CS 561, HW2

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1. You are doing a stress test for the new iPhone XC, and have a collection of identical prototypes. You have a ladder with n rungs. You want to determine the highest rung from which you can drop a prototype without it breaking. You want to do this with the smallest number of drops.

   (a) You have exactly 2 phones. Devise an algorithm that can determine the highest safe rung using $o(n)$ drops.

   (b) You have k phones, for any fixed k. Devise an algorithm that can determine the highest safe rung with the smallest number of drops. If $f(k, n)$ is the number of drops that your algorithm needs, what is $f(k, n)$ asymptotically? Hint: you should ensure that $f(k, n) = o(f(k - 1, n))$ for all k.

2. The game of Match is played with a special deck of 27 cards. Each card has three attributes: color, shape and number. The possible color values are {red, blue, green}, the possible shape values are {square, circle, heart}, and the possible number values are {1, 2, 3}. Each of the $3 * 3 * 3 = 27$ possible combinations is represented by a card in the deck. A match is a set of 3 cards with the property that for every one of the three attributes, either all the cards have the same value for that attribute or they all have different values for that attribute. For example, the following three cards are a match: (3, red, square), (2, blue, square), (1, green, square).

   (a) If we shuffle the deck and turn over three cards, what is the probability that they form a match? Hint: given the first two cards, what is the probability that the third forms a match?

   (b) If we shuffle the deck and turn over n cards where $n \leq 27$, what is the expected number of matches, where we count each match separately even if they overlap? Note: The cards in a match do not need to be adjacent! Is your expression correct for $n = 27$?

3. A *big* square with side length 1 is partitioned into x^2 *small* squares, each with side length $1/x$, for some positive x. Then n points are distributed independently and uniformly at random in the big square.

   (a) What is the expected number of pairs of points that are both in the same small square? For what value of x is this expected value greater than or equal to 1.

   (b) Use Markov's inequality and the expectation that you computed to bound the probability that there are at least 2 points in the same small square. For what values of x is your bound less than 1?