

CS 561, HW 11

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1. Prove by induction that any tree with n nodes has exactly $n - 1$ edges. Don't forget to include the Base Case (BC), Inductive Hypothesis (IH) and Inductive Step (IS).
2. Prove Claim 1 from Single Source Shortest Paths Lecture (SSSP Lecture)
3. Prove Claim 2 from Single Source Shortest Paths Lecture (SSSP Lecture)
4. Problem 22-3 Arbitrage
5. Saia Trucking is a very safety conscious – and algorithm loving – trucking company. Given a pair of cities, they always try to find the *safest* route between that pair. They are thus faced with the following problem.

There is a directed graph $G = (V, E)$, where the vertices represent cities and the edges represent roads. Each edge has a value associated with it that gives the probability of safe transport on that edge i.e the probability that there will be no accident when driving across that edge. The probability of safe transport along any path in the graph is the *product* of the probabilities of safe transport on each edge in that path.

The goal is to find a path from a given node s to a given node t that maximizes the probability of safe transport. Describe an efficient algorithm to solve this problem.