4. **P, NP, and NP-Hardness**

Consider the problem *4-Colorable* defined as follows: "Can a given graph G be colored using 4 colors?". Show that this problem is NP-Hard by giving a reduction from the problem *3-Colorable*. i.e. show that you could solve 3-Colorable in polynomial time if you had a polynomial time solution to 4-Colorable.

Solution: 4-Colorable is NP-Hard. The reduction is as follows. Assume we have some graph $G = (V, E)$ and we want to know if G is 3-Colorable. Let G' be G with one additional vertex, x, which is connected to every vertex in V. Then we return that G is 3-colorable iff G' is 4-colorable. Why does this work? Assume G is 3-colorable, then we can 4-color G' by taking the 3-coloring for all vertices in V and just coloring x the fourth color. Now assume that G' is 4-colorable - we know that x must be a different color than all vertices in V in the 4-coloring of G'. Hence only 3 colors are used to color the vertices in V and hence G must be 3-colorable.