CS 361, Pretest

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January 21, 2003, *Due: Tuesday, January 28*

This is a take-home "pretest" to test your math background for this course. It will be graded! However, if you don't see an answer to a question, you are free to ask a friend for help, or use a book or the web as a resource. Please show all your work.

This test will be used for two purposes: 1) to divide the class up into groups which are roughly balanced in terms of mathematical background (the class project and at least some hws will be done in groups) and 2) to see how much math review we need to do in class.

Graded Problems:

1. Find the solutions to this equation: $x^2 - 5x - 14 = 0$.

   Solution: $(x-7)(x+2) = 0$, *in other words,* $x = 7$ *or* $x = -2$. *Can solve this by "eyeballing" the equation or by using the quadratic formula (see below)*

2. Find the solutions to this equation $x^2 - x - 1 = 0$

   Solution: For equations of the form: $ax^2 + bx + c = 0$, *we can use the quadratic formula:* $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$. *The equation above can be written:* $1x^2 - 1x - 1$ *so* $a = 1$, $b = -1$, $c = -1$. *Plugging this into the quadratic formula, we get* $x = \frac{1 \pm \sqrt{1+4}}{2}$. *So one solution is* $x = \frac{1+\sqrt{5}}{2}$ *and the other is* $x = \frac{1-\sqrt{5}}{2}$

3. Assume that $x + y = 7$ and $-2x + 2y = 2$ for unknown variables x and y. What are the values of x and y?

   Solution: To solve this, use the linear algebraic trick for solving two equations in two unknowns (to be shown in class). The answer is $x = 3$, $y = 4$

4. Assume you know that some function f is of the form $f(x) = ax^2 + bx$, where the coefficients a and b are unknown. Assume further that $f(1) = 2$ and $f(2) = 10$. What are the coefficients a and b?

   Solution: $f(1) = 2$ implies that $a + b = 2$, $f(2) = 10$ implies that $4a + 2b = 10$. So we have two equations and two unknowns, and we can solve for a and b to get $a = 3$, $b = -1$. Thus, the unknown function is $f(x) = 3x^2 - x$

5. For each of the following equations, say whether it is always true, or if it may be false. If the equation is always true, say why. If it's false, give values for which it is false. All logs are base 2 unless stated otherwise.

   (a) $2^{\log n} = n$ *Solution: True by defn of log*

   (b) $a^{\log b} = b^{\log a}$. *Solution: True since $a = 2^{\log a}$, $b = 2^{\log b}$, and if we replace these quantities in the equation, we get the equation $2^{\log a \log b} = 2^{\log b \log a}$ which is trivially true*

   (c) $\log 2x = \log 2 + \log x$ *Solution: True since $\log xy = \log x + \log y$ for any x and y*

   (d) $\log x^2 = 2 \log x$ *Solution: True since $\log x^k = k \log x$ for any k and x*

   (e) $\log_8 x = (\log_2 x)/4$ (that is, *log* base 8 of x is *log* base 2 of x divided by 4.

   Solution: False: This is false for $x = 8$. The correct equation is $\log_8 x = \log_2 x / \log_2 8$ which reduces to $\log_8 x = \log_2 x/3$

6. What is $\sum_{i=0}^{\infty} 2^{-i} = 1 + 1/2 + 1/4 + 1/8 + \ldots$

   Solution: Let $S = \sum_{i=0}^{\infty} 2^{-i} = 1 + 1/2 + 1/4 + 1/8 + \ldots$. Then $2S = \sum_{i=0}^{\infty} 2^{-i+1} = 2 + 1 + 1/2 + 1/4 + 1/8 + \ldots$. Subtracting the first equation from the second, we get $S = 2$

7. Prove, by induction on n, that $\sum_{i=1}^{n} i = 1+2+3+\cdots+n = n(n+1)/2$.

   Solution: B.C. $\sum_{i=1^1 i=1(1+1)/2}$. Now assume, by the inductive hypoth-

esis that $\sum_{i=1}^{n-1} i = \frac{(n-1)n}{2}$. *Then*

$$\begin{aligned} \sum_{i=1}^{n} i &= \sum_{i=1}^{n-1} i + n && (1) \\ &= \frac{(n-1)n}{2} + n && (2) \\ &= \frac{n(n+1)}{2} && (3) \end{aligned}$$

8. Prove, by induction on n, that $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

   Solution: B.C. $\sum_{i=0}^{0} 2^i = 2^1 - 1$. *Assume by the inductive hypothesisthat* $\sum_{i=0}^{n-1} 2^i = 2^n - 1$. *Then*

$$\begin{aligned} \sum_{i=0}^{n} 2^i &= 2^n - 1 + 2^n && (4) \\ &= 2^{n+1} - 1 && (5) \end{aligned}$$

9. Let's say you have 2 blue blocks, and 2 green blocks, and 1 red block that are otherwise indistinguishable. How many different columns of height 5 can be built from these 5 blocks? For this problem, BRBGG and GGBRB will be considered to be two different columns (i.e. there is a bottom up ordering).

   Solution: There are $\binom{5}{2}$ *places to put the blue blocks initially. Then, there are three places remaining to put the red block, and the column is complete. So the total number of columns is* $3 * \binom{5}{2} = 30$

Ungraded Questions:

1. If there are any students in the class who you'd prefer to work with, or prefer not to work with, please list them here (every effort will be made to honor these requests):
   Prefer to work with:

   Prefer not to work with:

2. Put an "X" in any slot when you are normally *unavailable* to work on homeworks and projects i.e. the times when you have conflicts. (This info will be used to try to put you in a group with a compatible time schedule)

	Mon	Tues	Weds	Thurs	Fri	Sat	Sun
Morning							
Afternoon							
Evening							

3. Circle the algorithms and data structures that you could code in a language of your choice:

   Mergesort   Quicksort   Linked List   Binary Tree   Heap

4. Circle the languages that you are comfortable programming in:

   C   C++   Java