CS 362, HW 13

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1. An extendable array is a data structure that stores a sequence of items and supports the following operations.

   - **AddToFront**(x) adds x to the beginning of the sequence.
   - **AddToEnd**(x) adds x to the end of the sequence.
   - **LookUp**(k) returns the kth item in the sequence, or NULL if the current length of the sequence is less than k.

   Describe a simple data structure that implements an extendable array. Your **AddToFront** and **AddToBack** algorithms should take O(1) amortized time, and your LOOKUP algorithm should take O(1) worst-case time. The data structure should use O(n) space, where n is the current length of the sequence.

2. The **Subgraph Isomorphism** problem takes as input two undirected graphs G_1 and G_2 and returns TRUE iff G_1 is isomorphic to a subgraph of G_2. Prove that the Subgraph Isomorphism problem is NP-Complete.

3. Show that the next problem is NP-Hard via a reduction from one of the following problems: 3-SAT, VERTEX-COVER, INDEPENDENT-SET, 3-COLORABLE, HAMILTONIAN-CYCLE, or CLIQUE.
   WEIGHTED-ITEM-COVER: You are given (1) a set S of weighted items; (2) a set T of subsets of items; and (3) a number W. You are asked: can you choose a subset S' of items in S with total weight of items in S' no more than W, such that every subset in T contains at least one item in S'? As an example, let $S = \{a, b, c, d\}$, $w(a) = w(b) = w(c) = 1$ and $w(d) = 2$; $T = \{\{a, b, d\}, \{c, d\}, \{b, d\}, \{a, c\}\}$; and $W = 3$. Then the answer is YES since we can set $S' = \{a, d\}$, which has total weight 3 and also ensures that every set in T contains at least one item from S'.

4. **MAX-INSIDE-EDGES.** For a given set of nodes S in a graph G, call an edge of G *inside* S if both endpoints of the edge are nodes in S. In the MAX-INSIDE-EDGES problem, you are given a graph $G = (V, E)$, and a number x. You must output the *maximum* number of inside edges in any set S such that $S \subseteq V$ and $|S| \leq x$.

   (a) Prove that MAX-INSIDE-EDGES is NP-Hard by a reduction from one of the following: 3-SAT, VERTEX-COVER, CLIQUE, SUBGRAPH-ISOMORPHISM, INDEPENDENT-SET, 3-COLORABLE, HAMILTONIAN-CYCLE, or TSP.

   (b) Consider the randomized algorithm that picks a subset S of size x, uniformly at random from all subsets of size x. Compute the expected number of inside edges for this algorithm. Let $n = |V|$ and $m = |E|$. Hint: Use indicator random variables and linearity of expectation.