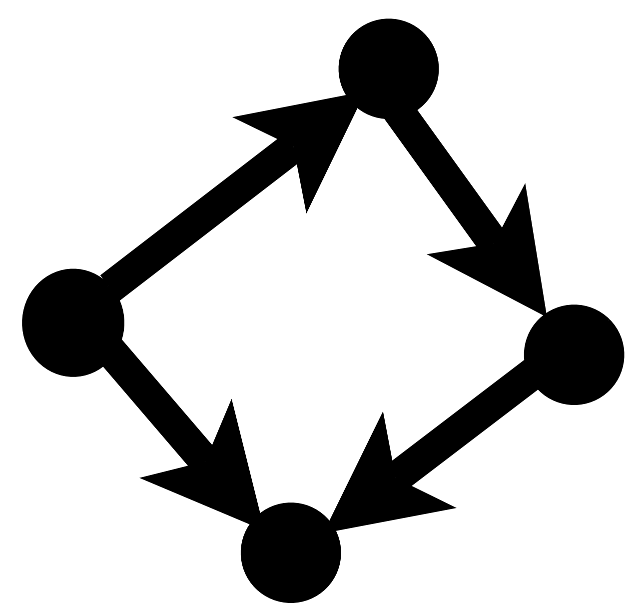


Searching DiGraphs

Question: Find all nodes reachable from s in a directed graph G.







Searching DiGraphs

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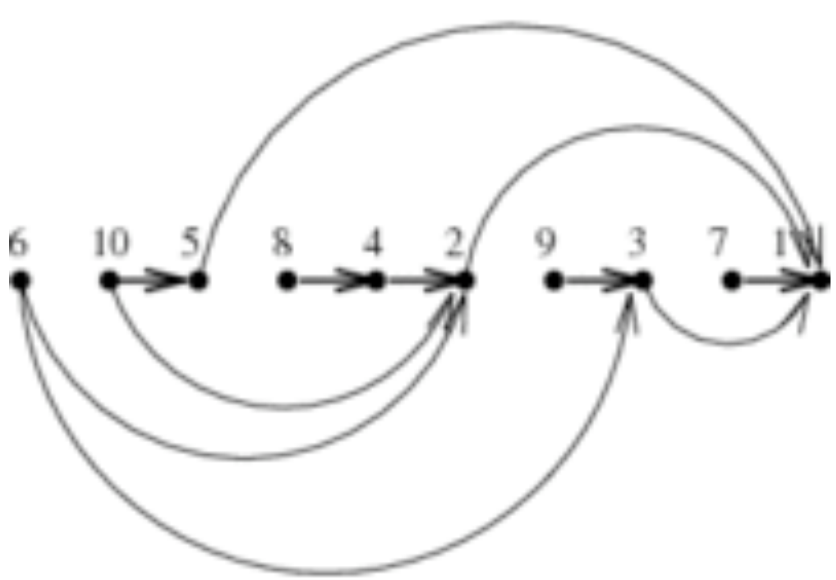
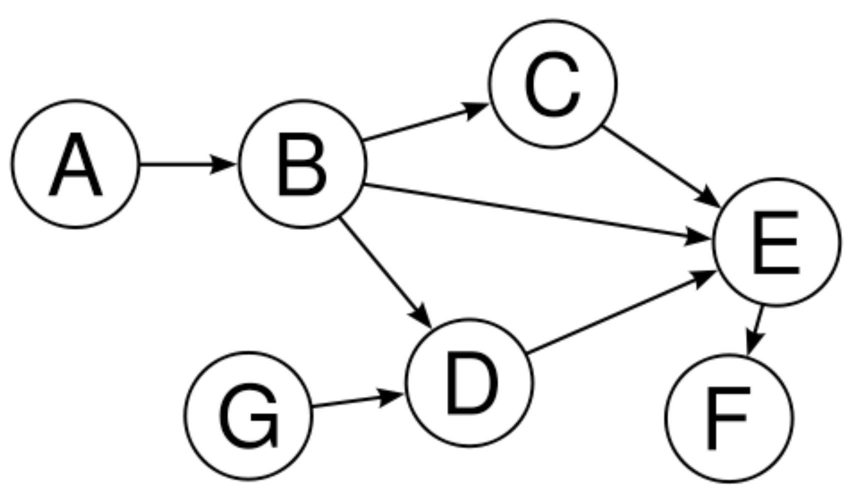
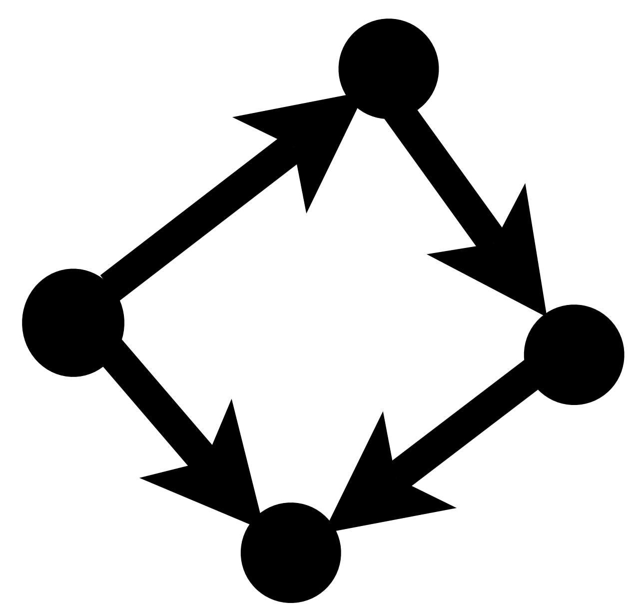
Idea: Slightly modify BFS and DFS algorithms: they should only “find” nodes along out-edges.

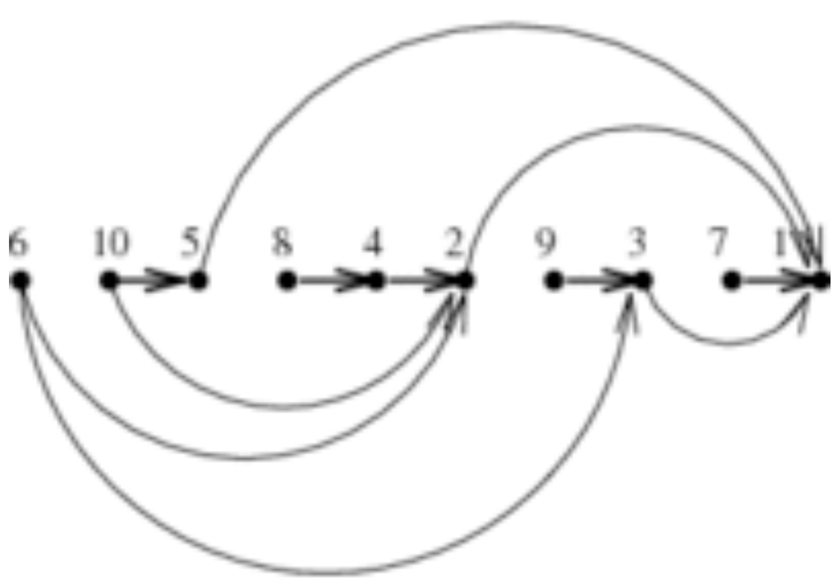


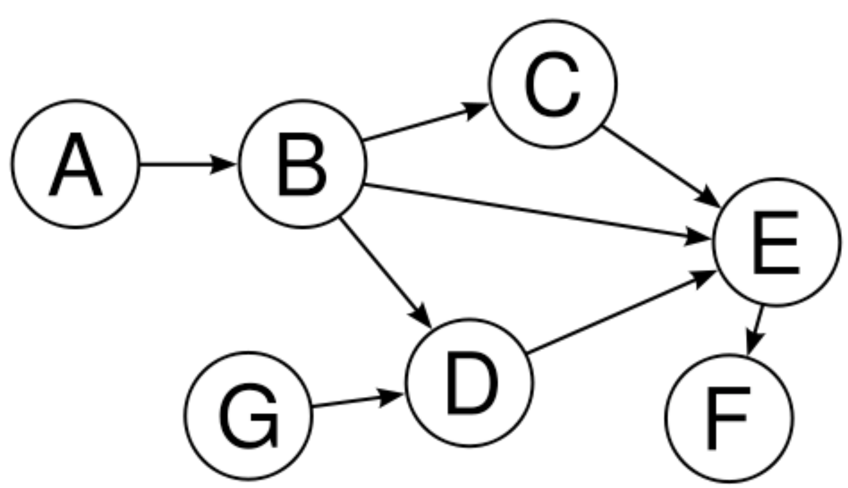


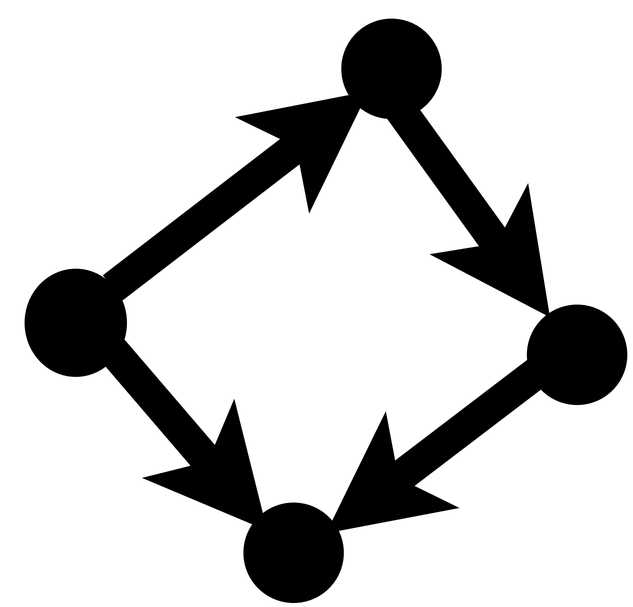


DFS and topo sort

Suppose we run DFS on a directed graph. What can we say about a node v when we mark it as INACTIVE?







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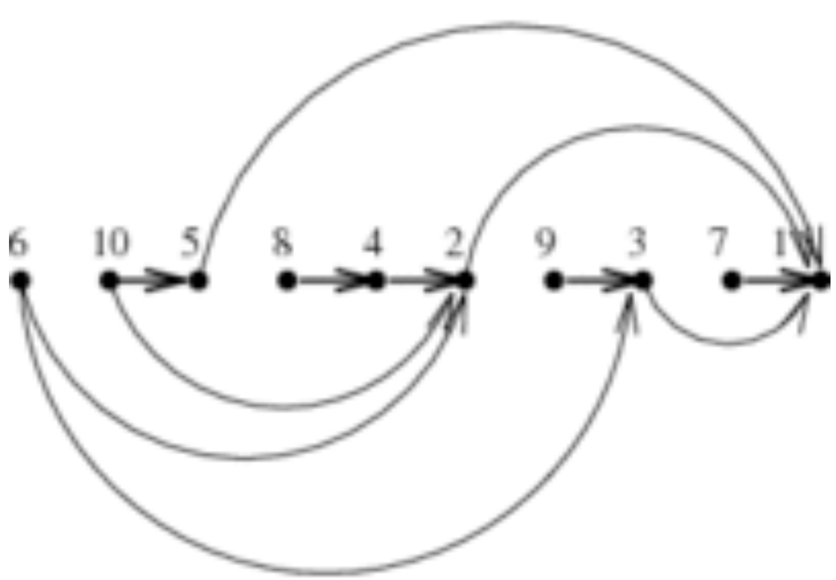

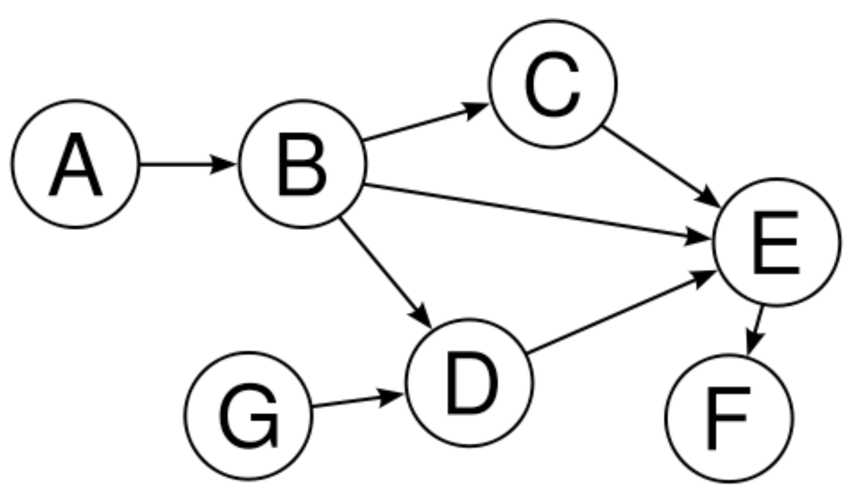

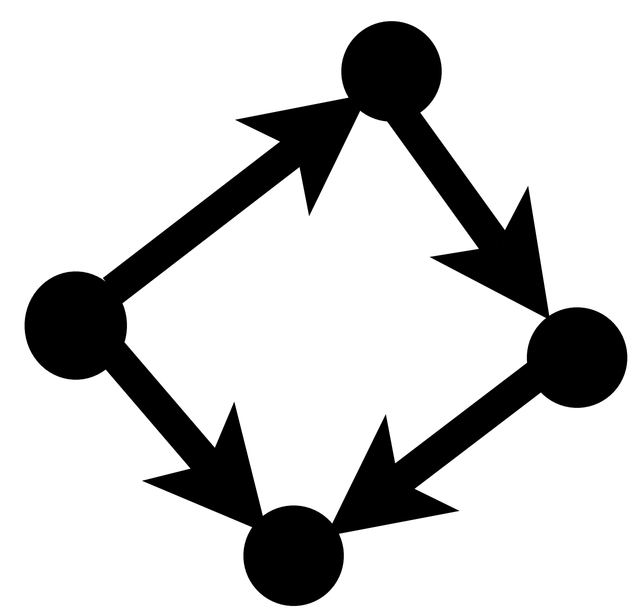

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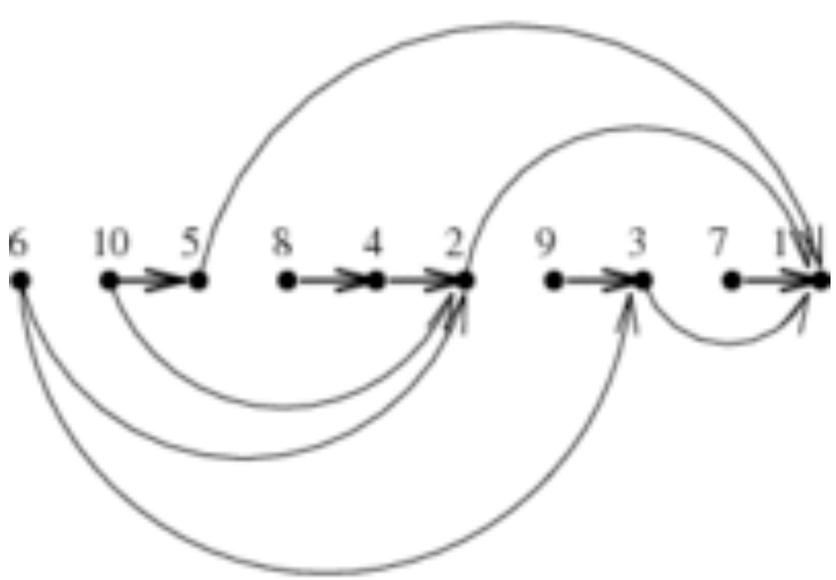

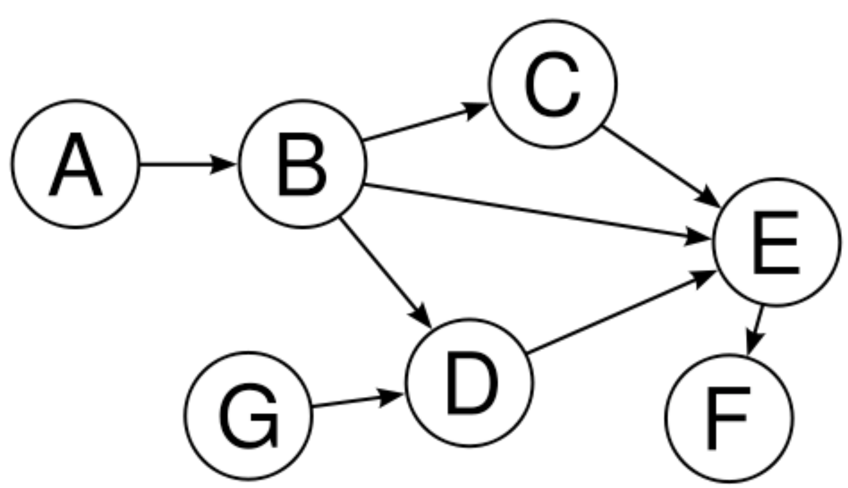

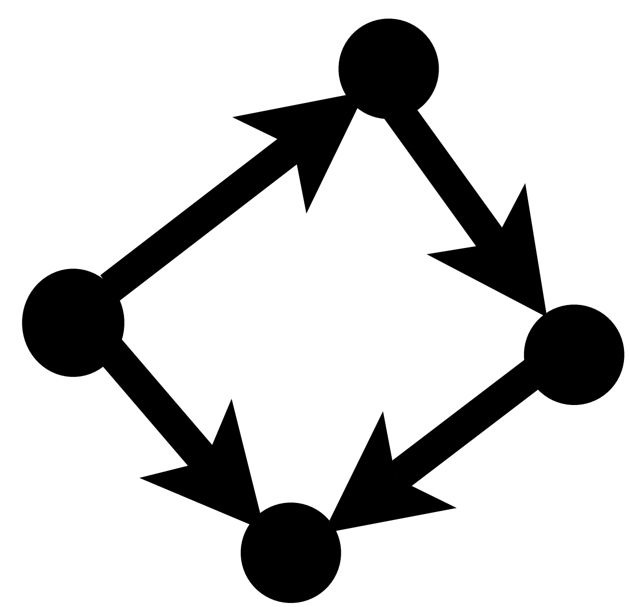

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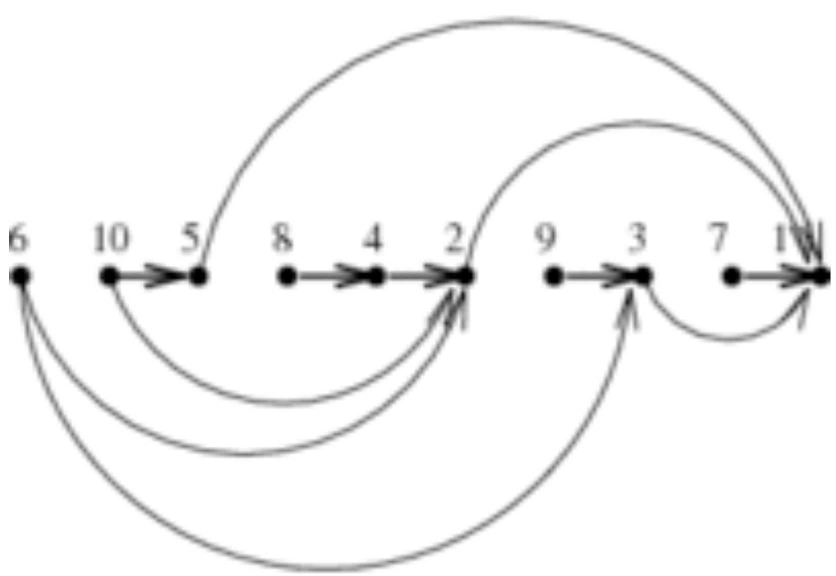
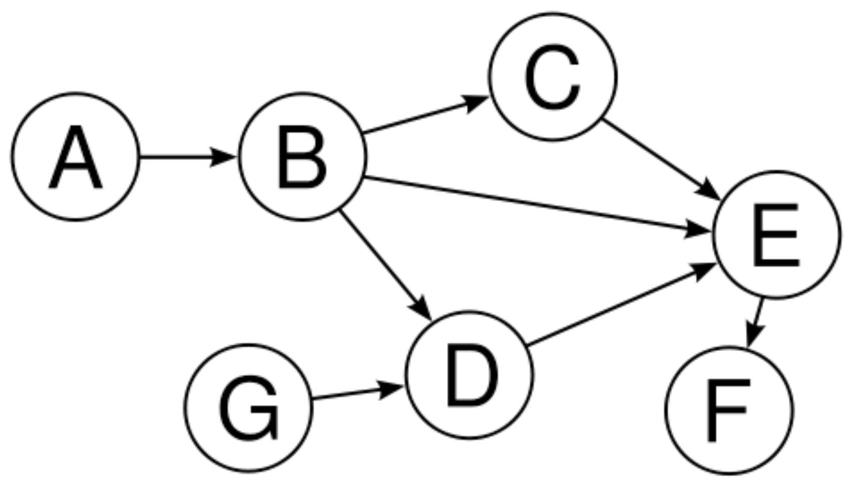
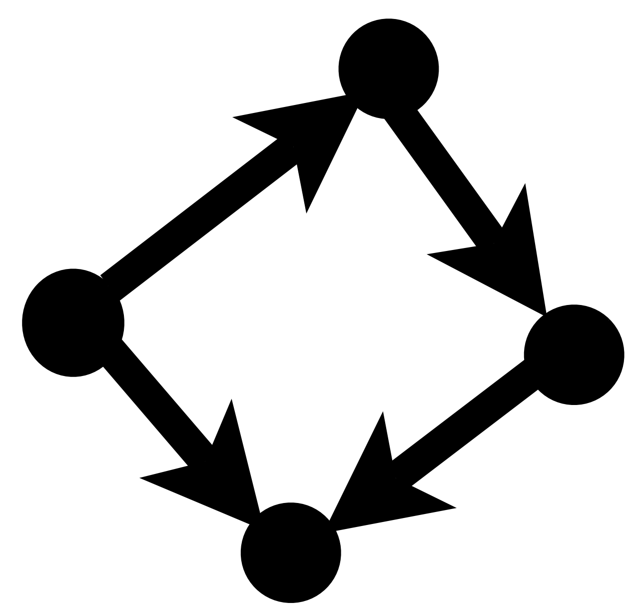
Did we fully explore all nodes reachable from v? YES, unless there is a back-edge to an active node (an ancestor of the current node). In this case there is an oriented cycle!.







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So: IF we ever find a back-edge to an active node, output the cycle. Otherwise, we know all nodes reachable from v were marked INACTIVE before v was.

Algorithm

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If not all nodes reached, go to next component.