CS 361, HW2

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You are free to work with your group, or use any book or the web as a resource in doing this homework assignment. However, you must write up the work yourself.

1. Consider the following problem: you are given an array of ints "arr" with size n, and you need to return the number of times that the number 0 appears in arr.

   - Give a (simple) algorithm to solve this problem
   - Give the run time of your algorithm in big-O notation
   - Show that your algorithm is correct, i.e. give a loop invariant and show initialization, maintenance and termination conditions for that loop invariant.

2. Consider the recurrence $T(n) = 3 * T(n/2) + n$, $T(1) = 1$. Use the recursion-tree method to get a good upperbound on this recurrence (i.e. in terms of big-O analysis). Now use induction to verify your answer.

3. Solve the following within big-Θ: $f(n) = f(n/2) + n$, $f(1) = 1$

4. Consider the recurrence $T(n) = 4T(n/2) + cn$, where c is a constant. Use the recursion-tree method to get a good tight asymptotic bound on its solution (i.e. in terms of Θ). Now use induction to verify your answer.

5. Solve the recurrence $T(n) = 3 * T(n-1)$, $T(0) = 4$ using annihilators. Do this by performing the following steps:

   - Find the annihilator for this sequence (you may find the example in slides 1-5 of lecture 10 helpful)

- Get the general form of the solution from the "Lookup Table" in Lecture 10
- Use the initial value ($T(0) = 4$) to solve for appropriate constants and get an exact solution.

6. Use annihilators to solve the recurrence $T(n) = 3T(n-1) - 2T(n-2)$, $T(0) = 0$, $T(1) = 1$. Do this by performing the following steps:

   - First write down the first four terms of T, $\mathbf{L}T$ and $\mathbf{L}^2T$ (as in slide 20 of lecture 10.)
   - Next find a linear combination of these three sequences which gives the all 0's sequence. Factor out the T term from this linear combination to get the annihilator for T
   - Now factor the annihilator for T you got in the previous step (using the quadratic formula) to get an annihilator of the form that can be looked up in the "Lookup Table" given in the class notes. From the "Lookup Table", get the general form of the solution to the recurrence.
   - Finally, use the initial values ($T(0)$ and $T(1)$) to the recurrence to solve for the appropriate constants in the general solution to get an exact solution