CS 461, Lecture 19

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Today's Outline

- BFS and DFS Wrapup
- Midterm Review

Generic Traverse

```
Traverse(s){
  put (nil,s) in bag;
  while (the bag is not empty){
    take some edge (p,v) from the bag
    if (v is unmarked)
      mark v;
      parent(v) = p;
      for each edge (v,w) incident to v{
        put (v,w) into the bag;
      }
    }
  }
}
```


DFS and BFS

- If we implement the "bag" by using a stack, we have *Depth First Search*
- If we implement the "bag" by using a queue, we have *Breadth First Search*

Analysis

- Note that if we use adjacency lists for the graph, the overhead for the "for" loop is only a constant per edge (no matter how we implement the bag)
- If we implement the bag using either stacks or queues, each operation on the bag takes constant time
- Hence the overall runtime is $O(|V| + |E|) = O(|E|)$

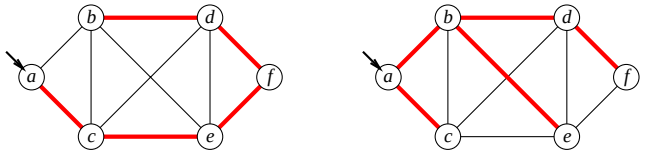
DFS vs BFS

- Note that DFS trees tend to be long and skinny while BFS trees are short and fat
- In addition, the BFS tree contains *shortest paths* from the start vertex s to every other vertex in its connected component. (here we define the length of a path to be the number of edges in the path)

Final Note

- Now assume the edges are weighted
- If we implement the "bag" using a *priority queue*, always extracting the minimum weight edge from the bag, then we have a version of Prim's algorithm
- Each extraction from the "bag" now takes $O(|E|)$ time so the total running time is $O(|V| + |E| \log |E|)$

Example



A depth-first spanning tree and a breadth-first spanning tree of one component of the example graph, with start vertex a.

Searching Disconnected Graphs

If the graph is disconnected, then Traverse only visits nodes in the connected component of the start vertex s. If we want to visit all vertices, we can use the following "wrapper" around Traverse

```
TraverseAll(){
  for all vertices v{
    if (v is unmarked){
      Traverse(v);
    }
  }
}
```


DFS and BFS

- Note that we can do DFS and BFS equally well on undirected and directed graphs
- If the graph is undirected, there are two types of edges in G: edges that are in the DFS or BFS tree and edges that are not in this tree
- If the graph is directed, there are several types of edges

DFS in Directed Graphs

- *Tree edges* are edges that are in the tree itself
- *Back edges* are those edges (u, v) connecting a vertex u to an ancestor v in the DFS tree
- *Forward edges* are nontree edges (u, v) that connect a vertex u to a descendant in a DFS tree
- *Cross edges* are all other edges. They go between two vertices where neither vertex is a descendant of the other

Acyclic graphs

- Useful Fact: A directed graph G is acyclic if and only if a DFS of G yeilds no back edges
- Challenge: Try to prove this fact.

Take Away

- BFS and DFS are two useful algorithms for exploring graphs
- Each of these algorithms is an instantiation of the Traverse algorithm. BFS uses a queue to hold the edges and DFS uses a stack
- Each of these algorithms constructs a spanning tree of all the nodes which are reachable from the start node s

Shortest Paths Problem

- Another interesting problem for graphs is that of finding shortest paths
- Assume we are given a weighted *directed* graph $G = (V, E)$ with two special vertices, a source s and a target t
- We want to find the shortest directed path from s to t
- In other words, we want to find the path p starting at s and ending at t minimizing the function

$$w(p) = \sum_{e \in p} w(e)$$

Example

- Imagine we want to find the fastest way to drive from Albuquerque,NM to Seattle,WA
- We might use a graph whose vertices are cities, edges are roads, weights are driving times, s is Albuquerque and t is Seattle
- The graph is directed since driving times along the same road might be different in different directions (e.g. because of construction, speed traps, etc)

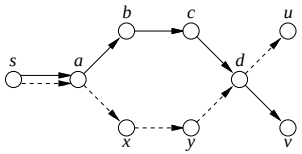
SSSP

- Every algorith known for solving this problem actually solves the following more general *single source shortest paths* or SSSP problem:
- Find the shortest path from the source vertex s to *every* other vertex in the graph
- This problem is usually solved by finding a *shortest path tree* rooted at s that contains all the desired shortest paths

Shortest Path Tree

- It's not hard to see that if the shortest paths are unique, then they form a tree
- To prove this, we need only observe that the sub-paths of shortest paths are themselves shortest paths
- If there are multiple shotest paths to the same vertex, we can always choose just one of them, so that the union of the paths is a tree
- If there are shortest paths to two vertices u and v which diverge, then meet, then diverge again, we can modify one of the paths so that the two paths diverge once only.

Example

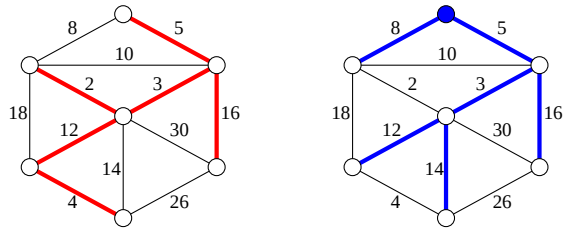


If $s \to a \to b \to c \to d \to v$ and $s \to a \to x \to y \to d \to u$ are both shortest paths,
then $s \to a \to b \to c \to d \to u$ is also a shortest path.

MST vs SPT

- Note that the minimum spanning tree and shortest path tree can be different
- For one thing there may be only one MST but there can be multiple shortest path trees (one for every source vertex)

Example



A minimum spanning tree (left) and a shortest path tree rooted at the topmost vertex (right).

Midterm Info

- Midterm will be Thursday, Nov. 13th at regular class time and place
- You can bring 2 pages of "cheat sheets" to use during the exam. You can also bring a calculator. Otherwise the exam is closed book and closed note.
- The web page contains *new* links to prior classes and their midterms. *Many of the questions on my midterm will be similar in flavor to these past midterms!*

Midterm Review Session

- I will have a review session Weds, Nov. 12th at 6:00pm in FEC 141 (the conference room on the first floor of FEC)
- Maxwell will also have a review session
- Please come with questions

Midterm

- 5 questions, about 20 points each
- Hard but fair
- There will be some time pressure, so make sure you can e.g. solve recurrences both quickly and correctly.
- I expect a class mean of between 60 :(and 70 :) points

New Topics

- Amortized Analysis: Aggregate Method, Accounting Method, Potential Method, Dynamic Array
- Disjoint Sets: Disjoint Set Operations, Representation as Forest, Union by Rank and Path Compression, Amortized Costs
- Graph Theory: Graph Representations, BFS, DFS
- MST: Definition, Kruskall's Algorithm, Prim's Algorithm, Safe Edge Theorem and Corollary
- Single-Source Shortest Paths: Definition and Algorithm

Problem 1

- Collection of true/false, multiple choice and short answer on topics we've covered
- Make sure you know resource bounds for all the algorithms we've covered so far
- Link on web page to MIT's algorithms class gives some good example problems

Problem 2 - Amortized Analysis

- I will give you a data structure and code for operations over that data structure. It will be a simple data structure, but not a stack, queue or bit counter
- You show the amortized cost per operation using both the accounting method and potential method
- Accounting method - you will give the charge for each operation and show how you can use these charges to pay for all operations
- Potential Method - I will give you a potential function and you will show that it's valid and will use it to calculate the amortized costs
- Like hw problems from Chapter 17 of text on Stacks and Bit Counters and Exercise 17.3-7

Problem 3 - Union Find

- A question about disjoint sets.
- Possibility 1: Simulate a disjoint set data structure as in Exercise 21.2-2
- Possibility 2: question about using the disjoint set data structure, similar to Exercise 21.1-3
- Possibility 3: ???

Problem 4 - Graph Theory

- Possibility 1: Computing the BFS and DFS trees of a graph
- Possibility 2: Questions about properties of BFS and DFS on certain types of graphs
- Possibility 3: Graph Theory proof, similar to in-class exercise

Problem 5 - MST

- Possibility 1: I give you an algorithm and ask you to either show that it always finds an MST or provide a counterexample where it doesn't, similar to Exercise 23.2-8
- Possibility 2: I give you a graph G and an edge set A and ask you to give me all the safe edges in G along with a cut for each edge which shows that it is safe.
- Possibility 3: A general question about MSTs, similar to Exercise 23.1-1, 23.1-3, 23.1-6
- Possibility 4: Simulation of Kruskal's and Prim's, questions about properties of these algorithms on certain types of graphs